# MARK SCHEME for the May/June 2012 question paper

# for the guidance of teachers

## 4037 ADDITIONAL MATHEMATICS

**4037/12**      Paper 1, maximum raw mark 80

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

Cambridge is publishing the mark schemes for the May/June 2012 question papers for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.

**Mark Scheme Notes**

Marks are of the following three types:

M   Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A   Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B   Accuracy mark for a correct result or statement independent of method marks.

• When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

• The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

• Note:   B2 or A2 means that the candidate can earn 2 or 0.
        B2, 1, 0 means that the candidate can earn anything from 0 to 2.

The following abbreviations may be used in a mark scheme or used on the scripts:

AG       Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD       Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO       Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

ISW       Ignore Subsequent Working

MR       Misread

PA       Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS       See Other Solution (the candidate makes a better attempt at the same question)

**Penalties**

MR –1       A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through $\sqrt{}$ " marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy.

OW –1,2       This is deducted from A or B marks when essential working is omitted.

PA –1       This is deducted from A or B marks in the case of premature approximation.

S –1       Occasionally used for persistent slackness – usually discussed at a meeting.

EX –1       Applied to A or B marks when extra solutions are offered to a particular equation. Again, this is usually discussed at the meeting.
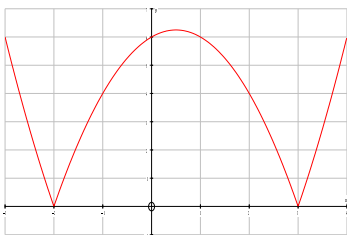
| 1 | (i) | $\frac{2}{21}(7x-5)^{\frac{3}{2}}$ $(+c)$ | **B1** | **B1** for multiplication by $\frac{2}{3}$, or division by $\frac{3}{2}$ |
|---|---|---|---|---|
| | | | **B1, B1** | **B1** for $(7x-5)^{\frac{3}{2}}$, **B1** for $\frac{1}{7}$ |
| | (ii) | $\frac{2}{21}\left(16^{\frac{3}{2}} - 9^{\frac{3}{2}}\right)$ $\left(=\frac{2}{21}(64-27)\right)$ | **M1** | **M1** for correct use of limits, must have attempted integration, must be using their |
| | | $= \frac{74}{21}$ or awrt 3.52 or $3\frac{11}{21}$ | **A1** [5] | $(7x-5)^{\frac{2n+1}{2}}$ from **(i)** |

| 2 | $4u^2 - 5u + 1 = 0$ <br> $(4u-1)(u-1) = 0$ | **B1, M1** | **B1** for $2^{2x+2} = 4u^2$ or $4 \times 2^{2x}$ or $2^2 \times 2^{2x}$ or $2^2 u^2$ <br> **M1** for attempt to obtain a 3 term quadratic equation in terms of either or, equated to zero. |
|---|---|---|---|
| | or $(4.2^x - 1)(2^x - 1) = 0$ | **DM1** | **DM1** for solution of quadratic equation |
| | $2^x = \frac{1}{4}$, $2^x = 1$ | **A1** | **A1** for both |
| | leading to $x = -2, 0$ | **A1** | **A1** for both |
| | Alternate scheme for one correct factor: | | |
| | $2^x = \frac{1}{4}$, leading to $x = -2$ | **[A1]** | |
| | $2^x = 1$, leading to $x = 0$ | **[A1]** [5] | |

| 3 | $\frac{\cos A}{\sin A} + \frac{\sin A}{1+\cos A}$ | **B1** | **B1** for $\cot A = \frac{\cos A}{\sin A}$ |
|---|---|---|---|
| | $= \frac{\cos A + \cos^2 A + \sin^2 A}{\sin A(1+\cos A)}$ | **M1** | **M1** for obtaining as a single fraction |
| | $= \frac{(1+\cos A)}{\sin A(1+\cos A)}$ | **M1** | **M1** for use of $\cos^2 A + \sin^2 A = 1$ |
| | $= \frac{1}{\sin A}$ $= \text{cosec}A$ | **A1** | **A1** for correct simplification – answer given. |
| | Alternate solution: | | |
| | $\cot A + \frac{\sin A(1-\cos A)}{(1+\cos A)(1-\cos A)}$ | **[M1]** | **M1** for multiplying by $(1 - \cos A)$ |
| | $= \cot A + \frac{\sin A(1-\cos A)}{\sin^2 A)}$ | **[M1]** | **M1** for use of $\cos^2 A + \sin^2 A = 1$ anywhere |
| | $= \cot A + \frac{1-\cos A}{\sin A}$ | **[M1]** | **M1** for cancelling $\sin A$ |
| | $= \cot A - \cot A + \frac{1}{\sin A}$ leading to cosec$A$ | **[A1]** [4] | **A1** for subtraction and simplification |

