ADDITIONAL MATHEMATICS

Paper 4037/11
Paper 11

Key messages

Candidates should be reminded to read each question carefully. They should pay attention to the required form of an answer. An exact answer is written either as a surd and/or in terms of π or a fraction and a decimal answers should not be given in these cases. If it is stated in a question that a calculator must not be used, candidates must show all steps of their working especially when rationalising surds.

General comments

The general quality of work showed great variation. Some candidates produced excellent answers. Their answers were clearly set out with easy-to-follow steps leading to a solution. Other candidates left several questions without valid attempts.

Topics on which candidates were generally well prepared included indices, simultaneous equations, composite and inverse functions. Sometimes little notice seems to have been taken of the mark allocation for a question, and the amount of work implied by its value. This was particularly true of questions worth two marks, for example. **Question 6(a)** and **Question 9(d)**. Sometimes the amount of work presented on such occasions was out of all proportion to the marks available. The adoption of an appropriate strategy in solving a mathematical problem was not always followed, especially with questions on trigonometry.

Comments on specific questions

Question 1

As expected for the first question, the vast majority of candidates scored full marks and showed their ability to manipulate indices. Those that did not score full marks generally made errors when working with negatives.

Question 2

(a) This part was generally well answered, but a significant number of candidates did not distinguish between velocity and speed and so gave a negative final answer.

(b) Of those candidates who successfully found the second derivative, the majority were then unable to show that acceleration is never zero. A common mistake was finding that the acceleration when t equals 0 is non-zero.

Few candidates thought to state that $(1 + 3t)$ is always positive, so acceleration can never be 0.

Question 3

(a) Few correct ranges were seen. Candidates are expected to know the properties of the graphs of logarithmic functions. The expectation is that the correct notation is also used when describing the range of a function.

(b) Most candidates made a good start and found the correct composite function and they successfully gained the first method mark. About half went on to provide good responses showing a good understanding of the order in which the equation had to be solved. Most responses with the order correct then went on to find a correct exact solution. Some wrote the answer correct to 3 significant figures without writing the exact answer and they lost the accuracy mark. If candidates went wrong, it was usually with the first step of the solution where it was common to multiply the 5 by the 1 and/or the 7.

(c) Many completely correct solutions were seen, but some candidates did not check that they had completely answered the question because they did not reject the negative solution. Candidates should take note of the hint that there was only one solution. Most candidates were able to find the inverse function, but some did not know how to differentiate the logarithmic function correctly and this hindered their progress in this question. It was also essential to show a method for solving the quadratic equation and to give solutions to the correct level of accuracy.

Question 4

(a) The majority of candidate had a good attempt at this part. Many candidates correctly identified that $\mathrm{f}(x) = (x+2)(x-1)(x-3)$. Some recognised that there was a stretch parallel to the *y*-axis but the most common error was having only $\mathrm{f}(x) = 4(x+2)(x-1)(x-3)$ as an answer and did not state the other solution $\mathrm{f}(x) = -4(x+2)(x-1)(x-3)$ as well. Again, candidates should read the question carefully as, in this part, they were asked to find possible expressions which should have alerted them to the fact that there was more than one expression.

(b) (i) Many candidates were able to produce a sketch of the correct shape with no part below the *x*-axis. The question required that sketches show the coordinates of the points where the graphs meet the coordinate axes. Frequently, well drawn graphs were unable to gain full credit as one of these points was not clearly labelled or that modulus graphs did not show an intercept in the first quadrant.

(ii) Success was most commonly achieved by candidates who squared both sides and worked on the resulting quadratic. These candidates tended to solve the equation. Those writing down the two possible linear equations indicated by the modulus statement often made a sign error somewhere which meant they could only find one solution.

Question 5

(a) This question required two elements to be put together. Firstly, that the vector was in the opposite direction requiring a sign change or negative in front of the given vector. A significant proportion of candidates understood this and scored accordingly. The second element required finding the magnitude of the given vector and comparing it with the given one. Many achieved $\frac{1}{2}$ but some did the division the wrong way around and got 2.

(b) To score on this question candidates, needed to show that outside of the vector equation the **i** and **j** components could be isolated and compared. Most understood this and achieved at least one correct equation. A correct attempt at simultaneous equations then scored full marks. Most candidates who lost marks either did not form one of the equations correctly or showed poor algebraic manipulation.

Question 6

(a) Candidates found this to be the most challenging part of the question. A small number of candidates gave concise, neat and accurate solutions. Not many candidates realised that when $y = k$ they need to consider the maximum and the minimum of $\sin\left(x + \frac{\pi}{4}\right)$ and so spent unnecessary time on this part.

(b) (i) A good number of correct and efficient responses were seen. Most candidates seemed to favour an initial step of combining the fractions and then simplifying the denominator to $1 - \cos^2\theta$ before using $\tan\theta\cos\theta = \sin\theta$ to continue and complete the solution. Some candidates made errors when dealing with fractions when they substituted $\tan\theta = \frac{\sin\theta}{\cos\theta}$ before expanding the brackets. A few candidates made little or no progress, undertaking incorrect substitutions and/or invalid operations or incorrectly cancelling terms.

(ii) Again, a good number of correct and concise responses were seen. A few candidates earned 3 marks only as they omitted one of the possible solutions. This was often because the negative base angle confused some candidates. A few candidates were not awarded full marks due to rounding errors. Some candidates struggled with the initial manipulation required to find an equation using **part (i)** hinted with the word 'Hence'. Weaker candidates were unable to deal with the trigonometric function after solving the quadratic and solved for $\sin\theta$ not θ. Some candidates made no attempt to answer this part.

Question 7

(a) There were few completely correct solutions to this question. Most candidates identified the common difference of the arithmetic progression correctly and went on to form an equation using the sum of the arithmetic equation. Some candidates did not use the exact form and were not able to cancel terms or divide by log3 correctly.

(b) Most candidates identified the common ratio of the given geometric progression as $\frac{1}{2}$. This gained 2 marks. Candidates obtained the second method mark when they substituted correctly in the sum to infinity formula. Some candidates did not give their final answer in the form $p\ln 2$ and so lost the final accuracy mark. Any errors were usually due to the use of an incorrect common ratio.

Question 8

(a) Many correct solutions were seen, with candidates showing a good understanding of solving simultaneous equations. Any errors were usually due to arithmetic slips in the simplification of the resulting equations or the substitution in the quadratic formula. The question instructed the candidates not to use their calculator, so a detailed working out needed to be evident to award the marks. Some candidates did not find the *y*-coordinates and lost the last accuracy marks.

