ADDITIONAL MATHEMATICS

Paper 4037/12
Paper 12

Key messages

For candidates to succeed in this paper, it is essential that they are familiar with the rubric on the front of the paper, taking into account the accuracy required. Careful reading of each question is necessary, together with the checking that the full demands of the question have been met. Setting work out in a clear and concise fashion with all the necessary steps is also essential.

The examination paper tests knowledge, understanding and skills, so familiarity with the syllabus and the assessment objectives is key. The questions cover simple application of standard techniques at a lower level with the expectation that candidates will be able to use these techniques to help with problem solving at a higher level.

General comments

There did not appear to be any timing issues, with the majority of candidates being able to attempt most, if not all, questions. Candidates should also be aware that if they run out of space, or need to make other attempts at a question, it is preferable to use any blank pages at the end of the question paper or ask for additional paper which may then be attached. This helps with the setting out of work in a clear and concise fashion. As in previous sessions, some candidates are still working in pencil first and then overwriting their work in ink. This very often makes the work difficult to read, which can result in marks not being awarded. It is preferable for a candidate to start again and use an additional sheet if necessary.

Many candidates are still unfamiliar with the word 'exact' and continue to give solutions which require an exact final answer in decimal form. Careful reading of a question and check as to what form the final answer should be in is essential.

Past examination papers are often used for examination practice and careful reading of the accompanying mark scheme and Examiner Report will help reinforce the necessity to give final answers in the required form.

Comments on specific question

Question 1

(i) Many candidates knew the general form of the graph of a cosine curve, but could not sketch the graph with the required transformation. The majority earned the mark for the *y*-intercept but very often did not gain any further marks as a common error was to start at $(-90°, -1)$ and finish at $(90°, 0)$.

(ii) Most candidates were able to identify the correct amplitude.

(iii) Finding the period of $2\cos 3x - 1$ was more problematic with many candidates not realising that an angle was required. Too many candidates gave 3 as an answer.

Question 2

Most candidates were able to find the gradient of the graph and most went on to find a correct vertical axis intercept of 32, but there were instances of arithmetic slips with 20 being obtained instead. The use of the equation $\lg y^2 = mx + c$ was recognised by most, with many candidates going on to obtain the correct equation $\lg y^2 = -4x + 32$. Some candidates did not attempt to rearrange this equation to obtain the required form. Candidates generally had more success if they wrote $\lg y^2$ as $2\lg y$ and then divided through by 2. Those that chose to write $y^2 = 10^{-4x+32}$ often made errors when attempting to find the square root.

Question 3

Most candidates were able to make reasonable attempts to expand $\left(1-\frac{x}{7}\right)^{14}$ and $(1-2x)^4$. There were occasional errors with signs and with the simplification of the coefficients. The number of correct solutions was unexpectedly low as it appeared that many candidates were unable to successfully multiply their two expansions out and simplify the resulting terms correctly. It was also apparent that some candidates thought that they just had to expand out $\left(1-\frac{x}{7}\right)^{14}$ and $(1-2x)^4$ and left these expansions as a final answer.

Question 4

(i) Most candidates recognised the correct shape of the graph. It was essential that candidates indicated the coordinates of the intercepts of the curve with the coordinate axes as required. Listing the coordinates was acceptable as was marking them in on the axes themselves. Many candidates, having obtained a correct shape and x-intercepts, then omitted to mark in or state the *y*-intercept.

(ii) It was intended that candidates make use of their graph in **part (i)** and identify either the position of the minimum point on the curve $y = 2x^2 - 9x - 5$ or the maximum point on the curve $y = \left|2x^2 - 9x - 5\right|$. Many candidates identified, either by observation, use of the discriminant or by calculus, that the values $\pm\frac{121}{8}$ were significant. Some correct solutions of $k > \frac{121}{8}$ were seen but the other solution of $k = 0$ was seldom obtained correctly.

Question 5

(a) Most candidates were able to identify at least two correct functions. Common errors included the incorrect order in **part (i)** and the erroneous f^2 as an answer to **part (iii)**. Many candidates chose to work out each of the functions listed and then match their responses to the appropriate question part.

