Cambridge Assessment International Education
Cambridge International Advanced Level

FURTHER MATHEMATICS **9231/23**

Paper 2 **May/June 2019**

MARK SCHEME

Maximum Mark: 100

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2019 series for most Cambridge IGCSE™, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

Abbreviations

AEF/OE Any Equivalent Form (of answer is equally acceptable) / Or Equivalent

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

CAO Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)

CWO Correct Working Only – often written by a ‘fortuitous’ answer

ISW Ignore Subsequent Working

SOI Seen or implied

SC Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Question	Answer	Marks	Guidance
1	Impulse = 0·2 × (250 – 40) = 42	**M1 A1**	Find impulse from change in momtm. (may be implied) (if sign of 40 wrong here or below, can allow M1)
	T = 42 / 1200 = 7/200 *or* 0·035 [s]	**M1 A1**	Find time T from impulse = Ft
	Alternative method for question 1		
	a = 1200 / 0·2 = 6000	**M1 A1**	Find acceleration a from $F = ma$ (may be implied)
	T = (250 – 40) / 6000 = 7/200 *or* 0·035 [s]	**M1 A1**	Find time T from $v = u + at$
	Alternative method for question 1		
	K.E. = ½ × 0·2 × ($250^2 - 40^2$) = 6090	**M1**	Find loss of K.E. (may be implied; ignore sign of K.E.)
	Work = 1200 × ½ (250 + 40) T = 174 000 T	**M1 A1**	Find work done from F × ½ ($u + v$) t (A1 for both)
	T = 6090 / 174 000 = 7/200 *or* 0·035 [s]	**A1**	Equate to find T
		4	

Question	Answer	Marks	Guidance
2(i)	$T_A = m\, v_A^2 / a - mg \cos \alpha$ $[= m\,(9ag/4) / a - mg \cos \alpha]$	B1	Find tension T_A at A from $F = ma$ radially
	$T_B = m\, v_B^2 / a + mg \cos \alpha$	B1	Find tension T_B at B from $F = ma$ radially
	$½mv_B^2 = ½m\, v_A^2 + 2mga \cos \alpha$ $[v_B^2 = ag\,(9/4 + 4 \cos \alpha)]$	M1 A1	Apply conservation of energy at B (A0 if no m)
	$v_B^2 = 4v_A^2 - 5ag \cos \alpha = v_A^2 + 4ga \cos \alpha$	M1	Combine using $T_B = 4\, T_A$ and $v_A^2 = 9ag/4$ to verify $\cos \alpha$
	$9ag \cos \alpha = 3v_A^2 = 27ag/4$, $\cos \alpha = 3/4$ **AG**	A1	
		6	
2(ii)	$T_B = 4\, T_A = (9 - 4 \cos \alpha)\, mg = 6mg$ *or* $v_B^2 = 21ag/4$, $T_B = (21/4 + \cos \alpha)\, mg = 6mg$	M1 A1	Find T_B
		2	

Question	Answer	Marks	Guidance
3(i)	$3mv_A + mv_B = 3mu$ [*or* $3v_A + v_B = 3u$] (AEF)	**M1**	Use conservation of momentum for A and B (correct masses)
	$v_B - v_A = eu$	**M1**	Use Newton's restitution law with consistent LHS signs
	$v_A = \frac{1}{4}(3 - e)u$ [$v_B = \frac{3}{4}(1 + e)u$]	**A1**	Combine to find speed of A
	$w_B + v_C = v_B$ *and* $v_C - w_B = ev_B$ (AEF)	**M1**	Use conservation of momentum for B and C *and* Newton's restitution law with consistent LHS signs
	$w_B = \frac{1}{2}(1 - e)v_B$ *and* $v_C = \frac{1}{2}(1 + e)v_B$ $w_B = (3/8)(1 - e^2)u$ *and* $v_C = (3/8)(1 + e)^2 u$ aef	**A1 A1**	Combine to find w_B *and* v_C in terms of v_B (may be implied) and with v_B replaced (note: $v_C = \frac{3}{8}e^2u + \frac{3}{4}eu + \frac{3}{8}u$)
		6	
3(ii)	$\frac{1}{4}(3 - e) = (3/8)(1 + e)^2$	**M1**	Find equation in e by equating their v_A and their v_C
	$3e^2 + 8e - 3 = 0 = (3e - 1)(e + 3)$	**M1**	Simplify and solve resulting quadratic eqn for e,
	$e = 1/3$	**A1**	(implicitly) rejecting $e = -3$
		3	