Although completing the square was a valid method for solving the quadratic equation in this question, not many candidates were successful using this method.

(b) This part of the question required the knowledge and use of two trigonometric identities: $\cot\theta = \frac{1}{\tan\theta}$ and $\mathrm{cosec}^2\theta = 1 + \cot^2\theta$. The most successful approach was to use $\cot\theta = \frac{1}{\tan\theta}$ first before rationalising.

Rationalisation was attempted by most candidates; however, some did not show sufficient detail to preclude the use of a calculator and some candidates showed no attempt at rationalisation at all. Candidates who showed insufficient detail in the rationalisation process were unable to gain the full marks.

Question 9

(a) A significant number of candidates did not read or take note of the wording of the question, so did not find *P* in terms of *r* only, but still had θ or even mixed degrees with radians in their expressions.

(b) Candidates who answered **part (a)** well, generally also answered **part** (**b)** well. The majority of those who answered correctly gave *r* in both exact and decimal form.

(c) Again, candidates who answered **parts (a)** and **(b)** well, were generally able to get the method mark here. Fewer candidates achieved the accuracy mark. Common errors were substituting their correct value of *r* into the second derivative but getting an incorrect value or an incorrectly rounded value.

(d) Surprisingly, fewer candidates found the correct value of θ. A significant proportion of candidates did not attempt this question.

Question 10

Most candidates correctly stated the derivative of the function – some omitting the 3 multiplier. Many found finding *p* challenging; they understood the process but did not match their found *p* with the limitations given in the question.

Having found their *p* and derivative, most attempted to find the gradient and applied the rule to find the normal gradient. They then went on with some success to find a value for the co-ordinates at *A* and *B*, although many resorted to decimals and so lost the final accuracy marks.

ADDITIONAL MATHEMATICS

Paper 4037/12
Paper 12

Key messages

Candidates are reminded that answers should be given to the required level of accuracy stated in the rubric on the front page of the examination paper, unless specified otherwise in the question. A careful check of each final solution should be made to ensure that it is complete and in the correct form. When being asked to show a given result, it is essential that every step of the solution is shown in full.

When sketching curves, it is usually required that the coordinates of the points where the curves or straight lines meet the coordinate axes are stated. Omission of these coordinates often result in the unnecessary loss of marks.

General comments

There did not appear to be any timing issues with the completion of the paper and candidates who required more room for their solutions made use of additional paper. It should be pointed out that it is best to use the blank page at the end of the examination paper first.

Most candidates were able to make a reasonable attempt at most of the topics with varying levels of success. Many candidates were well prepared and produced well set out solutions.

Comments on specific questions

Question 1

No method was required in this question. Candidates were expected to recognise the form of the trigonometric equation and relate the unknown constants to the amplitude, period and displacement of the curve shown by the graph. Many candidates were able to obtain at least 2 marks, with many gaining full marks. The most common error occurred when determining the value of *b*, which was associated with the period.

Question 2

It was important for candidates to realise that they needed to eliminate the trigonometric terms involving θ. Most candidates were able to gain a mark for attempting to use appropriate trigonometric properties, but often ended up with an equation involving x, y and $\sin^2\theta$, if they did not make use of the identity $\tan^2\theta + 1 = \sec^2\theta$. Other solutions leading to a correct equation in terms of *x* and *y* only were acceptable, but often more protracted.

Question 3

(a) Many completely correct solutions were seen, with candidates calculating a correct gradient and *y* intercept to form the equation $\lg(2y+1) = 4x^2 - 3$, and then re-arranging correctly to find *y* in terms of *x*. A few candidates incorrectly used ln rather than lg, but these candidates were still able to obtain method marks in subsequent parts.

(b) Even though a correct response in **part (a)** had been obtained, many candidates were unable to evaluate this correctly with a substitution of $x = \frac{\sqrt{3}}{2}$.

(c) Provided their answer to **part (a)** was in the correct form, most candidates were able to obtain at least one mark for the attempt to obtain the value of *x*. An exact answer was acceptable as well as an answer correct to three significant figures. Too many candidates gave answers of 0.96 or 0.961.

Question 4

(a) Most candidates were able to find the magnitude of the vector and gain a mark, but too many of them are still unaware of what a unit vector is and gave an incorrect response as a result.

(b) Most candidates realised the necessity to equate like vectors and solve the resulting simultaneous equations. Many correct solutions were seen although there were arithmetic errors made by some candidates.

Question 5

Most candidates were able to obtain correct expansions for $\left(1+\frac{x}{6}\right)^{12}$ and $(2-3x)^3$. There were occasional errors with signs and evaluation of some terms. Multiplication of the two obtained expansions sometimes caused problems with some candidates not being able to identify all the terms needed to calculate the values of *p* and *q*.

Candidates who used the expansions to three terms were usually more successful than candidates who chose to attempt to identify and evaluate the terms individually. Often the work of these candidates was difficult to follow if no clear method had been shown. It is suggested that in questions of this type, candidates attempt to expand the binomial expressions to at least three terms even if they are unsure of how to proceed, as this will usually mean that they will obtain some marks.

Many clear, correct and well set out solutions were seen.

Question 6

(a) Many correct solutions were seen, with candidates showing a good understanding of both the remainder theorem and the factor theorem. The question states that *a* and *b* are integers, so if a non-integer result is obtained, candidates are advised to check their working for errors.

(b) The wording of the question suggests that little or no working is required. Many candidates did not realise that a division by *x* implies a substitution of $x = 0$ into the given polynomial and wrote down their result from **part (a)**.

(c) Most candidates recognised the notation used meant that they were to use differentiation twice followed by a substitution of $x = 0$. As a result, many candidates obtained full marks for this part.

Question 7

(a) This question part was meant to be a simple algebraic exercise, the result of which was intended to be used in **part (b)**. Although there were many correct solutions, too many candidates did not know how to deal with the separate fractions correctly. Some candidates did not show enough detail to warrant the awarding of both marks.

(b) Too many candidates did not recognise that the integrand could be written in a different form using **part (a)**. These candidates were unable to gain any further marks. Of those that did write the integrand using the three fractions from **part (a)**, most were able to identify at least one correct logarithm term. Some candidates had difficulty with the integration of $\frac{1}{(x-1)^2}$, with some mistakenly thinking a logarithm was involved. There were of course sign errors and coefficient errors. There were errors made in the correct simplification of correct integrals to obtain the required final form. These usually resulted from sign errors and algebraic slips.

