CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge Ordinary Level

MARK SCHEME for the October/November 2015 series

4037 ADDITIONAL MATHEMATICS

4037/22 Paper 2, maximum raw mark 80

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Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfww	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied
www	without wrong working

1 (i)	$f(-2) = -32 - 16 + 30 + 18 = 0$	B1	All four evaluated terms must be seen. Allow if correct long division used
(ii)	$f(x) = (x+2)(4x^2 - 12x + 9)$	M1 A1	Coefficients 4 and 9 Coefficient –12
	$= (x+2)(2x-3)(2x-3)$	A1	All three factors together
	$f(x) = 0 \to x = -2,\ 1.5$ nfww	A1	Allow 1.5 mentioned just once
2 (i)	$(2-3x)^6 = 64 - 576x + 2160x^2$ isw	B1B1B1	
(ii)	$2160 - 2 \times 576 = 1008$	M1 A1	*their* final 2160 + 2 × *their* final –576
3 (i)	$\overrightarrow{AB} = \begin{pmatrix} -15 \\ 8 \end{pmatrix}$	B1	Allow $\overrightarrow{BA}$ May be implied by later work.
	$\lvert AB \rvert = \sqrt{15^2 + 8^2} \quad (=17)$ Speed $= 17 \times 3 = 51 \text{km/hr}$	M1 A1	Use of Pythagoras on *their AB* Must be exact
(ii)	$\overrightarrow{BC} = \begin{pmatrix} 16 \\ -30 \end{pmatrix}$	B1	Allow $\overrightarrow{CB}$
	$\lvert BC \rvert = \sqrt{16^2 + 30^2} \quad (=34)$	M1	Use of Pythagoras on *their BC*
	Time taken $= \frac{34}{51} \times 60 = 40$ mins (or $\frac{2}{3}$ hrs)	A1	Allow answers which round to 40 to 2sf. Accept 0.66 or 0.67 hrs. Mark final answer.

4 (a)	$2\mathbf{BA} = 2\begin{pmatrix}1 & -2 & 4\\ -2 & 3 & 0\end{pmatrix}\begin{pmatrix}2 & -1\\ 3 & 5\\ 7 & 4\end{pmatrix}$ $= 2\begin{pmatrix}24 & 5\\ 5 & 17\end{pmatrix} = \begin{pmatrix}48 & 10\\ 10 & 34\end{pmatrix}$	B3,2,1,0	–1 each error in 2 × 2 result. Failure to multiply by 2 is one error
(b) (i)	$\mathbf{C}^{-1} = \frac{1}{8}\begin{pmatrix}6 & -2\\ 1 & 1\end{pmatrix}$ isw	B1 B1	$\frac{1}{8}$ Matrix
(ii)	$\mathbf{I}-\mathbf{D} = \begin{pmatrix}-2 & 2\\ -1 & -3\end{pmatrix}$	B1	
	$\mathbf{X} = \mathbf{C}^{-1}(\mathbf{I}-\mathbf{D}) = \frac{1}{8}\begin{pmatrix}6 & -2\\ 1 & 1\end{pmatrix}\begin{pmatrix}-2 & 2\\ -1 & -3\end{pmatrix}$	M1	Pre multiply *their* **I – D** with *their* $\mathbf{C}^{-1}$
	$= \frac{1}{8}\begin{pmatrix}-10 & 18\\ -3 & -1\end{pmatrix}$ isw	A1	
5 (a)	$2^{3(q-1)} \times 2^{2p+1} = 2^{14}$	B1	Correct powers of 2 allow unsimplified isw
	$3^{2(p-4)} \times 3^{q} = 3^{4}$	B1	Correct powers of 3 allow unsimplified isw
	Solve $3q + 2p = 16$ $q + 2p = 12$	M1	Attempt to solve *their* linear equations by eliminating one variable
	$p = 5, \quad q = 2$	A1	Both correct
(b)	$(3x-2)(x+1)$ $= 50$	M1 A1	LHS oe isw 50 from correct processing of $2 - \lg 2$
	$3x^2 + x - 52 = 0 \rightarrow (3x+13)(x-4)$	M1	Solution of *their* three term quadratic Roots must be obtained from correct quadratic
	$x = 4$ $x = -\frac{13}{3}$ discarded	A1 A1	