Question	Answer	Marks	Guidance
4(i)	$I_{AB} = \frac{1}{3}\, 5M\,(7a/2)^2 + 5M\,(7a/2)^2$ *or* $(4/3)\, 5M\,(7a/2)^2$ $[= (245/3)\, Ma^2]$	**B1**	Find or state MI of rod *AB* about axis at *A*
	$I_M = \frac{2}{3}\, M\,(2a)^2 + M\,(7a)^2$ $[= (155/3)\, Ma^2]$	**M1 A1**	M1 for one term correct, A1 for both terms correct
	$I_{kM} = \frac{2}{3}\, kM\, a^2 + kM\,(4a)^2$ $[= (50/3)\, kMa^2]$	**M1 A1**	M1 for one term correct, A1 for both terms correct
	$I = [\,(245/3 + 155/3 + 50k/3)\, Ma^2 =]\ (50/3)\,(8 + k)\, Ma^2$	**A1**	Find MI of object about axis at *A,* simplified to 2 terms aef
		6	
4(ii)	$\frac{1}{2} I\,\omega^2 = 5Mg\,(7a/2)\,(1 - \cos 60°) + Mg\,(7a)\,(1 - \cos 60°)$	***M1 A1**	Find eqn. for ω^2 when *AB* vertical by energy (3 terms on RHS, same trig expression in each for M1) A1 for 2 terms correct on RHS
	$+\, kMg\,(4a)\,(1 - \cos 60°\,)$	**A1**	A1 for RHS and LHS all correct
	$\frac{1}{4}\,(49 + 8k)\,/\,\frac{1}{2}\,(50/3)\,(8 + k) = 81/400$	**DM1**	Equate ω^2 to $81g\,/\,400a$ to find k
	$4\,(49 + 8k) = 27\,(8 + k)$, $k = 4$	**A1**	
	Alternative method for question 4(ii)		
	$6 + k)\, Mg \times \{(49 + 8k)/2(6 + k)\}\,(1 - \cos 60°)$ $= \frac{1}{2}\,(49 + 8k)\, Mga\,(1 - \cos 60°) = \frac{1}{4}\,(49 + 8k)\, Mga$	***M1 A1** **A1**	M1 for mass × com × trig expression, A1 for 2 parts correct A1 all correct
	$\frac{1}{4}\,(49 + 8k)\,/\,\frac{1}{2}\,(50/3)\,(8 + k) = 81/400$	**DM1**	Equate ω^2 to $81g\,/\,400a$ to find k
	$4\,(49 + 8k) = 27\,(8 + k)$, $k = 4$	**A1**	
		5	