Question 8

(a) Many correct solutions were seen, with most candidates considering two separate cases as required.

(b) (i) Very few incorrect answers were seen.

(ii) Many correct solutions were seen with correct approaches evident in the working shown.

Question 9

Most candidates were able to make a reasonable start to this unstructured question, by attempting to find the gradient of the given function. Most realised that they needed to differentiate a quotient, and there was only the occasional sign error or incorrect differentiation of the logarithmic function. It was essential that candidates realised early on in their working that they needed to keep their working exact and not resort to using decimals as an answer in terms of ln 2 was required. Very many correct approaches were seen with candidates attempting to find the equation of the normal and the coordinates of the point *B*. Marks for the subsequent attempt to find the required gradient depended on candidates using correct logarithmic notation. Too many candidates wrote $(\ln 2)^2$ as $\ln 2^2$, $2\ln 2$ or $\ln 4$. Other errors included using the equation of the tangent rather than the normal and using the point *C* in the normal equation rather than the point *A*.

Question 10

(a) (i) It is essential that candidates read a question carefully. Too many candidates solved the given equation rather than show that the two given equations could be used to form it. There were many correct solutions, but some candidates obtained the result fortuitously by stating incorrectly that $x + y^2 = 10$ and then using $y^2 = \frac{x}{10}$ as it could be seen that this would give the required result. Some candidates obtained an equation in terms of just *y*.

(ii) All that was required was for candidates to solve the equation given in **part (i)** and discount those apparent solutions that were impossible. Many candidates wrote down that $x = 15$ and $x = -2$, without discounting the latter. When solving equations, candidates should always check to see if their solutions are feasible. The same applied when calculating the *y* values, although some candidates then realised that $x = -2$ gave an invalid *y* value and then discounted it. Some candidates also gave two possible *y* values erroneously.

(b) There were many correct solutions to this part, with candidates recognising that a change to a common logarithm base was needed. Most candidates did this, although some did not form a quadratic equation after the change of base. Some candidates were unable to obtain a final accuracy mark when they used base *x* logarithms, not being able to deal correctly with $x^{\frac{1}{3}} = a$.

Question 11

(a) Most candidates attempted to differentiate the given displacement equation in order to find the velocity. Some chose to expand out the given expression and then differentiate, while others chose to differentiate the given expression as a product. Both methods were equally common and equally successful. Many fully correct solutions were seen.

(b) This question part was meant to test the candidates' skills of curve sketching. Many did not associate the given displacement expression as a cubic function that most would have had more success at sketching had it been in terms of *x* and *y*. Of those that did recognise the cubic form and attempted a sketch of a cubic graph, most did not relate their answers to **part (a)** to the sketch, omitting the maximum point at the point where $x = \frac{1}{3}$. It was required that the coordinates of the points where the graph meets the coordinate axes were stated. Too many candidates did not do this in this part and subsequent parts.

(c) The sketching of the quadratic function representing the velocity was required and candidates appeared to be more successful at this proving they had a correct expression for the velocity. Again, many candidates omitted stating the points where the curve met the coordinate axes, most commonly $x = \frac{1}{3}$.

(d) (i) Of those candidates who had a correct expression for the velocity, most were able to obtain a correct expression for the acceleration. This was intended to help candidates with the next part.

(ii) Many candidates did realise that a straight line graph was needed, especially if they had the correct form for the acceleration in **part (d)**. Again, many candidates omitted stating the points where the curve met the coordinate axes, most commonly $x = \frac{8}{3}$.

ADDITIONAL MATHEMATICS

Paper 4037/21
Paper 21

Key messages

- In order to do well in this paper, candidates need to show full and clear methods in order that marks can be awarded.
- In questions where the final answer is required in a given form, candidates should be aware that full credit cannot be awarded for otherwise fully correct work unless this is done.
- Candidates should be aware of the general guidance on the cover sheet and ensure that all answers are given to the accuracy indicated.
- In questions where the answer is given, candidates are required to show that it is correct and fully explained solutions with all method steps shown are needed. Repeating information given in the question cannot be credited. In such questions candidates are encouraged to use consistent notation such as using the same variable throughout a solution and should avoid replacing a function of a variable with the variable itself.
- In questions that state that a calculator should not be used, omitting method steps often results in full credit not being given for a solution. Combining the steps directly into a simplified form, such as that produced by a calculator, cannot be credited. In questions that require a solution of several steps, clearly structured and logical solutions are more likely to gain credit.
- Candidates should be encouraged to write down any general formula they are using as this reduces errors and is likely to improve the accuracy of their solutions. In particular, method marks cannot be given for solving an incorrect equation when the solutions are taken directly from a calculator, without showing any working.

General comments

Some candidates produced high quality work displaying wide-ranging mathematical skills, with well-presented, clearly organised answers. This meant that solutions were generally clear to follow. Other candidates produced solutions with a lot of unlinked working, often resulting in little or no credit being given. More credit was likely to be given when a clear sequence of steps was evident.

Several questions were unstructured and candidates needed to plan their method carefully. There were many good solutions to these questions. Some candidates wrote down a few relevant steps but did not link them together.

Questions which required the knowledge of standard methods were done well. Candidates had the opportunity to demonstrate their ability with these methods in many questions. Most candidates showed some knowledge and application of technique. The majority of candidates attempted most questions, demonstrating a full range of abilities.

Some candidates needed to take more care when reading questions and keep their working relevant in order to improve their solutions. Candidates should also read the question carefully to ensure that, when a question requests the answer in a particular form, they give the answer in that form. This is particularly the case when the question states that an exact answer is required. Candidates should ensure also that each part of a question is answered and the answer clearly identified. When a candidate uses the blank page or an additional booklet they should make it clear which question their work relates to. It is not possible in most cases to connect work otherwise to a specific question, which can lead to the loss of potential credit. When a question demands that a specific method is used, candidates must realise that little or no credit will be given for the use of a different method. This is also the case when a part of a question is linked to the previous part by 'Hence…' They should also be aware of the need to use the appropriate form of angle measure within a question. When a question indicates that a calculator should not be used, candidates must realise that clear and complete method steps should be shown and that the sight of values clearly found from a calculator will result in the loss of marks. When a candidate is given or has derived an expression and subsequently needs to make a substitution into this expression the substitution should be clearly shown. This is also the case when applying limits to an integral.