(b) (i) Many candidates were under the misapprehension that they had to show that the function was not a one-one function. It was intended that candidates realised that when $x = 0$ the function was not defined and hence the given domain was unsuitable. There were few correct responses.

(ii) It was hoped that candidates would recognise the notation h' being used for the derivative of the function h especially as the notation for the inverse of a function had been used in **part (a)**. Candidates who did not recognise this were unable to gain any marks. Those that did sometimes made errors in the differentiation of $\frac{b}{x^2}$ which often led to a fortuitously correct answer being obtained.

Question 6

(a) Slips in the simplification of indices and sign errors meant that not all candidates were able to gain marks in this part of the question.

(b) Most candidates attempted to change the base of either $\log_7 x$ or $\log_x 7$. The coefficient in $2\log_x 7$ caused problems when it was used to write the term as $\log_x 49$. Having made use of a change of base, most attempted to multiply through by the logarithmic term in the denominator of their equation. Too many, however, were under the misapprehension that $(\log_7 x)^2$ was equal to $\log_7 x^2$

or that $(\log_x 7)^2$ was equal to $\log_x 7^2$. Further progress was not then possible. A quadratic equation in terms of either $\log_7 x$ or $\log_x 7$ was required. Many candidates were able to gain fully correct solutions.

Question 7

(i) Most candidates realised that they had to use the product rule, even if an expansion of the terms was attempted first. Some candidates were unable to gain full marks if they were unable to differentiate the exponential term correctly.

(ii) Errors in simplification of an initially correct derivative in **part (i)** and/or errors in substitution of $x = 0.5$ meant that many candidates were unable to obtain the accuracy mark in this part. There were also candidates who were unable to deal correctly with the idea of small changes.

(iii) Candidates were able to gain follow through marks in this part, making use of the numerical part of their answer to **part (ii)**. There were some solutions when an incorrect rates of change equation was used e.g. 2 being multiplied by, rather than being divided by, the numerical part of the answer to **part (ii)**.

Question 8

(a) (i) It was expected that matrices that were comparable i.e. could be multiplied together, were written down in the correct order. Many candidates were unable to do this. It appeared that many did not really understand the demands of the question.

(ii) Unless a correct pair of matrices had been written down in **part (i)**, it was not possible to gain any marks in this part. Of those candidates that did have a correct matrix pair, some did not answer the question completely, omitting to state which team was awarded the most points. Others made an error, which was all too common, in evaluating the matrix product, obtaining an element of 8 rather than the correct element of 6.

(b) (i) Most candidates were able to write down a completely correct inverse matrix with very few errors seen.

(ii) Many correct solutions to this part were also seen, with most candidates making use of their inverse matrix from **part (i)** and pre-multiplication of the given matrix **B** to obtain the matrix **C**. There were some errors in the evaluation of the matrix product, but most candidates were able to gain marks. There were some who attempted a non-matrix method, but these candidates were unable to gain marks as the question specified 'Hence', meaning that their answer to **part (i)** had to be used.

Question 9

(i) It was essential that each step of working be shown as candidates were working towards a given answer. It appeared that many candidates were unable to recall the formula for the volume of a cylinder. Knowledge of this is assumed to be prior knowledge for this examination. There were slips with terms of π in the simplification of the surface area equation after a substitution had been made. Many candidates were able to produce a completely correct and well set out solution.

(ii) Many candidates realised the process of solution required differentiation and equating the resulting derivative to zero. There were many correct results of $r = 8.43$ or equivalent. There were candidates who, having obtained a correct equation from differentiation and equating to zero, were unable to solve it correctly. As has been common in this type of question in the past, many candidates have not answered the question completely by omitting in this case to find the stationary value of S. Most candidates made use of the second derivative with varying levels of success to determine the nature of the stationary point.

Question 10

(i) Most candidates were able to use either the cosine rule or basic trigonometry involving a suitable right-angled triangle, to show that angle AOB was equal to 2.24 radians. Not all of them however, justified this value to 2 decimal places. It was expected that an answer to greater accuracy, in this case 2.2395…, be shown first which then provides justification of 2.24 to 2 decimal places.