| 4 | Using $y = \dfrac{2-5x}{3}$ or, using $x = \dfrac{2-3y}{5}$ <br><br> $5x^2 - 21x + 4 = 0$ or $3y^2 + 17y - 6 = 0$ <br><br> $(5x-1)(x-4) = 0$ or $(3y-1)(y+6) = 0$ <br><br> $x = \dfrac{1}{5}, \ y = \dfrac{1}{3} \qquad x = 4, \ y = -6$ <br> Alternate substitutions: <br><br> $x = \dfrac{2y}{3+y}$ or $y = \dfrac{3x}{2-x}$ | M1 <br><br> M1 <br><br> DM1 <br><br> A1, A1 <br> [5] | M1 for substitution to get an equation in terms of one variable <br> M1 for attempt to form a 3 term quadratic equation = 0 <br> DM1 for solution of quadratic equation <br><br> A1 for each 'pair' |
|---|---|---|---|
| 5 (i) | $(2-x^2)\dfrac{3}{(3x+1)} - 2x\ln(3x+1)$ | B1 <br> M1 <br> A1 | B1 for differentiating $\ln(3x+1)$ correctly <br> M1 for correct attempt at product <br> A1 for all else correct |
| (ii) | $\dfrac{5x(-2\sec^2 2x) - 5(4 - \tan 2x)}{25x^2}$ <br><br> or $\dfrac{5x(-2\sec^2 2x) - 5(4 - \tan 2x)}{(5x)^2}$ | B1 <br> M1 <br> A1 <br> [6] | B1 for differentiating $\tan(4-2x)$ correctly <br> M1 for correct attempt at quotient or product <br> A1 for all else correct |
| 6 (i) | $\dfrac{8(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)} = 4(\sqrt{3}-1)$ <br><br> or $\dfrac{8}{\sqrt{3}+1} = a(\sqrt{3}-1)$, <br> $8 = a(\sqrt{3}-1)(\sqrt{3}+1)$ <br> $a = 4$ | M1 <br><br><br><br><br> A1 | M1 for rationalisation or attempt to form equation |
| (ii) | $\sin 60 = \dfrac{\sqrt{3}}{2} = \dfrac{h}{4(\sqrt{3}-1)}$ <br><br> $\tan 60 = \sqrt{3} = \dfrac{h}{2(\sqrt{3}-1)}$ <br><br> Or $(4(\sqrt{3}-1))^2 = h^2 + (2(\sqrt{3}-1))^2$ <br> $h = 6 - 2\sqrt{3}$ ANSWER GIVEN | M1 <br><br><br><br><br> A1 | M1 for use of sine or tangent and their value of $a$ from (i) or $\dfrac{8}{\sqrt{3}+1}$ <br><br> or Pythagoras, <br> A1 for rearranging and simplifying correctly to obtain given answer. |
| (iii) | Area $= \dfrac{1}{2} 4(\sqrt{3}-1)(6-2\sqrt{3})$ <br><br> or $\dfrac{1}{2} 4(\sqrt{3}-1)4(\sqrt{3}-1)\sin 60°$ <br><br> $= 16\sqrt{3} - 24$ | M1 <br><br><br><br><br> A1 <br> [6] | M1 for valid method for area using their $a$ from (i) or $\dfrac{8}{\sqrt{3}+1}$ <br><br> A1 working must be seen |

**7 (i)**



B1     **B1** for shape

B1     **B1** for $x = -2, 3$

B1     **B1** for $y = 6$

**(ii)** $x^2 - x - 6 = 6$, leading to     B1     **B1** for one correct answer

$x = -3, 4$   (www)     B1     **B1** for a second correct answer

$x^2 - x - 6 = -6$, leading to     B1     **B1** for a third and fourth correct answer

$x = 0, 1$   (www)     **[6]**

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**8 (i)**   arc $AB = \dfrac{20\pi}{3}$   or 20.94, 20.9     B1     **B1** for arc length correct