Candidates should take care with the accuracy of their answers. Centres are advised to remind candidates of the rubric printed on the front page of the examination paper, which clearly states the requirements for this paper. Candidates need to ensure that their working values are of a greater accuracy than is required in their final answer.

Candidates should be advised that any work they wish to delete should be crossed through with a single line so that it can still be read. There are occasions when such work may be marked and it can only be marked when it is readable. Where a candidate feels they have made an error and is unable to offer any alternative work they are advised not to cross out their work as the work they have done may be creditworthy. Rubbing work out then writing over it can sometimes result in the work being difficult to read.

Comments on specific questions

Question 1

(a) This part was well attempted by most candidates working mainly with logarithms to base 5. Some candidates rewrote the equation by changing the left hand side to $\frac{5^w}{5}$ first, but finding an expression equal to $w - 1$ was the most popular method. It was important to note that the question required an answer correct to 2 decimal places and this was not always given. This was particularly an issue for those who arrived at a value without working, presumably using their calculator and therefore having no method which could be implied by their answer.

(b) Those candidates who rewrote the equation with another variable replacing $x^{\frac{1}{3}}$ were usually successful, forming and solving a three-term quadratic. The final step of cubing the roots of these equations was frequently completed correctly, although some found the cube roots and some did not carry out the final step. Those candidates who chose to use x as the replacing variable often found themselves unable to complete the question and it is inadvisable to do this unless clearly stating the substitution first. A common error was to initially cube each term separately and some candidates assumed that, as the equation involved indices, taking logs was a starting point.

Question 2

(a) This proved a good test of the application of the laws of logarithms. There were some very efficient solutions using just two of these laws, although some candidates preferred an extra step combining three terms one at a time. Many candidates made an incorrect and unrecoverable first step of rewriting $\log(x+6)$ as $\log x + \log 6$ or had a final step of $\frac{\log x^2}{\log 3x + 18}$. Where there are several steps which can be carried out in a variety of orders it is imperative that no errors are introduced as accuracy of application can only be awarded if the statement involved is fully correct.

(b) This was an example of a question where 'Hence' indicated that candidates should use their previous result and not start again. This allowed candidates to illustrate their knowledge that $\log 1 = 0$. Those who rearranged the original equation and then removed the logs could therefore not gain full credit. Of those who arrived at the correct quadratic and solved it correctly, a significant number retained the negative solution. This was not appropriate as the first term in the original equation would not have been possible. There was a strong correlation between successful outcomes in the two parts of the question.

Question 3

Nearly all candidates correctly found the gradient of the line with only a few having the *x* and *y* coordinates reversed. Using one of the points to find the equation of a line usually followed this correctly. A few found and used the mid-point which, whilst correct, was unnecessary. Some candidates stopped at this stage or wrote the equation in terms of $\sqrt[3]{y}$ and x^2 and then stopped. Candidates who took the final step and cubed both sides were not penalised on this occasion for the incorrect expansion of this term. Common errors were using values for $\sqrt[3]{1}$ and 9^2, or similar, in the line equation or cubing each term as the final step.

Question 4

Most candidates made a very good attempt at this question leading to many fully correct solutions. Some made minor numerical errors in producing the initial equations or solving them simultaneously, so gained few marks. A very common mistake was to equate the second expression to 12 or to 0 instead of –12. Those who made little meaningful progress generally either began by having f(2) = 0 as a starting point or tried using long division at the very start rather than the factor and remainder theorems. Long division was often used accurately in finding the remainder having correctly identified *m* and *n*.

Question 5

(a) (i) The binomial expansion is given on the formulae sheet so most candidates, though not all, were able to give a correct starting expansion. The key part of the question was to simplify each term and very few attempted to do this or did it successfully. The first few terms were the ones most likely to be simplified. It is expected that, when asked to simplify, candidates do not have terms such as 1^n or do not leave $\frac{16}{2}$ as part of the third term. Some candidates chose a value for *n* rather than using a general expansion.

(ii) Many candidates were able to identify the appropriate two terms in x^2 and combine them to equal 6032. At this stage it was required to simplify these terms, using the previous part as an aid, to form a quadratic equation. This needed to be solved to find the positive root. As the general rubric suggests, all steps should be shown, so stating the correct solution without at least stating a correct quadratic was not accepted as a valid method. It is also worth candidates noting that solving an incorrect quadratic without an attempt at factorisation or a correct and clear substitution into the quadratic formula will not be considered as a valid method. On this occasion a correct value of 29 did not always warrant full marks. It was rare to see the negative solution not discarded and many correct solutions did not even mention it.

(b) Many candidates were able to identify the correct term sometimes using an algebraic method to find the correct value for *r* in the general term. Some did not expand this but those who did usually found the value $\frac{45}{4}$. A final answer of $-\frac{45}{4}$ was not uncommon due to not squaring the negative term correctly. The term needed had to be clearly identified to gain any credit. Some candidates were evidently unclear of which term they were trying to find and wrote down a multi-term expansion only, although some did then indicate which term was required.

Question 6

Questions involving permutations and combinations such as this one always provide a multitude of potentially correct methods. Candidates who did not arrive at correct values for the final answer for each part were only able to gain partial credit if they made their working clear, ideally with a diagram or description in words, e.g. ending in zero, starting with an odd number.

(a) (i) The most common method seen was to find the number of digits which could occupy each position and multiply these i.e. $6 \times 6 \times 5 \times 4 \times 3$. Some found the total number of permutations and subtracted those beginning with zero which was also carried out successfully on the whole. Others used combinations which was inappropriate here or presented methods, sometimes at length, which showed little understanding of the question. There were also a large number of candidates who made no attempt to answer.

(ii) Whilst there were far fewer fully correct solutions than for the previous part, there was more opportunity to identify some of the permutations which could be combined to find the answer. Perhaps the most straightforward method was to find how many numbers ended in 0, 2, 4 or 6 and adding these, but this was rarely seen in full. Similarly, finding the numbers which began with an even digit and adding to those which started with an odd digit worked well but was equally rare. Too often candidates left this blank or multiplied a series of digits often starting with 7 or containing 2 which were very unlikely to be appropriate. Again combinations were inappropriate.

(b) Candidates were generally more successful in this part than in **part (a)**. There were two common methods which led to the correct answer and these were rarely miscalculated having been set out clearly. One method was to calculate the total number of teams possible and then deduct those with no men. The other was to calculate each possible combination of men and women and to then add these.