6 (i)	$a=3, \quad b=2, \quad c=4$	B1B1B1	
(ii)	$\frac{\mathrm{d}y}{\mathrm{d}x}=8\cos 4x$ isw	M1 A1FT	$\pm k\cos cx$ and no other term in x $\quad c\neq 1$ $bc\times\cos cx$ and no other term
(iii)	$x=\frac{\pi}{2}\to\frac{\mathrm{d}y}{\mathrm{d}x}=8\cos 2\pi=8$	DM1	Find *their* correct numerical $\frac{\mathrm{d}y}{\mathrm{d}x}$
	Eqn: $\frac{y-3}{x-\frac{\pi}{2}}=-\frac{1}{8} \quad \left(\to y=-\frac{1}{8}x+3.20\right)$	M1	Find equation with *their* numerical normal gradient ie $\frac{-1}{\frac{\mathrm{d}y}{\mathrm{d}x}}$ and point $\left(\frac{\pi}{2},\ 3\right)$
		A1	All correct isw
7 (i)	$\frac{h}{8}=\frac{6-r}{6}\to h=\frac{4}{3}(6-r)$	M1 A1	Uses correct ratio. Cannot be implied
(ii)	$V=\pi r^2h=\pi r^2\times\frac{4}{3}(6-r)$ $=8\pi r^2-\frac{4}{3}\pi r^3$	B1	**AG** all steps must be seen Penalise missing brackets at any point in working
(iii)	$\frac{\mathrm{d}V}{\mathrm{d}r}=16\pi r-4\pi r^2$	M1 A1	Differentiate at least one power reduced by one
	$\frac{\mathrm{d}V}{\mathrm{d}r}=0\to r=4$	M1 A1	Attempt to solve – must get $r=\ldots$ Correct value of r. Ignore $r=0$
	$V=\frac{128}{3}\pi \quad (=42.7\pi)$	A1	Correct value of V. Condone 134.
	$\frac{\mathrm{d}^2V}{\mathrm{d}r^2}=16\pi-8\pi r<0$ when $r=4\to$ max	B1	$\frac{\mathrm{d}^2V}{\mathrm{d}r^2}$ must be correct and some indication of a negative value seen plus maximum stated

8 (i)	Gradient $AB = \frac{8-2}{9+3}$ $\left(=\frac{1}{2}\right)$ isw	B1	
	Equation AB and $x = 0 \to \frac{y-2}{0+3} = \frac{1}{2}$ $\left(\to y = \frac{1}{2}x + 3.5\right)$	M1	Find equation with *their* gradient and set $x = 0$
	$\to y = 3.5$	A1	
(ii)	D is (3, 5)	B1	
(iii)	Gradient perpendicular = –2	M1	Use of $m_1 \times m_2 = -1$ on gradient used for *their* line in **(i)**
	Equation perpendicular $\frac{y-5}{x-3} = -2$ $\to (y = -2x + 11)$	A1	
(iv)	E is (0, 11)	A1FT	
(v)	Area of $ABE = \frac{1}{2}\begin{vmatrix} -3 & 9 & 0 & -3 \\ 2 & 8 & 11 & 2 \end{vmatrix}$	M1	For area of ABE or ECD. $\frac{1}{2}$ and *their* correct 8 elements must be seen.
	$= \frac{1}{2}\lvert -24 + 99 - 18 + 33 \rvert = 45$	A1	45 condone from E (0, –4)
	Area of $EDC = \frac{1}{2}\begin{vmatrix} 3 & 0 & 0 & 3 \\ 5 & 3.5 & 11 & 5 \end{vmatrix}$		
	$= \frac{1}{2}\lvert -10.5 + 33 \rvert = 11.25$	A1	11.25 condone from E(0, –4)

9 **(i)**	$\tan 2x = -\frac{5}{4}$ $(2x = 128.7,\ 308.7)$	M1	For obtaining and using $\tan 2x = \pm\frac{5}{4}$ or $\pm\frac{4}{5}$ resulting in $2x =$
	$x = 64.3$ awrt 154.3 awrt	A1 A1FT	$\tan x = \ldots$ gets M0 *their* $64.3° + 90°$
(ii)	$\text{cosec}^2 y + 3\text{cosec}y - 4 = 0$ or $4\sin^2 y - 3\sin y - 1 = 0$ $(\text{cosec}y + 4)(\text{cosec}y - 1) = 0$ or $(4\sin y + 1)(\sin y - 1) = 0$	B1	In any form as a three term quadratic.
	$\sin y = -\frac{1}{4}$ or $\sin y = 1$	M1	Solve three term quadratic in $\text{cosec}\, y$ or $\sin y$
	$y = 194.5,\ 345.5,\ 90$	A1A1A1	Answers must be obtained from the correct quadratic
(iii)	$z + \frac{\pi}{4} = \pi - \frac{\pi}{3}$ or	B1	Accept 2.09, 2.10, $\pi - 1.05$, $\pi - 1.04$ on RHS. Could be implied by final answer
	$z + \frac{\pi}{4} = \pi + \frac{\pi}{3}$	B1	Accept 4.19, 4.18, $\pi + 1.05$, $\pi + 1.04$ on RHS. Could be implied by final answer
	$z = \frac{5\pi}{12},\ \frac{13\pi}{12}$	B1B1	Answers must be correct multiples of π.
10 **(i)**	$s = \frac{1}{2}e^{2t} + 3e^{-2t} - t + (c)$	M1	Integrate : coefficient of $\frac{1}{2}$ or 3 seen with no change in powers of e. Ignore $-t$
	$t = 0,\ s = 0 \to c = -3.5$ $\left(s = \frac{1}{2}e^{2t} + 3e^{-2t} - t - 3.5\right)$	A1 A1	All correct and simplified
(ii)	$v = 0 \to u^2 - u - 6 = 0$ oe	M1	Obtain three term quadratic in u or e^{2t} Condone sign errors.
	$(u-3)(u+2) = 0$	DM1	Solve three term quadratic
	$\to u = 3 \to t = \frac{1}{2}\ln 3$ or 0.549	A1	Accept 0.55 No second answer
(iii)	$t = \frac{1}{2}\ln 3 \to a = 2e^{2t} + 12e^{-2t}$	B1	Correct differentiation
	$= 6 + 4 = 10$	B1	Allow awrt 10.0 or 9.99. No second answer.