(ii) Many candidates were unable to gain many marks as they had not read the question carefully enough. They incorrectly made use of the angle found in **part (i)** as angle *AOC* rather than using the angle found in **part (i)** to help find angle *AOC*. This clearly had an effect on the next part of the question as well. Other errors included the inappropriate use of 18 instead of 10 when attempting to find the arc length *AC*. Most candidates were able to gain method marks for a correct process and many did obtain a correct perimeter.

(iii) Few completely correct solutions were seen, although many candidates were able to obtain method marks when finding the area of the triangle *AOC* and the sector *AOC*. Common errors included the use of 18 instead of 10 for the lengths involved in both calculations and an inaccurate final answer due to premature approximation involving angle *AOC* in **part (ii)**.

Question 11

As a completely unstructured question, candidates were expected to work out a plan of action. It was expected that integration be used to find an expression for the gradient function together with the use of the given information, to find the value of a first arbitrary constant. Most candidates realised that they had to integrate, and this was done with varying levels of success. The most common error was in the coefficient of $(3x-1)^{\frac{1}{3}}$. Some candidates made arithmetic slips when attempting to find the value of the arbitrary constant, others did not consider an arbitrary constant at all. Fewer candidates continued and attempted to integrate a second time, with similar errors occurring. Whilst there were few completely correct solutions, many candidates knew exactly the correct process to take, but made arithmetic slips and sign errors.

ADDITIONAL MATHEMATICS

Paper 4037/22
Paper 22

Key messages

The instruction to not use a calculator in certain questions needs to be adhered to as marks are not awarded when there is compelling evidence that a calculator has been used.

General comments

There was a wide range of marks achieved on this paper with a number of candidates being awarded full marks. A few candidates seemed to run out of time whilst working on the last question, but most completed as much as they were able.

Comments on specific questions

Question 1

The first diagram proved most difficult with incorrect responses usually shading the area outside both sets. The third diagram was most frequently correct. A fairly common approach was to number all the regions of a diagram and then identify the required set by analysing these numbers. This time-consuming method seemed to work for some but there was no real evidence that it led to fewer mistakes.

Question 2

There were many completely correct solutions to this question. Many candidates were able to correctly differentiate twice and insert their expressions to obtain the required form. A number did not include the expression for $3y$ whilst others made errors in differentiating but realised the need to combine the three expressions. Sign errors were not uncommon and there were some who replaced $3x$ with x. A few considered the given function to be a product and as a result made the question more demanding than necessary.

Question 3

Those who were successful in **part (i)** usually went on to score full marks in the other two parts. A number of candidates did not obtain any marks and there were some who did not attempt the question at all. The greatest source of error was due to candidates believing that the question required combinations rather than permutations. A few candidates managed to miscount the number of symbols in each group and others added correct terms rather than multiply. The simplest successful solution avoided factorials completely and just multiplied the number of options for each choice, that is 14.13.12.11.10 for **part (i)** and similar products for the other two parts.

Question 4

Most candidates were able to eliminate y successfully and obtain a quadratic in x. This was usually correct as was the discriminant that followed and the solution of the quadratic in k. Most were then able to establish a correct range between the two values of –1 and 11. A number of candidates stopped having found the values and others did not express the region correctly. Connecting the two inequalities with ‘or’, a comma or blank space was not accepted. A few gave incorrect inequalities suggesting the outer regions or overlapping regions.

Question 5

This question proved very challenging for all except the most able. The significance of the gradient of the normal being $\frac{1}{3}$ was widely overlooked and the differentiation, when attempted, was demanding. The difficulty with the derivative arose due to unnecessary use of the quotient rule and errors resulting from this by not realising that the derivative of *k* was 0.

In **part (ii)** candidates obtained credit for finding *y* using their value of *k*. Even if a correct derivative had been found in **part(i),** it was often ignored in favour of belatedly finding and using a gradient of –3 from information given in **part (i).**

Question 6

(i) There were a large number of completely correct proofs in this question although some lost credit due to poor notation such as omission of brackets or omission of *x* from multiple terms. There were a variety of routes to the proof and some were quite long-winded whilst others were succinct. At some point the fractions needed to be combined and $\tan x$ and $\sec x$ needed to be rewritten in terms of $\sin x$ and $\cos x$. Subsequently a correct Pythagorean identity had to be used and correct cancellation performed. Incorrect solutions included errors at various stages usually due to incorrect knowledge of an identity or overcomplicated expressions making progress impossible. As is often the case there were some who jumped to the correct final statement from a totally incorrect preceding statement.