(c) (i) As the factorial version for $\binom{n}{r}$ was given on the formulae sheet it was usual for most candidates to start this part correctly. Knowing how to rewrite the terms without factorials proved more challenging but was achieved by a large number of candidates. Having achieved this stage most were able to complete successfully usually by combining as a single fraction but occasionally by first taking out common factors. Carelessness with bracketing and other slips were penalised here as the final simplification was given. This was an example of a question where a result is given. It is not good practice to use the given result as part of the solution and so multiplying across the equals sign was instantly penalised. Instead candidates should always start from the left-hand side and conclude with an expression equal to that which they have been asked to show on the right-hand side. Writing the right-hand side throughout raises the temptation to include it in the solution so this should be avoided. Using a value for n and showing that this works is insufficient as the question required a general solution.

(ii) Those who followed the guidance and used the previous result found this relatively straightforward, only occasionally giving –5 or 0 as erroneous final additional solutions. Some used an earlier part of their solution to the previous part rather than the given result and this approach met with mixed results. Some candidates apparently ‘spotted’ the answer and as a consequence did not show use of **part (i)** and could gain no credit.

Question 7

There were effectively two parts to this question with a degree of problem solving required.

The first step was to differentiate y and the structure of this, usually using the quotient rule but sometimes the product rule, was well presented. Errors, if they appeared, were most often found in differentiating $\sin 3x$, with $\cos 3x$ most common along with 3, $\cos 3$ and $3\sin 3x$. There was also some misapplication of the chain rule, usually not reducing the power from 4 to 3.

The second step was less well done often due to not explicitly showing the substitution of 1.9 for x throughout their derivative. This was condoned if a fully correct derivative led to a correct value to 3 or more decimal places and multiplied by h. There were very few who managed this and the majority found the complex calculation hard to complete accurately. Candidates who insisted on unnecessarily simplifying their derivative before substitution rarely left themselves with a correct expression to use.

Question 8

This question, especially the first and last parts, was found very challenging.

There were many parts left blank; in some cases all parts, and it was rare for a candidate to score full marks on all parts.

(a) Attempts, when offered, frequently tried to work from the position vector rather than the velocity vector. Those who did attempt to find the components of the velocity vector using trigonometry rarely justified the negative sign for the **j** component, often just adding the sign as an afterthought. Some reference to moving south or similar was necessary.

(b) There was more success here. Candidates should be careful in both the presentation of their answer and avoid simplification, which was not required and often resulted in errors in **part (d)**. Poor bracketing often led to 16 rather than 16**j** as a final answer and there were also sign errors. A reasonably common error was to multiply the velocity vector by 12 rather than 3.

(c) There were some good attempts here too with similar errors in bracketing and signs to **part (b)**. The most common error was to omit *t* or to misplace it. Candidates should be careful also that when using column vectors these should not include **i** and **j**.

(d) Fully complete solutions were rare for this part. There were a few who followed through from the previous parts to find the vector from *A* to *B* or the reverse but stopped without finding the magnitude. Errors in the value of *t* (often using 3 rather than 1), if it was included, were usually the cause of incorrect solutions. Some candidates tried to find the magnitude with *t* still included.

Question 9

(a) In questions like this, where the required answer is given, candidates should understand that trying to contrive the components of the formula or working backwards from the answer to find a potential starting point is not acceptable. The best policy is to try to derive, in this case *V*, and use the given answer as something to check against. It was required to first find a connection between *x* and *h* and then substitute this into the formula for the area of the triangle forming the cross-section and multiply this by 5. There were numerous ways of finding the initial connection usually using trigonometry, Pythagoras or two versions of the area formula.

(b) This question required a clear use of the chain rule using the variables given in the question and using *t* for time as is the convention. Use of a general statement of the chain rule was only credited when it was made relevant to the question. No credit was given for treating this as a small increment problem other than for correctly finding $\frac{dV}{dh}$. Several candidates treated *V* as a quotient when differentiating rather than a simple term of the form kh^2. Whilst many of these were eventually correct this method provided many possibilities for slips. The other key error was in the final step with numerous candidates multiplying $\frac{dh}{dt}$ by $\frac{dh}{dV}$. Candidates who simply substituted 0.1 into *V* made no progress of note.

Question 10

(a) Answers to this part were very mixed with some very good concise solutions while other candidates did not realise that the first term was a product and made no meaningful progress. Of those who attempted to work with the product, common errors were to leave 2*x* undifferentiated or to leave 1 – 2 unsimplified.

(b) This question required candidates to work through two stages of integration to arrive at *y* =…. Before the first integration could be attempted the function for the second derivative needed to be expanded and simplified to three terms. The first integration was then relatively straightforward. If the expansion was not attempted or incorrect then candidates were unlikely to have appropriate terms and the rest of the question became inaccessible. While candidates who had the appropriate form often made reasonable progress, completely correct solutions were extremely rare. Errors in finding constants of integration were not uncommon and some candidates tried to introduce their answer to **part (a)** too early. If the answer to **part (a)** was not of the correct form then it was likely that the second integration, if attempted, could not succeed.

Question 11

Many candidates appreciated that to find the area under a curve, the equation of the curve required integrating. This allowed many to gain credit for integrals of the correct form and often for correct functions. Substituting in the limits was less successful, frequently as this was done on a calculator without clear indication of how the limits were used. This is an example of a case where, as indicated in the rubric on the cover sheet, all necessary working must be shown. This also has the effect that the final answer is rarely in an exact form as was required. The *x*-coordinate of *B* also needed to be found and was sometimes the only correct part of the solution. To be appropriate as a limit this needed to be in radians and 18 was not credited.

The plan needed to find the shaded area was fairly straightforward as this could be found by finding the difference in the areas under the two curves. There were lengthier alternatives. Common errors in integration suggested that differentiation may have taken place. Another error which often cost the final mark was either to give the answer as a decimal, without first expressing it exactly, or to calculate $2\frac{11}{5}$ as $\frac{9}{5}$.

ADDITIONAL MATHEMATICS

Paper 4037/22
Paper 22

Key messages

To be successful in this paper, candidates should read each question carefully and identify any key words or phrases, making sure they answer each question fully. When a question indicates that a calculator is not to be used, candidates need to show sufficient and convincing method to be credited. Also, when values are incorrect and the method from which they arise is not seen, marks cannot be given. Method should always be shown and sufficient method needs to be shown so that marks can be awarded. For example, candidates should not rely on calculators to solve quadratic equations or to work out the values of derivatives or integrals for particular values. Calculators are an excellent checking tool and candidates should be encouraged to use them in this way. Candidates should be able to write well-formed expressions using brackets where necessary and conventional ordering of terms in products involving functions with arguments, such as trigonometric functions, to avoid ambiguity of meaning.