$\tan \dfrac{\pi}{3} = \dfrac{AX}{10}$,   AX $= 10\sqrt{3}$, 17.3   (or $XB$)     B1     **B1** for $AX/XB$

Perimeter = awrt 55.6 or $20\sqrt{3} + \dfrac{20\pi}{3}$     B1     **B1** for final answer

**(ii)** Area of sector $AOB = \dfrac{1}{2}10^2 \dfrac{2\pi}{3}$ or 104.7     B1     **B1** for sector area correct

or 105

Area of $OAXB = 100\sqrt{3}$   or 173.2     M1     **M1** for valid attempt at area $OAXB$, using their $BX$ from part **(i)** $(10 \times$ their $BX)$

Shaded area = awrt 68.5 or $100\sqrt{3} - \dfrac{100\pi}{3}$     M1     **M1** for area $OAXB$ – sector area used (independent)

A1     Must be considering a quadrilateral, not a

**[7]**     triangle.

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**9 (i)**   250     B1     **B1** for 250

**(ii)** $8 = e^{\frac{x}{100}}$     B1     **B1** for $8 = e^{\frac{x}{100}}$

$\dfrac{x}{100} = \ln$ 'their 8' or $x = 100 \ln$ their 8     M1     **M1** for dealing with e correctly, using ln

$x = 208$   or awrt 208     A1     **A1** for awrt 208

**(iii)** $\dfrac{\mathrm{d}N}{\mathrm{d}x} = \dfrac{1}{2}e^{\frac{x}{100}}$     B1, B1     **B1** for $e^{\frac{x}{100}}$, B1 for $\dfrac{1}{2}e^{\frac{x}{100}}$ or $\dfrac{50}{100}e^{\frac{x}{100}}$

$45 = \dfrac{1}{2}e^{\frac{x}{100}}$     M1     **M1** for equating their $\dfrac{\mathrm{d}N}{\mathrm{d}x}$ to 45 and attempt to solve

$e^{\frac{x}{100}} = 90$,   so $N = 4700$     A1     **A1** for 4700

(awrt 4700)     **[8]**

| | | |
|---|---|---|
| **10 (a) (i)** $f'(x) = -(2+x)^{-2}$<br><br>$f''(x) = 2(2+x)^{-3}$ | **B1**<br><br>**B1** | First **B1** may be implied by a correct answer for $f''(x)$<br>If done by quotient rule, allow unsimplified |
| **(ii)** $y = \dfrac{1}{2+x}, \quad x = \dfrac{1}{y} - 2$<br><br>$f^{-1}(x) = \dfrac{1}{x} - 2$ or $\dfrac{1-2x}{x}$ | **M1**<br><br>**A1** | **M1** for a valid attempt at the inverse<br><br>**A1** must be in correct form, allow $y = \ldots$ |
| **(iii)** $f^2(x) = \left(\dfrac{1}{2+\dfrac{1}{2+x}}\right) = \dfrac{2+x}{5+2x}$<br><br>Equating to $-1$ leads to $x = -\dfrac{7}{3}$ or $-2.33$ | **M1**<br><br>**DM1**<br><br>**A1** | **M1** for correct attempt at $f^2(x)$<br><br>**DM1** for attempt at solution of $f^2(x) = -1$<br><br>**A1** for $x = -\dfrac{7}{3}$ or equivalent |
| **(b) (i)** $gh(x)$ or $gh$ | **B1** | **B1** for either form |
| **(ii)** $kg(x)$ or $kg$ | **B1**<br>**[9]** | **B1** for either form |
| **11 (i)** $P(3, 1)$<br><br>Grad $AB = \dfrac{18}{12}$<br><br>$\perp$ grad $-\dfrac{2}{3}$<br><br>$PQ: y - 1 = -\dfrac{2}{3}(x-3)$ $\quad (2x + 3y = 9)$ | **B1, B1**<br><br>**B1**<br><br>√**B1**<br><br>√**B1** | **B1** for each coordinate<br><br>**B1** for gradient of $AB$<br><br>√**B1** for perpendicular gradient<br><br>√**B1** on their perp gradient and their point $P$<br>Must be $y = \ldots$ |
| **(ii)** $Q(-15, 13)$ | **M1**<br>**A1** | **M1** for use of $y = 13$ and their $PQ$ equation.<br>**A1** for both coordinates (can be implied) |
| **(iii)** Area $= \dfrac{1}{2}\sqrt{18^2 + 12^2}\ \sqrt{8^2 + 12^2}$<br><br>or Area $= \dfrac{1}{2}\begin{vmatrix} 3 & 11 & -15 & 3 \\ 1 & 13 & 13 & 1 \end{vmatrix}$<br><br>or Area $= \dfrac{1}{2} \times 26 \times 12$<br>$= 156$ | **M1**<br><br><br><br><br>**A1**<br>**[9]** | **M1** for a valid attempt at area $\dfrac{1}{2} \times PQ \times PB$<br><br>Matrix method using their coordinates correctly<br><br>$\dfrac{1}{2} \times QB \times$ vertical perp height |

