



Cambridge International AS & A Level

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FURTHER MATHEMATICS

9231/11

Paper 1 Further Pure Mathematics 1

May/June 2020

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Blank pages are indicated.

1 Let a be a positive constant.

(a) Sketch the curve with equation $y = \frac{ax}{x+7}$.

[2]

- (b) Sketch the curve with equation $y = \left| \frac{ax}{x+7} \right|$ and find the set of values of x for which $\left| \frac{ax}{x+7} \right| > \frac{a}{2}$.
[4]

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2 The cubic equation $6x^3 + px^2 - 3x - 5 = 0$, where p is a constant, has roots α, β, γ .

(a) Find a cubic equation whose roots are $\alpha^2, \beta^2, \gamma^2$. [3]

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(b) It is given that $\alpha^2 + \beta^2 + \gamma^2 = 2(\alpha + \beta + \gamma)$.

(i) Find the value of p . [3]

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(ii) Find the value of $\alpha^3 + \beta^3 + \gamma^3$. [2]

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3 The curve C has equation $y = \frac{x^2}{2x+1}$.

(a) Find the equations of the asymptotes of C . [3]

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(b) Find the coordinates of the stationary points on C . [3]

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(c) Sketch *C*.

[3]

(b) Deduce the value of $\sum_{r=2}^{\infty} \frac{1}{r^2 - 1}$. [1]

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(c) Find the limit, as $n \rightarrow \infty$, of $\sum_{r=n+1}^{2n} \frac{n}{r^2 - 1}$. [4]

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5 The lines l_1 and l_2 have equations $\mathbf{r} = 3\mathbf{i} + 3\mathbf{k} + \lambda(\mathbf{i} + 4\mathbf{j} + 4\mathbf{k})$ and $\mathbf{r} = 3\mathbf{i} - 5\mathbf{j} - 6\mathbf{k} + \mu(5\mathbf{j} + 6\mathbf{k})$ respectively.

(a) Find the shortest distance between l_1 and l_2 . [5]

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The plane Π contains l_1 and is parallel to the vector $\mathbf{i} + \mathbf{k}$.

(b) Find the equation of Π , giving your answer in the form $ax + by + cz = d$. [4]

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(c) Find the acute angle between l_2 and Π . [3]

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6 Let $\mathbf{A} = \begin{pmatrix} 2 & 0 \\ 1 & 1 \end{pmatrix}$.

(a) The transformation in the x - y plane represented by \mathbf{A}^{-1} transforms a triangle of area 30 cm^2 into a triangle of area $d \text{ cm}^2$.

Find the value of d . [3]

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(b) Prove by mathematical induction that, for all positive integers n ,

$$\mathbf{A}^n = \begin{pmatrix} 2^n & 0 \\ 2^n - 1 & 1 \end{pmatrix}. \quad [5]$$

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7 The curve C_1 has polar equation $r = \theta \cos \theta$, for $0 \leq \theta \leq \frac{1}{2}\pi$.

(a) The point on C_1 furthest from the line $\theta = \frac{1}{2}\pi$ is denoted by P . Show that, at P ,

$$2\theta \tan \theta - 1 = 0$$

and verify that this equation has a root between 0.6 and 0.7. [5]

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The curve C_2 has polar equation $r = \theta \sin \theta$, for $0 \leq \theta \leq \frac{1}{2}\pi$. The curves C_1 and C_2 intersect at the pole, denoted by O , and at another point Q .

(b) Find the polar coordinates of Q , giving your answers in exact form. [2]

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(c) Sketch C_1 and C_2 on the same diagram.

[3]

(d) Find, in terms of π , the area of the region bounded by the arc OQ of C_1 and the arc OQ of C_2 . [7]

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Additional Page

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A series of horizontal dotted lines providing space for writing answers.

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