General comments

Many candidates seemed to be fairly well-prepared for this examination. Candidates were often able to demonstrate knowledge and understanding of mathematical techniques and were able to apply these to solve problems. Some questions required multiple skills. This was evident in **Questions 2, 8** and **12**, for example, in this examination.

When a question uses the key phrase 'Show that', showing clear and complete method for every step is essential. Candidates are expected to state **all** method steps to justify that a result is correct or an expression is of a particular form. The marks are awarded for the method. This was required in **Question 9(a)** for example. Also in **Question 10(a)**, some candidates did not understand the phrase 'Show that *r* satisfies the equation …'. This is commonly used in mathematics and it would have been helpful if candidates had understood that this meant they needed to use the information given to derive the given equation in *r*.

Some candidates gave neat and clear responses, making their work easy to follow. These candidates were much less likely to make an error such as miscopying one of their own figures. Other candidates would have improved, perhaps, if they had presented their work in a more logical and less haphazard way. Presentation was often poor in **Questions 6**, **9(a)** and **10(a)**, for example and work was often difficult to follow, especially if candidates had overwritten pencil with pen.

Candidates seemed to have sufficient time to attempt all questions within their capability.

Comments on specific questions

Question 1

Candidates were instructed not to use a calculator in this question. It was therefore important that sufficient method was shown so that working without a calculator was successfully demonstrated. A good proportion of fully correct responses were seen. Most candidates indicated a correct first step and stated clearly their intention to rationalise the denominator. Some candidates needed to take a little more care with presentation here as statements which were not correctly formed, such as $\dfrac{(6+\sqrt{6})^{\times 3-\sqrt{6}}}{(3+\sqrt{6})_{\times 3-\sqrt{6}}}$, were not credited until candidates had shown the correct expansion of terms in the next step. It is expected that candidates will write $\frac{(6+\sqrt{6})(3-\sqrt{6})}{(3+\sqrt{6})(3-\sqrt{6})}$ or similar. To show evidence of non-calculator use, candidates should have stated at least 3 terms in the expansion of the numerator. Most candidates did this and only a few slips in sign or arithmetic were seen. A few candidates made an error in the final step by cancelling incorrectly, for example $\frac{\overset{4}{\cancel{12}}-3\sqrt{6}}{\cancel{3}}=4-3\sqrt{6}$ was occasionally seen. Candidates who stated nothing more than the answer were not credited.

Question 2

This question was multi-technique involving coordinate geometry and absolute value functions. Candidates needed to find equations of lines and solve them to find points of intersection, taking into account the modulus of f(x). Many candidates solved this problem successfully and gave answers that were fully correct. A few candidates found the correct critical values but were unable to determine the correct inequality even though the diagram was there to assist with this. Most candidates formed a pair of linear equations or inequalities and solved them. Some candidates were working with correct expressions and the method attempted was correct, but they made sign or arithmetic errors when finding their critical values. Candidates who squared each expression and equated were commonly successful in finding the correct critical values. Other candidates who attempted this method only squared one expression before equating and rearranging. Sometimes presentation of solutions was poor. Candidates whose work was poorly presented made more errors than those whose work was neat and logical. Some candidates would have done better to have rewritten their solutions on extra paper as they then might have spotted slips they had made. A few candidates made sign errors with the expressions for f(x) or g(x) or both of these. In weaker responses, common errors seen were $f(x) = 2x + 5$ and $g(x) = -x - 1$ for the initial functions.

Question 3

A good number of fully correct solutions were seen to this question and most candidates made an attempt to answer. A few candidates did not interpret the condition ‘has real roots’ correctly and stated $k < -25$ or $k > -1$. Some other candidates stated incorrectly formed answers such as $-1 \leqslant k \leqslant -25$. Other candidates chose the incorrect set of values stating $-25 \leqslant k \leqslant -1$. A few candidates made sign or arithmetic errors at some stage but most were able to take an initial correct step in the method, forming a correct expression for the discriminant. A small number of candidates squared $k + 5$ correctly but were unable to collect the terms in k accurately, with $26k$ sometimes being written as $36k$. Some candidates confused k and x at some stage in their solution. In weaker responses, candidates generally either applied the quadratic formula to the equation without identifying the discriminant as being the expression of interest, or attempted trials with various values of k.

Question 4

Most candidates understood that finding a derivative was a key step in the method of solution. A few candidates attempted to rearrange to make x the subject and then find $\frac{dx}{dy}$. Success with this method was varied, although some fully correct answers were seen using this approach. Most candidates attempted to find $\frac{dy}{dx}$. Many were able to find a correct derivative, although some candidates integrated at least one of the terms or simply rearranged the equation and labelled the rearrangement $\frac{dy}{dx}$. The next step was to find the appropriate value of x by substituting $y = 4$ into the given equation, rearranging and solving the quadratic equation that resulted. Some candidates formed a cubic equation, which was permitted but these candidates clearly needed to reject the solution $x = 0$ for this method to be accepted, as $x = 0$ was invalid for this equation regardless of the condition on x stated. Candidates who worked with both $x = 1$ and $x = -\frac{1}{3}$ were not able to earn full credit unless the work with the negative value was rejected. Some candidates may have done better if they had written $\frac{0.01}{\delta x} = \left.\frac{dy}{dx}\right|_{x=1}$ or similar before attempting to find δx, as occasional slips in method were made at this stage. Candidates should know that they need to show the substitution of their value into their derivative to be sure of being credited should either the value or derivative be incorrect. A correct method is generally not deduced from an incorrect value. A small number of candidates used $\delta y = 4.01$ instead of 0.01. A few candidates attempted to manipulate $\frac{\mathrm{d}y}{\mathrm{d}x}$ into $\frac{\mathrm{d}x}{\mathrm{d}y}$ in algebraic form, but this was not always successful as some simply took the reciprocal of each term. In weaker responses candidates substituted $x = 4$ into $\frac{\mathrm{d}y}{\mathrm{d}x}$. These candidates may have benefitted if they had reread the question or read the question more carefully in the initial stages, as it was y that was equal to 4 and not x. In other weak responses, candidates found the value of x when $y = 4$ and when $y = 4.01$ and then calculated the difference in these values of x. These candidates did not use differentiation and were unable to be awarded many marks.

