UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Ordinary Level

MARK SCHEME for the May/June 2011 question paper

for the guidance of teachers

4037 ADDITIONAL MATHEMATICS

4037/11 Paper 1, maximum raw mark 80

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- Cambridge will not enter into discussions or correspondence in connection with these mark schemes.

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International Examinations

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Accuracy mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2, 1, 0 means that the candidate can earn anything from 0 to 2.

The following abbreviations may be used in a mark scheme or used on the scripts:

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

Penalties

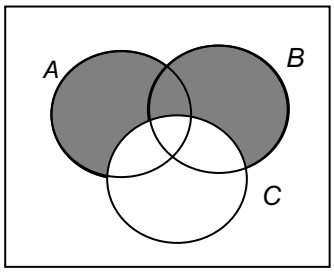
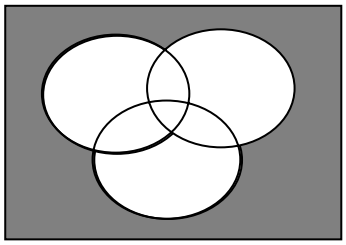
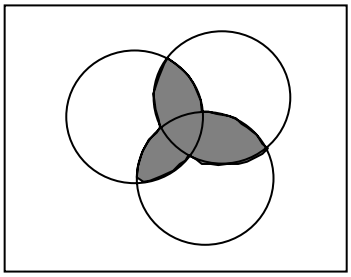
MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √ " marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy.

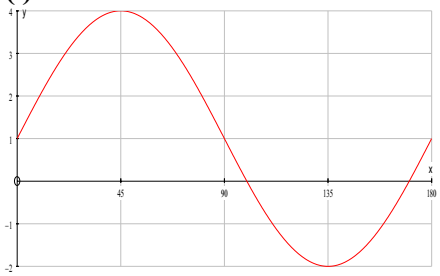
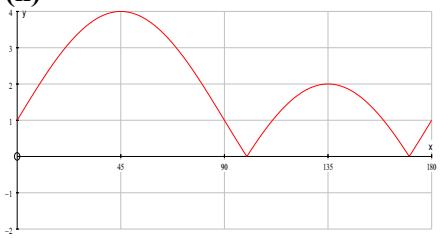
OW –1,2 This is deducted from A or B marks when essential working is omitted.

PA –1 This is deducted from A or B marks in the case of premature approximation.

S –1 Occasionally used for persistent slackness – usually discussed at a meeting.

EX –1 Applied to A or B marks when extra solutions are offered to a particular equation. Again, this is usually discussed at the meeting.

1	$\dfrac{(1+\cos\theta)+(1-\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}$ $= \dfrac{2}{1-\cos^2\theta}$ $= \dfrac{2}{\sin^2\theta}$ $= 2\,\text{cosec}^2\,\theta$	M1 M1 A1 [3]	M1 for attempt to deal with fractions M1 for attempt at simplification and use of $1-\cos^2\theta = \sin^2\theta$ in denominator
2	$\lg ab^3 - \lg 1000$ $= \lg \dfrac{ab^3}{1000}$	B1B1 B1 [3]	B1 for $\lg ab^3$, B1 for $\lg 1000$
3	**(a) (i)**  **(ii)**  **(iii)**  **(b)** n(P) = 3	B1 B1 B1 B1 [4]	B1 for each region shaded correctly
4	**(a)** Powers of 2: $4(3x-2) = 3(2x)$ or equivalent for powers of 4, 8 or 16 $x = \dfrac{4}{3}$, allow 1.33 **(b)** p = 1, $q = -\dfrac{4}{5}$	M1 A1 A1 B1B1 [5]	M1 for powers of 2, 4, 8 or 16 A1 for all powers correct

5	**(i)** 	B1 B1 B1	B1 for shape B1 for 1 cycle between 4 and –2 B1 all correct
	(ii) 	√B1	√B1 for modulus of (i)
	(iii) 5	√B1 [5]	√ on their graph
6	**(i)** $3x^2 = -2x^2 + 20x - 20$ and verification. Or $(x-2)^2 = 0$, $x = 2$	B1 B1	substitution of $x = 2$ B1 for solution of equation
	(ii) for OA, $\frac{dy}{dx} = 2x$ when $x = 2$, grad = 4 for other curve, $\frac{dy}{dx} = -\frac{4}{3}x + \frac{20}{3}$ when $x = 2$, grad = 4 Or $2x = -\frac{4}{3}x + \frac{20}{3}$ leading to $x = 2$	B1 M1 A1 M1 M1 A1	B1 for grad at A from OA M1 for attempt to differentiate the other curve and substitute $x = 2$ M1 for differentiation of both M1 for equating and attempt to solve
	(iii) tangent $y - 4 = 4(x-2)$	B1 [5]	