Question	Answer	Marks	Guidance
5(i)	$R_B \cos\theta + F_B \sin\theta = W\sin\theta + \frac{1}{4} W \sin\theta$	*M1 A1	Resolve forces along rod AB ($\sin\theta = 3/5$, $\cos\theta = 4/5$)
	$F_B = \frac{1}{3} R_B$	B1	Relate F_B and R_B (may be implied)
	$(4/5 + \frac{1}{3} \times 3/5)\, R_B = (1 + \frac{1}{4})\,(3/5)\, W$, $R_B = \frac{3}{4} W$ **AG**	DM1 A1	Combine using $\tan\theta = \frac{3}{4}$ to verify R_A
		5	
5(ii)	$R_C = F_B \cos\theta - R_B \sin\theta + W\cos\theta + \frac{1}{4} W \cos\theta$	*M1 A1	Resolve forces perpendicular to rod AB, where C denotes rim Substitute to find R_C
	$= (\frac{1}{3} \times \frac{3}{4} \times 4/5)\, W - (\frac{3}{4} \times 3/5)\, W + (5/4 \times 4/5)\, W$	DM1	
	$= (1/5 - 9/20 + 1)\, W = \frac{3}{4} W$	A1	
		4	
5(iii)	A: $R_C \times AC + (R_B \sin\theta - F_B \cos\theta) \times 2x - W\cos\theta \times x$ $[= W(\frac{3}{4} \times 2x - \frac{3}{4} \times 8a/5 + 9/20 \times 2x - 1/5 \times 2x - 4x/5)]$ C: $(R_B \sin\theta - F_B \cos\theta) \times 2a\cos\theta - W\cos\theta \times CG + \frac{1}{4}W\cos\theta \times AC$ $[= W(9/20 \times 2a - 1/5 \times 2a - 8a/5 + x + \frac{1}{4} \times 2x - \frac{1}{4} \times 8a/5)\cos\theta]$ G: $R_C \times CG - (R_B \sin\theta - F_B \cos\theta) \times x - \frac{1}{4} W\cos\theta \times x$ $[= W(\frac{3}{4} \times 8a/5 - \frac{3}{4} \times x - 9/20 \times x + 1/5 \times x - 1/5 \times x)]$ B: $R_C \times 2a\cos\theta - W\cos\theta \times x - \frac{1}{4}W\cos\theta \times 2x$ $[= W(\frac{3}{4} \times 2a - x - \frac{1}{4} \times 2x)\cos\theta]$	M1 A1	Find total moments about any point, denoting rod's centre by G (taken on either side of C) where $AC = 2x - 2a\cos\theta = 2x - 8a/5$ $CG = x - AC = 2a\cos\theta - x = 8a/5 - x$
	$x = a$	A1	Substitute and equate to zero to find x
		3	

Question	Answer	Marks	Guidance
6(i)	$f(t) = (1/400)\exp(-t/400)$ *or* $0{\cdot}0025\exp(-0{\cdot}0025t)$ [$= 0\ (t < 0)$]	**B1**	State probability density function f(t) for $t \geqslant 0$
		1	
6(ii)	$P(T < 500) = \int_0^{500} f(t)\,dt = [\exp(-t/400)]_0^{500}$ $= 1 - e^{-5/4} = 0{\cdot}7135$ *or* $0{\cdot}713$ (allow 0·714)	**M1 A1**	Find $P(T < 500)$: (M0 for $1 - F(500) = 0.287$)
		2	
6(iii)	$1 - e^{-m/400}$ *or* $e^{-m/400} = ½$, $e^{m/400} = 2$	**M1**	Find median value m from $F(m) = ½$
	$m = 400 \ln 2$ *or* 277	**M1 A1**	
		3	

Question	Answer	Marks	Guidance
7(i)	$\mathrm{E}(X) = 1 / (½ \times ½) = 4$	**B1**	Find or state $\mathrm{E}(X)$
		1	
7(ii)	$\mathrm{P}(X = 3) = q^2 p$ (with $p = ¼$, $q = ¾$) $= 9/64$ *or* $0{\cdot}141$	**M1 A1**	Find prob. of exactly 3 throws needed
		2	
7(iii)	$\mathrm{P}(X < 4) = 1 - q^3 = 37/64$ *or* $0{\cdot}578$	**M1 A1**	Find prob. of fewer than 4 throws needed
		2	
7(iv)	$1 - q^{N-1} > 0{\cdot}95$ (AEF)	**M1**	Formulate condition for N ($1 - q^N$ is M0)
	$0{\cdot}05 > (3/4)^{N-1}$, $N - 1 > \log 0{\cdot}05 / \log 0{\cdot}75$	**M1**	Set $q = 3/4$, rearrange and take logs (any base) to give bound
	$N - 1 > 10{\cdot}4$, $N_{\min} = 12$	**A1**	Find $N_{\max}$ (< or = can earn M1 M1 A0, max 2/3)
		3	