(ii) Some started from scratch not realising the significance of 'hence' and others just solved $1+3\sin x=0$. Those who used the given identity as expected usually obtained a three term quadratic and solved it correctly. The solutions of $\sin x=\frac{2}{3}$ were invariably correct but many candidates quoted a solution of $x = 90°$ from $\sin x = –1$.

Question 7

(a) More able candidates realised that the product of roots was 40 and hence the third root was 5. Finding *a* and *b* was then done by multiplying out the factors. The vast majority of candidates solved the simultaneous equations generated from f(2) = 0 and f(4) = 0. Some were successful but many made algebraic mistakes and did not obtain the correct values. The request to find the third root was often overlooked and if attempted was sometimes given as ($x – 5$). Similar confusion between roots and factors led to some candidates solving equations obtained from f(–2) = 0 and f(–4) = 0.

(b) This was very well attempted, and the vast majority of candidates produced thorough and succinct solutions. However, not all candidates demonstrated that they had found their first root legitimately without using a calculator. There was also an occasional doubt regarding use of a calculator to solve $x^2 – 6x – 40 = 0$ using the quadratic formula. Candidates should ensure that they show sufficient working in such questions to make it clear that they have not used a calculator.

Question 8

The whole of this question seemed unfamiliar to a large number of candidates many of whom did not respond at all.

(i) Many candidates found the magnitude of the given velocity vector but did not know how to link it with the speed of $6.5\,\text{m}\,\text{s}^{-1}$ given in the question.

(ii) This required candidates just to find the modulus but many proceeded to do more work than was required.

(iii) There was a very mixed response here. Many answers did not mention time at all or attached time to the wrong vector. Others ignored the velocity vectors completely.

(iv) Where candidates had vectors in the correct form in **part (iii)** progress was possible. Only the most able candidates achieved complete success.

Question 9

(i) Full marks were obtained by a large number of candidates who recognised how to tackle this question. The mistakes that occurred were mainly due to mishandling of negative coordinates when adding or subtracting for the mid-point or gradient. Some candidates did not write the equation in the requested form.

(ii) Few candidates realised that the answer could be written directly by replacing x and y by r and s in their answer to **part (i)**. Many started again and others just left the answer space blank.

(iii) Almost all candidates were able to obtain a correct equation from the information that PM was of length 10 units. Many candidates made errors in algebra with the quadratic involving r and s. One popular choice was to equate the r terms to 100 and solve and then do the same for s. Other candidates opted to remove the squares leading to $(r - 1) + (s - 2) = 10$.

Some candidates did realise that they had to use the linear expression from **part (ii)** and found that the algebra required was very difficult when substituting into their quadratic expression. Only the more able candidates managed to obtain the correct quadratic equation, solve and reject the negative values of r and s.

Question 10

(i) There were many good solutions here particularly using the quotient rule which was often quoted correctly prior to substitution. Subsequent simplification was sometimes incorrect and this often prevented progress in **part (ii)**.

(ii) Most equated their answer to **part (i)** to 0 and attempted to solve. Many did get the correct value for x but a number did not find y or did not do so to the required accuracy. A number still had a factor of x in the numerator that had not been cancelled in **part (i)** and this created problems when they were trying to solve.

(iii) There were very few candidates who managed a completely correct answer to this part. Most did not fully grasp the link with **part (i)**. Those that did write down the correct relationship subsequently multiplied or divided by x and then attempted to rearrange.

(iv) It was not possible to progress here without a viable attempt at the previous part, so very few marks were awarded. Many used their calculators to obtain an answer.

Question 11

The vast majority knew that it was expected to use the formula but correct simplification of the discriminant was a problem. Some candidates obtained $\sqrt{25}$ but a few then replaced this with $\sqrt{5}$. Errors in the denominator were not as common although $2\sqrt{5} - 3$ was seen a number of times. Most candidates knew how to rationalise the denominator and did so quite well. The final problem came when dealing with the minus sign to get the final answers and it was quite common to see $-\frac{1}{4}\sqrt{5} + \frac{3}{4}$ or $\sqrt{5} - 3$.