**12  EITHER**

**(i)**  velocity = 12**i** + 16**j**                                       **M1**
        position = ( 54**i** + 16**j**) + ( 36**i** + 48**j**)
        = 90**i** + 64**j**       ANSWER GIVEN              **A1**

**M1** for
(3 × their velocity ( must in numeric vector form)) + (54**i** + 16**j**)

**(ii)**  (54**i** + 16**j**) + (12*t***i** + 16*t***j**)              **M1, A1**

**M1** for position vector + (their numeric velocity vector × time)

**(iii)**  At 16 00,
         ship has 'travelled'   (102**i** + 80**j**)              **B1**

**B1** for (102**i** + 80**j**)

         boat needs to do this in 2 hours                  **M1**
         so velocity of boat (51**i** + 40**j**)
         speed $\sqrt{51^2 + 40^2}$
         = 64.8                                             **A1**

**M1** for attempt at velocity of boat and speed

**(iv)**  (51**i** + 40**j**) – (12**i** + 16**j**)              **B1**

**B1**, allow unsimplified  but must be correct

         = 39**i** +24**j**

**(v)**  $\tan \alpha = \dfrac{51}{40}$                         **M1**
        angle = 51.9                                        **A1**
                                                            **[10]**

**M1** for use of tan and their velocity vector

**12 OR**

**(i)** $\overrightarrow{OQ}$ $\mathbf{a} + \dfrac{1}{3}(\mathbf{b} - \mathbf{a})$          **B1**    Allow unsimplified

$\qquad = \dfrac{2}{3}\mathbf{a} + \dfrac{1}{3}\mathbf{b}$

$\overrightarrow{PQ} = -\dfrac{5}{4}\mathbf{b} + \mathbf{a} + \dfrac{1}{3}(\mathbf{b} - \mathbf{a})$      √**B1**    Follow through on their $\overrightarrow{OQ}$, allow unsimplified

$\qquad = \dfrac{2}{3}\mathbf{a} - \dfrac{11}{12}\mathbf{b}$

**(ii)** $\overrightarrow{QR} = \lambda\,\mathbf{a} - (\mathbf{a} + \dfrac{1}{3}(\mathbf{b} - \mathbf{a}))$      **M1**    **M1** for λa – their $\overrightarrow{OQ}$

$\qquad = \lambda\,\mathbf{a} - \dfrac{2}{3}\mathbf{a} - \dfrac{1}{3}\mathbf{b}$      **A1**    **A1** – allow unsimplified

**(iii)** $\overrightarrow{QR} = \mu(\overrightarrow{PQ} + \overrightarrow{QR})$      **M1**    **M1** for attempt to obtain $\overrightarrow{QR}$ in terms of $\overrightarrow{PQ}$

$\quad (1 - \mu)\overrightarrow{QR} = \mu\overrightarrow{PQ}$      **M1**    **M1** for attempt to simplifiy

$\quad QR = \dfrac{\mu}{1 - \mu}\left(\dfrac{2}{3}\mathbf{a} - \dfrac{11}{12}\mathbf{b}\right)$      **A1**

**(iv)** Equating **b**'s $\quad -\dfrac{11}{12}\dfrac{\mu}{1 - \mu} = -\dfrac{1}{3}$      **M1**    **M1** for equating like vectors and attempt to solve

$\quad \mu = \dfrac{4}{15}$      **A1**    **A1** for each

$\quad \lambda = \dfrac{10}{11}$      **A1**    **[10]**