Question	Answer	Marks	Guidance
8	H_0: Holidays are independent of salesman *or* no association between holidays and salesman (AEF)	**B1**	State (at least) null hypothesis Find expected values (lose A1 if rounded to integers) Find value of X^2 from $\Sigma\,(E_i - O_i)^2 / E_i$ [or $\Sigma\,O_i^2/E_i - n$] State or use correct tabular χ^2 value
	E_i: 29·68 33·04 21·28 23·32 25·96 16·72 (to 1 d.p.)	**M1 A1**	
	X^2 = 0·7380 + 0·7446 + 0·0037 + 0·9392 + 0·9477 + 0·0047	**M1**	
	= 3·38 (to 3 s.f.)	**A1**	
	$\chi_{2,\,0.9}{}^2$ = 4·605 (to 3 s.f.)	**B1**	
	Accept H_0 if X^2 < tabular value	**M1**	Compare their calculated value with their X^2 value and appropriate conclusion
	Type of holidays is independent of salesman (AEF)	**A1**	Correct conclusion, from correct values $(3.37 - 3.39)$
		8	

Question	Answer	Marks	Guidance
9(i)	$\bar{x} = 0{\cdot}785$	**B1**	Find sample mean
	$s^2 = (6{\cdot}19 - 7{\cdot}85^2/10) / 9$ $[= 37/12\,000$ *or* $0{\cdot}003083$ *or* $0{\cdot}05553^2]$	**M1**	Estimate population variance (allow biased here: $0{\cdot}002775$ *or* $0{\cdot}05268^2$)
	H_0: $\mu = 0{\cdot}75$, H_1: $\mu > 0{\cdot}75$ (AEF)	**B1**	State hypotheses (B0 for $\bar{x}$ ...)
	$t_{9,\,0.95} = 1{\cdot}83[3]$	**B1**	State or use correct tabular t-value
	$t = (\bar{x} - 0{\cdot}75)/(s/\sqrt{10}) = 1{\cdot}99$ [Reject H_0 and accept H_1]	**M1 A1**	Find value of t (or compare $\bar{x}$ with $0{\cdot}75 + 0{\cdot}032 = 0{\cdot}782$) Consistent conclusion
	Claim (of yield per plant increased) is justified (AEF)	**B1**	**FT** on both t-values (must be t value)
		7	
9(ii)	$\bar{x} \pm t\sqrt{(s^2/10)}$	**M1**	Find confidence interval (must be a t value)
	$t_{9,\,0.975} = 2{\cdot}26[2]$	**B1**	State or use correct tabular value of t
	$0{\cdot}785 \pm 0{\cdot}04$ *or* $[0{\cdot}745, 0{\cdot}825]$	**A1**	Evaluate confidence interval (either form)
		3	

Question	Answer	Marks	Guidance
10(i)	$b \quad = S_{xy} / S_{xx}$ and $r = S_{xy} / \sqrt{(S_{xx} S_{yy})}$ so	**M1**	Relate gradient b in $y = bx + c$ to r, S_{xx} , S_{yy}
	$b \quad = r \sqrt{(S_{yy} / S_{xx})}$	**A1**	
	$= 0{\cdot}5815 \sqrt{(7{\cdot}9473 / 3{\cdot}3086)} = 0{\cdot}901[2]$	**M1 A1**	Find S_{xx} and S_{yy} and hence find b to 3 s.f.
	$(y - 6{\cdot}7375) = b\,(x - 3{\cdot}3125)$, $y = 0{\cdot}901x + 3{\cdot}75$	**M1 A1**	Find equation of regression line of y on x
	Alternative method for question 10(i)		
	$S_{xy} \quad = r \sqrt{(S_{xx} S_{yy})} = r \sqrt{\{(8s_x^2)\,(8s_y^2)\}}$ $= 8 \times 0{\cdot}5815 \sqrt{(7{\cdot}9473 \times 3{\cdot}3086)}$	**M1**	Find S_{xy} (allow values consistently scaled by factor 8)
	$= 23{\cdot}855$ (allow $2{\cdot}9818$) (to 4 s.f.)	**A1**	
	$b \quad = S_{xy} / S_{xx} = 23{\cdot}85 / (8 \times 3{\cdot}3086) = 0{\cdot}901[2]$	**M1 A1**	Hence find b to 3 s.f.
	$(y - 6{\cdot}7375) = b\,(x - 3{\cdot}3125)$, $y = 0{\cdot}901x + 3{\cdot}75$	**M1 A1**	Find equation of regression line of y on x
		6	