7	Grad of $AB = -2$, perp grad $= \frac{1}{2}$ Eqn of perp $y - 15 = \frac{1}{2}(x+2)$ $C(0, 16)$ Area $= \frac{1}{2}\sqrt{125}\sqrt{5}$ $= 12.5$ (or $\frac{1}{2}\begin{vmatrix} -2 & 3 & 0 & -2 \\ 15 & 5 & 16 & 15 \end{vmatrix} = \frac{1}{2}(38-13)$)	B1M1 M1 A1 M1 A1 [6]	B1 for grad AB M1 use of $m_1 m_2 = -1$ M1 for correct attempt to find the equation of AC and hence to find C M1 for a valid method to find area
8	**(a)** **AB**, **AC** **(b)** Either: $\mathbf{Y} = \mathbf{X}\begin{pmatrix} -12x+3y & 6 \\ -7x+3y & 6 \end{pmatrix}$ $\begin{pmatrix} 2x & 3y \\ x & 4y \end{pmatrix} = \begin{pmatrix} -52x+18y & 36 \\ -71x+24y & 48 \end{pmatrix}$ leading to $y = 12$ and $x = 4$ Or $\frac{1}{10-12}\begin{pmatrix} 5 & -4 \\ -3 & 2 \end{pmatrix}\mathbf{Y} = \begin{pmatrix} -12x+3y & 6 \\ -7x+3y & 6 \end{pmatrix}$ $-\frac{1}{2}\begin{pmatrix} 6x & -y \\ -4x & -y \end{pmatrix} = \begin{pmatrix} -12x+3y & 6 \\ -7x+3y & 6 \end{pmatrix}$ leading to $y = 12$ and $x = 4$	B2,1,0 M1 M1A1 M1 A1A1 B1 B1 M1 M1 A1A1 [8]	–1 each one incorrect or extra M1 for pre-multiplying by **X** M1 for multiplication of matrices A1 for correct product M1 for equating like elements B1 for determinant for inverse B1 for 'matrix part' of inverse M1 for multiplication of matrices M1 for equating like elements
9	**(i)** 5 **(ii)** $a = -20\sin 4t$ $\sin 4t = -0.5$ $t = \frac{7\pi}{24}$ (allow 0.916) **(iii)** $s = \frac{5}{4}\sin 4t(+c)$ When $t = 5$, $s = 1.14$	B1 M1A1 DM1 A1 M1A1 DM1 A1 [9]	M1 for attempt to differentiate DM1 for attempt to solve for $4t$ M1 for attempt to integrate DM1 for substitution of t in radians

10	**(a)** **(i)** $2 = a - 3$, $a = 5$	B1	
	(ii) $y = -5e^{1-x} - x^3 + c$ $c = 10$ $y = -5e^{1-x} - x^3 + 10$	√B1 B1B1 M1 A1	√B1for first term using their a B1 for $-x^3$, B1 for $+c$ M1 for attempt to find c
	(b) **(i)** $\frac{1}{7}\frac{3}{4}(7x+8)^{\frac{4}{3}}$	B1B1	B1 for $\frac{1}{7}$, B1 for $\frac{3}{4}(7x+8)^{\frac{4}{3}}$
	(ii) $\left[\frac{3}{28}(7x+8)^{\frac{4}{3}}\right]_0^8$ $= \frac{180}{7}$ or 25.7	M1A1 [10]	M1 for use of limits
11	**(i)** $2(x-2)^2 - 3$	B1B1	B1 for –2, B1 for –3
	(ii) $x \geq 2$ or equivalent	√B1	√ on their '–2'
	(b) **(i)** $g(x) \geq 4$, $h^{-1}(x) \geq 0$	B1B1	B1 for each
	(ii) Correct sketch	B1 B1 B1	B1 for g(x) B1 for $g^{-1}(x)$ B1 for idea of symmetry
	(iii) $g(4x-25) = 85$ $(4x-25)^2 + 4 = 85$ $x = \frac{17}{2}$, $x = 4$ Discarding $x = 4$	M1 DM1 A1 B1 [12]	M1 for correct order DM1 for attempt to solve A1 for both B1 for discarding $x = 4$

12 EITHER		
(i) $\frac{dy}{dx} = 3x^2 - 14x + 8$	M1A1	M1 for attempt to differentiate
When $\frac{dy}{dx} = 0$, $x = \frac{2}{3}, 4$	M1A1	M1 for attempt to equate to zero and solve
$\frac{d^2y}{dx^2} = 6x - 14$,	M1	M1 for attempt to differentiate (or other valid method)
$x = \frac{2}{3}$ max, $x = 4$ min	A1	A1 correct from correct working for both
(ii) Use of $\frac{dy}{dt} = \frac{dy}{dz} \times \frac{dz}{dt}$, leading to	M1	M1 for attempt to use rates of change
$\frac{dy}{dt} = -\frac{5}{6}$ allow –0.833	A1	
(iii) Use of $\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$ leading to	M1	M1 for attempt to use rates of change
$\frac{dx}{dt} = \frac{5}{48}$	√A1 [10]	ft on $\frac{dy}{dt}$
12 OR		
(i) $2x^2y = 72$, $A = 4x^2 + 6xy$	B1M1	B1 for $2x^2y = 150$,
leading to given answer	A1	M1 for $A = 4x^2 + 6xy$
(ii) $\frac{dA}{dx} = 8x - \frac{216}{x^2}$	M1A1	M1 for attempt to differentiate
When $\frac{dA}{dx} = 0$, $x = \sqrt[3]{27} = 3$	M1	M1 for attempt to equate to zero and solve
Dimensions are 3 by 6 by 4	A1	A1 for dimensions
(iii) Use of $\partial A \approx \frac{dA}{dx} \times \partial x$ leading to	M1	M1 for attempt to use small changes
$\partial A = -38p$, decrease	A1√A1 [10]	A1 for $-38p$, √A1 on their ∂A