Cambridge Assessment International Education

# Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER



**FURTHER MATHEMATICS** **9231/13**

Paper 1 Further Pure Mathematics 1 **October/November 2024**

**2 hours**

You must answer on the question paper.

You will need: List of formulae (MF19)

**INSTRUCTIONS**

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

**INFORMATION**

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages.




**[Turn over**

**1** The matrix **M** represents the sequence of two transformations in the $x$-$y$ plane given by a stretch parallel to the $x$-axis, scale factor $k$ ($k \neq 0$), followed by a shear, $x$-axis fixed, with (0, 1) mapped to ($k$, 1).

**(a)** Show that $\mathbf{M} = \begin{pmatrix} k & k \\ 0 & 1 \end{pmatrix}$. [4]

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**(b)** The transformation represented by **M** has a line of invariant points.

Find, in terms of $k$, the equation of this line. [3]

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The unit square $S$ in the $x$-$y$ plane is transformed by **M** onto the parallelogram $P$.

**(c)** Find, in terms of $k$, a matrix which transforms $P$ onto $S$. [1]

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**(d)** Given that the area of $P$ is $3k^2$ units$^2$, find the possible values of $k$. [2]

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**2** Prove by mathematical induction that, for all positive integers $n$,

$$\frac{\mathrm{d}^n}{\mathrm{d}x^n}\left(\tan^{-1}x\right) = P_n(x)\left(1+x^2\right)^{-n},$$

where $P_n(x)$ is a polynomial of degree $n-1$. [6]

**3** The quartic equation $x^4 + 2x^3 - 1 = 0$ has roots $\alpha, \beta, \gamma, \delta$.

**(a)** Find a quartic equation whose roots are $\alpha^4, \beta^4, \gamma^4, \delta^4$ and state the value of $\alpha^4 + \beta^4 + \gamma^4 + \delta^4$. [5]

**(b)** Find the value of $\alpha^5 + \beta^5 + \gamma^5 + \delta^5$. [3]

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**(c)** Find the value of $\alpha^8 + \beta^8 + \gamma^8 + \delta^8$. [2]

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**4** **(a)** Use the method of differences to find $\sum_{r=1}^{n} \frac{5k}{(5r+k)(5r+5+k)}$ in terms of $n$ and $k$, where $k$ is a positive constant. [4]

It is given that $\sum_{r=1}^{\infty} \frac{5k}{(5r+k)(5r+5+k)} = \frac{1}{3}$.

**(b)** Find the value of $k$. [2]

**(c)** Hence find $\sum_{r=n}^{n^2} \frac{5k}{(5r+k)(5r+5+k)}$ in terms of $n$. [2]

**5** **(a)** Show that the curve with Cartesian equation

$$\left(x^2+y^2\right)^2 = 6xy$$

has polar equation $r^2 = 3\sin 2\theta$. [2]

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The curve $C$ has polar equation $r^2 = 3\sin 2\theta$, for $0 \leqslant \theta \leqslant \frac{1}{2}\pi$.

**(b)** Sketch $C$ and state the maximum distance of a point on $C$ from the pole. [3]

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**(c)** Find the area of the region enclosed by $C$. [2]

**(d)** Find the maximum distance of a point on $C$ from the initial line. [6]

**6** The curve $C$ has equation $y = \dfrac{4x^2 + x + 1}{2x^2 - 7x + 3}$.

**(a)** Find the equations of the asymptotes of $C$. [2]

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**(b)** Find the coordinates of any stationary points on $C$. [4]

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**(c)** Sketch $C$, stating the coordinates of any intersections with the axes. [5]

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**(d)** Sketch the curve with equation $y = \left|\dfrac{4x^2+x+1}{2x^2-7x+3}\right|$ and state the set of values of $k$ for which $\left|\dfrac{4x^2+x+1}{2x^2-7x+3}\right| = k$ has 4 distinct real solutions. [2]

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**7** The lines $l_1$ and $l_2$ have equations $\mathbf{r} = \mathbf{i} + 3\mathbf{j} - 2\mathbf{k} + \lambda(2\mathbf{i} + \mathbf{j} + \mathbf{k})$ and $\mathbf{r} = \mathbf{i} - 2\mathbf{j} + 9\mathbf{k} + \mu(\mathbf{i} - 4\mathbf{j} + 2\mathbf{k})$ respectively. The plane $\Pi_1$ contains $l_1$ and is parallel to $l_2$.

**(a)** Find the equation of $\Pi_1$, giving your answer in the form $ax + by + cz = d$. [4]

The plane $\Pi_2$ contains $l_2$ and the point with coordinates $(2, -1, 7)$.

**(b)** Find the acute angle between $\Pi_1$ and $\Pi_2$. [4]

The point $P$ on $l_1$ and the point $Q$ on $l_2$ are such that $PQ$ is perpendicular to both $l_1$ and $l_2$.

**(c)** Find a vector equation for $PQ$. [7]

## Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

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