Question	Answer	Marks	Guidance
10(ii)	H$_0$: $\rho = 0$, H$_1$: $\rho > 0$	**B1**	State both hypotheses (B0 for r …)
	$r_{8,\,5\%} = 0{\cdot}621$	**B1**	State or use correct tabular one-tail r-value
	Accept H$_0$ if $0{\cdot}5815 <$ tab. r-value (AEF)	**M1**	State or imply valid method for conclusion
	No evidence of positive correlation (AEF)	**A1**	Correct conclusion
	Alternative method for question 10(ii)		
	H$_0$: $\rho = 0$, H$_1$: $\rho > 0$	**B1**	State both hypotheses (B0 for r …)
	$t_r = r\sqrt{((n\text{-}2) / (1 - r^2))} = 1{\cdot}75$, $t_{6,0{\cdot}95} = 1{\cdot}943$	**B1**	
	Accept H$_0$ if $\lvert t_r \rvert <$ tab. t-value (AEF)	**M1**	State or imply valid method for conclusion
	No evidence of positive correlation (AEF)	**A1**	Correct conclusion
		4	
10(iii)	$y = 9{\cdot}16$	**B1**	Find y when $x = 6{\cdot}0$
	Unreliable since r is small *or* r is not close to 1 *or* no correlation *or* $6{\cdot}0$ is not close to mean of x (AEF)	**B1**	Reasonable comment on reliability
		2	

Question	Answer	Marks	Guidance
11E(i)	$\frac{1}{2} mu^2 = \frac{1}{2} kmg (\frac{1}{4} a)^2/a$, $k = 16u^2/ag$ **AG**	**M1 A1**	Verify k by using conservation of energy (M0 if ω is found and SHM formula $v = a\omega$ then used)
		2	
11E(ii)	$\pm\, m\, \mathrm{d}^2x/\mathrm{d}t^2 = kmg\, x/a$	***M1**	Apply Newton's law at general point (e.g. $OP = a + x$), requires m
	$\mathrm{d}^2x/\mathrm{d}t^2 = -(kg/a)\, x$ *or* $-(16u^2 / a^2)\, x$ $[\omega = \sqrt{(kg/a)}$ *or* $4u/a]$	**A1**	Derive standard SHM form (requires minus sign)
	Period is $2\pi\sqrt{(a / kg)} = 2\pi\sqrt{(a^2 / 16u^2)} = \pi a / 2u$	**DM1 A1**	Find period from $2\pi/\omega$
		4	
11E(iii)	$[\frac{1}{2} mv^2 = \frac{3}{4} \times \frac{1}{2} mu^2]$ $v^2 = \frac{3}{4} u^2$ *or* $v = (\sqrt{3}/2)\, u$	**M1**	Relate v and u using given loss in energy
	$x = (\frac{1}{4} a) \sin \omega t$, $v = (\frac{1}{4} a)\, \omega \cos \omega t$ *or* $u \cos \omega t$	**M1**	Relate v and t
	$\cos \omega t = (\sqrt{3}/2)\, u / (\frac{1}{4} a)\, \omega = (\sqrt{3}/2)$	**M1 A1**	Combine to find $\cos \omega t$
	$t = (\pi/6) / (4u/a) = \pi a / 24u$ *or* $0{\cdot}0417\, \pi a / u$ *or* $0{\cdot}131\, a/u$	**M1 A1**	and hence t
		6	