Question 5

(a) The majority of candidates rewrote the equation so that all terms were expressed as powers of 5 and then combined terms. A good proportion of candidates earned all 3 marks in this way for correct and accurate work. A small number of candidates used logarithms or other valid methods, as detailed in the mark scheme. A few candidates found $\sqrt[3]{-3}$ and then rejected it, indicating there was no solution. This misconception was not condoned. A few candidates took a correct first step but then combined powers and equated incorrectly. For example, brackets were expanded as $2x^3 - 1$ or 5 was written as 5^0. A few other candidates made a sign or arithmetic slip in the first step but were able to combine powers and equate correctly. In weaker responses, candidates either stated that $\frac{2x^3 - 2}{3x^3} = 1$ or 5 or made fundamental process errors such as $625^{\frac{x^3-1}{2}} = 625^{x^3}$, from which there was no recovery.

(b) For full credit, the exponential graph drawn needed to have the correct curvature, tend to $y = 3$ and pass through (0, 7), which needed to be indicated. A small proportion of candidates earned both marks. A reasonable number of candidates earned one of the two marks available but not both. Whilst it was not necessary to mark $y = 3$, candidates who did this almost always earned both marks. Candidates who plotted points rather than sketching, as requested, often drew graphs that were insufficiently smooth to be awarded both marks. Some candidates only drew the part of the graph in either the first or the second quadrant, this was not sufficient. A few candidates had a y-intercept of 3 and others had no intercept marked with a graph that tended to the x-axis. These did not score. In the weakest responses, candidates drew straight lines or graphs that were of completely incorrect curvature or that passed through the third or fourth quadrants.

Question 6

Candidates found this question assessing the composing and resolving of vectors to be a challenge. In the first part of the question, candidates needed to form a resultant vector and many struggled with this. In the second part of the question, it was not necessary to form a resultant vector, as a simple sketch and use of the cosine and sine rules was sufficient to produce a fully correct solution. Not many candidates observed this and many, again, attempted to form the resultant vector, a method that was very prone to error. Good diagrams were essential but these were not always seen.

(a) It was very clear at the start of this question that **j** was the unit vector due north. Many candidates did not take note of this and used **i** as the unit vector due north. This resulted in the components being reversed and this was not condoned. A reasonable number of fully correct solutions were seen. A few solutions were spoiled by rounding or premature rounding errors. If sufficiently accurate values had already been stated, these errors were ignored, but some candidates had not stated any more-accurate values and could not be credited or fully credited. These candidates may have improved if they had taken note of the instructions on the front of the examination paper and stated final values to 3 significant figures, understanding that any working values needed to have greater accuracy. A few candidates who used exact values in surd form omitted to use correct brackets. Again, this error was ignored if it had been clearly stated that $x = \sqrt{6} - \sqrt{2}$ and $y = \sqrt{6} + \sqrt{2}$, otherwise it was penalised.

(b) As has been stated, the simplest method of solution in this part was to make a simple sketch and use the cosine rule and sine rule or cosine rule again, to find the magnitude and hence the angle and bearing needed, respectively. Those who attempted this approach were more successful, although not all had the correct angle 110°, with 70° being a fairly common error. Very many candidates did not attempt this approach and chose to form a resultant vector. Often these candidates made sign errors in the components which resulted in magnitudes and bearings which seemed to be correct but were from an incorrect vector and so earned only method marks. A few candidates attempted to find the resultant vector and made no attempt to continue their solution beyond that point. These candidates may have improved if they had reread the question. In the weakest responses, candidates applied Pythagoras or trigonometry to a right-angled triangle with sides 2 and 6, which was not credited. A large proportion of candidates made no attempt to answer.

Question 7

A good number of candidates earned 3 or 4 marks for their solution to this question. Those who lost the final mark often did so as they had at least one ambiguous term in the derivative, such as $\sec^2 xe^{4x}$ which candidates should understand is not acceptable and should be written in the conventional way $e^{4x}\sec^2 x$. A few candidates did not observe the need to use the product rule in the numerator and gave the derivative of the numerator as $4e^{4x}\sec^2 x$, for example. A few other candidates attempted the product rule but subtracted instead of adding the terms. Some candidates applied the product rule correctly to the numerator but were unable to correctly recall the correct structure of the quotient rule, with some dividing by $\left(\frac{1}{x}\right)^2$.

Some candidates differentiated the numerator, divided by ln*x* and then applied the quotient rule to this new quotient, differentiating the numerator again. In the weakest responses, candidates often did little more of value than differentiate e^{4x} and often not even that.

Question 8

This question combined functions with stationary points and functions with solving trigonometric equations. In each case, values that were outside the domain stated needed to be discarded. It was necessary for angles to be in radians and this was stated in the question to assist candidates, though not all observed this.

(a) A small number of fully correct solutions were seen to this part of the question. It should have been clear to candidates from the wording of the question that there was in fact only one stationary point. It was necessary to apply the chain rule to the first term in order to differentiate it successfully. Some candidates did find a correct derivative, or a derivative of similar structure, and factored out the sin*x*.

Some of these candidates then ignored the factor sin*x* and only worked with 6cos*x* + 2 = 0 or the equivalent. Other candidates divided through by sin*x* instead of factorising. A special case mark was available for candidates who did this and then discarded the pair of values they found as being outside the domain of the function. The most common derivative stated was $3\cos^2 x + 2\sin x$. Candidates who did this were unable to progress in the solution beyond this point. In the weakest responses, candidates rewrote the expression for f using a substitution such as *u* = cos*x* and then completed the square on the quadratic form that resulted. This was not a valid solution for a function of this type. In other weak solutions, candidates commonly equated f(*x*) to 0 and solved.

(b) Candidates were more successful in this part with a reasonable number earning at least 4 of the 5 marks available. To earn full credit, the values which were outside the domain of the function needed to be discarded. An initial step of using $\cos^2 x = 1 - \sin^2 x$ to rewrite the function in terms of cos*x* only was valid in this part of the question and was the approach used by most candidates. Many candidates correctly manipulated the equation they had formed and derived the correct quadratic equation in cos*x*. Often candidates stated and used a substitution to make the factorising easier. This was a good approach. Most candidates worked in radians and found at least one valid solution. Some candidates gave angles in degrees and these were not credited for accuracy. In weaker responses candidates solved 1 – 3cos*x* = 0, completely ignoring f(*x*), or candidates made no real attempt to answer.

