Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

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FURTHER MATHEMATICS **9231/12**

Paper 1 **May/June 2017**

3 hours

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF10)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a calculator is expected, where appropriate.
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

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1 It is given that $\sum_{r=1}^{n} u_r = n^2(2n+3)$, where n is a positive integer.

(i) Find $\sum_{r=n+1}^{2n} u_r$. [2]

(ii) Find u_r. [3]

2 Prove, by mathematical induction, that $5^n + 3$ is divisible by 4 for all non-negative integers n. [5]

3 A curve C has equation $\tan y = x$, for $x > 0$.

(i) Use implicit differentiation to show that

$$\frac{d^2y}{dx^2} = -2x\left(\frac{dy}{dx}\right)^2.$$ [3]

...

(ii) Hence find the value of $\frac{d^2y}{dx^2}$ at the point $\left(1, \frac{1}{4}\pi\right)$ on C. [2]

...

4 **(i)** Find the value of k for which the set of linear equations

$$\begin{aligned} x + 3y + \quad kz &= 4, \\ 4x - 2y - 10z &= -5, \\ x + \quad y + \quad 2z &= 1, \end{aligned}$$

has no unique solution. [3]

(ii) For this value of k, find the set of possible solutions, giving your answer in the form

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \mathbf{a} + t\mathbf{b},$$

where **a** and **b** are vectors and t is a scalar. [3]

5 The matrix **A**, given by

$$\mathbf{A} = \begin{pmatrix} 1 & 2 & -2 \\ 6 & 4 & -6 \\ 6 & 5 & -7 \end{pmatrix},$$

has eigenvalues 1, −1 and −2.

(i) Find a set of corresponding eigenvectors. [4]

(ii) The matrix **B** is given by $\mathbf{B} = \mathbf{A} - 2\mathbf{I}$, where **I** is the 3×3 identity matrix. Write down the eigenvalues of **B**, and state a set of corresponding eigenvectors. [2]

6 Let $I_n = \int_0^{\frac{1}{2}\pi} x^n \sin x \, dx$.

(i) Prove that, for $n \geqslant 2$,

$$I_n + n(n-1)I_{n-2} = n\left(\tfrac{1}{2}\pi\right)^{n-1}.$$ [4]

(ii) Calculate the exact value of I_1 and deduce the exact value of I_3. [3]

7 By finding a cubic equation whose roots are α, β and γ, solve the set of simultaneous equations

$$\alpha + \beta + \gamma = -1,$$
$$\alpha^2 + \beta^2 + \gamma^2 = 29,$$
$$\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = -1.$$

[8]

8 **(i)** Let $z = \cos\theta + \mathrm{i}\sin\theta$. Show that $z - \dfrac{1}{z} = 2\mathrm{i}\sin\theta$ and hence express $16\sin^5\theta$ in the form $\sin 5\theta + p\sin 3\theta + q\sin\theta$, where p and q are integers to be determined. [6]

(ii) Hence find the exact value of $\int_0^{\frac{1}{3}\pi} 16\sin^5\theta\,d\theta$. [3]

9 The curve C has equation $y = \dfrac{x^2 - 3x + 6}{1 - x}$.

(i) Find the equations of the asymptotes of C. [3]

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(ii) Find the coordinates of the turning points of C. [3]

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(iii) Find the coordinates of any intersections with the coordinate axes. [2]

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(iv) Sketch C. [3]

10 It is given that $x = t^{\frac{1}{2}}$, where $x > 0$ and $t > 0$, and y is a function of x.

(i) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 2t^{\frac{1}{2}}\dfrac{\mathrm{d}y}{\mathrm{d}t}$ and $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} = 2\dfrac{\mathrm{d}y}{\mathrm{d}t} + 4t\dfrac{\mathrm{d}^2y}{\mathrm{d}t^2}$. [3]

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(ii) Hence show that the differential equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - \left(8x + \frac{1}{x}\right)\frac{\mathrm{d}y}{\mathrm{d}x} + 12x^2y = 4x^2\mathrm{e}^{-x^2} \qquad (*)$$

reduces to the differential equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}t^2} - 4\frac{\mathrm{d}y}{\mathrm{d}t} + 3y = \mathrm{e}^{-t}.$$ [1]

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(iii) Find the general solution of $(*)$, giving y in terms of x. [7]

11 The curve C has polar equation $r = a(1 + \sin\theta)$ for $-\pi < \theta \leqslant \pi$, where a is a positive constant.

(i) Sketch C. [2]

(ii) Find the area of the region enclosed by C. [4]

(iii) Show that the length of the arc of C from the pole to the point furthest from the pole is given by

$$s = (\sqrt{2})a \int_{-\frac{1}{2}\pi}^{\frac{1}{2}\pi} \sqrt{(1 + \sin\theta)}\, \mathrm{d}\theta. \qquad [3]$$

(iv) Show that the substitution $u = 1 + \sin\theta$ reduces this integral for s to $(\sqrt{2})a \int_0^2 \frac{1}{\sqrt{(2-u)}}\, du$. Hence evaluate s. [4]

12 Answer only **one** of the following two alternatives.

EITHER

The curve C has equation $y = \frac{1}{2}\left(e^{x} + e^{-x}\right)$ for $0 \leqslant x \leqslant 4$.

(i) The region R is bounded by C, the x-axis, the y-axis and the line $x = 4$. Find, in terms of e, the coordinates of the centroid of the region R. [10]

(ii) Show that $\frac{\mathrm{d}s}{\mathrm{d}x} = \frac{1}{2}\left(\mathrm{e}^{x} + \mathrm{e}^{-x}\right)$, where s denotes the arc length of C, and find the surface area generated when C is rotated through 2π radians about the x-axis. [4]

OR

The position vectors of the points A, B, C, D are

$$\mathbf{i} + \mathbf{j} + 3\mathbf{k}, \quad 3\mathbf{i} - \mathbf{j} + 5\mathbf{k}, \quad 3\mathbf{i} - \mathbf{j} + \mathbf{k}, \quad 5\mathbf{i} - 5\mathbf{j} + \alpha\mathbf{k},$$

respectively, where α is a positive integer. It is given that the shortest distance between the line AB and the line CD is equal to $2\sqrt{2}$.

(i) Show that the possible values of α are 3 and 5. [7]

(ii) Using $\alpha = 3$, find the shortest distance of the point D from the line AC, giving your answer correct to 3 significant figures. [3]

(iii) Using $\alpha = 3$, find the acute angle between the planes ABC and ABD, giving your answer in degrees. [4]