Question	Answer	Marks	Guidance
11O	$\bar{x} - \bar{y} = 0{\cdot}448 - 0{\cdot}48 = [-]\ 0{\cdot}032$	**B1**	Find difference in sample means (either sign; may be implied)
	$s_X^2 = (10{\cdot}1 - 22{\cdot}4^2/50) / 49 = 0{\cdot}0648 / 49$ [$= 0{\cdot}001322$]	**M1 A1**	Estimate both population variances (may be implied)
	$s_Y^2 = (16{\cdot}3 - 28{\cdot}8^2/60) / 59 = 2{\cdot}476 / 59$ [$= 0{\cdot}04197$]	**A1**	(allow biased here: 0·001296 *and* 0·04127)
	$s_C^2 = s_X^2/50 + s_Y^2/60$	**M1**	Estimate combined variance
	$= 0{\cdot}0007259$ *or* $0{\cdot}02694^2$ (to 3 s.f. throughout)	**A1**	
	$z = 0{\cdot}032 / s_C$ [or $0{\cdot}032 / s_P \sqrt{(1/50 + 1/60)}$] $= 1{\cdot}188$ [or $1{\cdot}0895$]	**M1 A1**	Find value of z (either sign)
	$\Phi(z) = 0{\cdot}8826$ [or $0{\cdot}8620$]	**A1**	Find $\Phi(z)$
	$100 \times (1 - \Phi(z)) = 11{\cdot}7$ [or $13{\cdot}8$]	**M1 A1**	Find limiting value for α, based on one-tail test (M0 for basing on two-tail test)
	$\alpha >$ (*or* $\geqslant$) $11{\cdot}7$ [or $13{\cdot}8$]	**A1**	Find set of possible values of α Allow 11.8 if 11.74 seen (Misreading α% as α loses only last A1)

Question	Answer	Marks	Guidance
11O	**Alternative method for question 11O**		
	$\overline{x} - \overline{y} = 0{\cdot}448 - 0{\cdot}48 = [-]\ 0{\cdot}032$	B1	Find difference in sample means (either sign; may be implied)
	$s_X^2 = (10{\cdot}1 - 22{\cdot}4^2/50) / 49 = 0{\cdot}0648 / 49$ [= 0·001322]	M1 A1	Estimate both population variances (may be implied)
	$s_Y^2 = (16{\cdot}3 - 28{\cdot}8^2/60) / 59 = 2{\cdot}476 / 59$ [= 0·04197]	A1	(allow biased here: 0·001296 *and* 0·04127)
	Assume equal [population] variances	B1	State assumption
	$s_P^2 = (49\, s_X^2 + 59\, s_Y^2) / 108$ *or* $(0{\cdot}0648 + 2{\cdot}476) / 108$ $= 0{\cdot}02353$ *or* $0{\cdot}1534^2$	B1	(Find pooled estimate of common variance s_X^2 and s_Y^2 not needed explicitly so may be implied by result)
	$z = 0{\cdot}032 / s_C$ [or $0{\cdot}032 / s_P \sqrt{(1/50 + 1/60)}$] $= 1{\cdot}188$ [or 1·0895]	M1 A1	Find value of z (either sign)
	$\Phi(z) = 0{\cdot}8826$ [or 0·8620]	A1	Find $\Phi(z)$
	$100 \times (1 - \Phi(z)) = 11{\cdot}7$ [or 13·8]	M1 A1	Find limiting value for α, based on one-tail test (M0 for basing on two-tail test)
	$\alpha >$ (*or* $\geqslant$) 11·7 [or 13·8]	A1	Find set of possible values of α Allow 11.8 if 11.74 seen (Misreading α% as α loses only last A1)
		12	