Question 9

(a) This was an example of a question where candidates had to show that a particular result was true. It was necessary for candidates to provide all method steps. The angle used should have been ϕ, although other angles were accepted as long as they were consistently used. It was not acceptable, for example, to use a mixture of angles and then in the last step define one of them as being whatever was needed to make their expression the same as the given expression. This was not at all convincing. It was also necessary for candidates to use a rather than the general r, as using the general case did not show engagement with the question. Use of r was only acceptable if it was clear that r had been linked correctly to a, either by adding it to the diagram or by a clear statement that $a = r$ as was seen on occasion. Some fully correct and neat solutions were seen. The most common approach was to form the area of the sector, then the shaded area, using the area of the kite less the area of the sector, and equate them. Various other equally successful methods were also seen and many of these are detailed in the mark scheme. A few candidates made dimension errors and had one or more parts of their equation that were one dimensional. Some candidates made no real progress beyond stating a correct sector area. In weaker responses, candidates either made no real attempt to answer or they treated 2ϕ radians as 2 radians and proceeded to find values for lengths and other angles. This was not accepted.

(b) Some fully correct answers were seen to this part. Candidates needed to form an expression for the perimeter of each of the sector and the shaded area and either double the sector perimeter or find half of the perimeter of the shaded area and equate. A good number of candidates did this and a good number of these manipulated the resulting equation correctly to find the correct expression for $\tan\phi$. A few candidates made slips with a or ϕ when factoring out 2 or $2a$, for example, and so lost accuracy. A few other candidates found an expression for ϕ in terms of $\tan\phi$. In weak responses, candidates tended to start with the arc length only, treating this as the perimeter. In other weak responses, candidates incorrectly used the result from **part (a)** or added extra terms such as $2a$ to the perimeter of the shaded area.

Question 10

Presentation was often poor in this question and work was often difficult to follow, especially if candidates had overwritten pencil with pen.

(a) (i) This question required candidates to form a quadratic equation, using knowledge of geometric progressions and the information given. Most candidates made some attempt to answer. Some candidates offered neat, clear and fully correct solutions. These candidates commonly started with $ar = 8$ and $ar^2 + ar^3 = 160$ and then made a substitution to eliminate a. Many of these candidates were able to then manipulate the result correctly into the required form. Candidates who started with $S_4 - S_2 = 160$ very rarely offered fully correct solutions, as they did not know how to proceed. Progress could be made from this starting point, particularly if candidates used the difference of two squares to simplify, but this was not commonly seen. These candidates often derived a cubic equation. In some weak responses, candidates confused 'terms' with 'sums' and $S_3 + S_4 = 160$ was seen on occasion. In other weak responses, candidates attempted to solve the equation without deriving it and then sometimes showed that the values of r they had found were solutions of the equation. These candidates did not understand the key phrase 'Show that r satisfies the equation...'.

(ii) Candidates were much more successful in this part of the question. The wording of the question should have been a reminder to candidates that only one value of a was needed. Many did find the correct value of r and discarded the negative value, finding only the positive value of a that resulted. A few candidates stated both values of a and lost the accuracy mark. Some candidates did not factorise the correct equation, even though this was given in the question.

(b) A good number of candidates offered fully correct solutions to this part of the question. Most candidates interpreted the information in the question correctly, formed a correct pair of equations that they then solved simultaneously. Those candidates who wrote the sum as $q\{p+2(q-1)\}=168$ and then substituted $p+2(q-1)=14$ had a very neat and simple end to the question and almost always earned full marks. A few candidates made arithmetic slips or slips in algebraic manipulation in their solution. In weaker responses, candidates did not use the sum formula but rather used two formulae for terms or did not link p and q to a and n or did not use the given common differences correctly.

Question 11

There were many successful approaches seen when answering this question. A reasonable number of fully correct solutions were seen. To earn full credit, candidates needed to work with exact values throughout, work in radians and show all key method steps in a correct plan, including the correct substitution of the limits, however simple, into the integrated expression. Some candidates used rounded decimals as part of their solution and, depending on when this occurred, they were variously penalised. Those who made a reasonable attempt at a method often were able to find the key x values $\frac{\pi}{2}$ and $\frac{3\pi}{2}$ correctly, although this was not always the case. Some candidates needed to detail their plan more fully as it was common for candidates, for example, when finding the area A, to find the area under the curve but omit to subtract it from π. A correct plan was essential and some candidates, who were successful, kept track of what they were doing by making sketches of the relevant areas they were adding or subtracting before they found them. Other candidates added labels to the diagram and referenced them as they went through their solutions, this was very helpful and kept them focused. In weaker solutions, candidates integrated $1 + \cos x$ as $x - \sin x$ or thought that the coordinates of R were $(\pi, 0)$, not understanding the information given in the question regards the 'one complete period of the curve'. In other weak solutions, candidates worked only in degrees or thought that area A was 1, from the given ratio not from any working, or made no real attempt to answer.

Question 12

This question differentiated well and good responses were offered generally from those still capable of working at this level. It was expected at this stage of the paper that not all candidates would find this question accessible. Candidates needed to determine a strategy and understand that they could not integrate until they had rewritten the expression given for the second derivative. Some candidates did attempt to manipulate the expression first and a reasonable number were successful in doing this. These candidates usually went on to provide correct or almost fully correct solutions. Those who wrote the expression with powers rather than roots were generally more successful with the integration than those who still had some roots in their expression. A few slips were made with cross terms being omitted or an error made when simplifying one of the terms. An error or omission was acceptable and some further progress could still be made by these candidates. Those candidates who progressed were credited for integrating valid terms and usually stated a constant of integration. These generally went on to find the value of this constant using a correct method, although occasional slips were seen. Candidates who started with a correct expression were able to earn further marks by integrating correctly for a second time and using a second constant of integration. Again many of these did this successfully. The question asked candidates for the equation of the curve and it was necessary to have $y = ….$ or $f(x) = ….$ to earn the final mark, an expression only was not sufficient. In weak solutions, candidates generally did not attempt to expand the given expression for the second derivative and instead either integrated the expression as if it were x^2 or integrated term by term within the expression. Some candidates who were successfully squaring the numerator were unsure of how to manipulate the denominator. For example, in other weak solutions, candidates simplified the given expression for the second derivative as something similar to $\left(x^{\frac{1}{2}} + 1 \times x^{-\frac{1}{4}}\right)^2$ or misinterpreted the 4th root of x as $4\sqrt{x}$ which they then squared to become $16x$.