Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

FURTHER MATHEMATICS **9231/13**

Paper 1 **October/November 2016**

3 hours

Additional Materials: List of Formulae (MF10)

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READ THESE INSTRUCTIONS FIRST

An answer booklet is provided inside this question paper. You should follow the instructions on the front cover of the answer booklet. If you need additional answer paper ask the invigilator for a continuation booklet.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a calculator is expected, where appropriate.
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

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1 Use the method of differences to find $\sum_{r=1}^{n} \frac{1}{(2r)^2 - 1}$. [4]

Deduce the value of $\sum_{r=1}^{\infty} \frac{1}{(2r)^2 - 1}$. [1]

2 Find the cubic equation with roots α, β and γ such that

$$\alpha + \beta + \gamma = 3,$$
$$\alpha^2 + \beta^2 + \gamma^2 = 1,$$
$$\alpha^3 + \beta^3 + \gamma^3 = -30,$$

giving your answer in the form $x^3 + px^2 + qx + r = 0$, where p, q and r are integers to be found. [6]

3 Find a matrix **A** whose eigenvalues are -1, 1, 2 and for which corresponding eigenvectors are

$$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix},$$

respectively. [7]

4 Using factorials, show that $\binom{n}{r-1} + \binom{n}{r} = \binom{n+1}{r}$. [2]

Hence prove by mathematical induction that

$$(a + x)^n = \binom{n}{0} a^n + \binom{n}{1} a^{n-1} x + \ldots + \binom{n}{r} a^{n-r} x^r + \ldots + \binom{n}{n} x^n$$

for every positive integer n. [4]

5 The linear transformation $\mathrm{T} : \mathbb{R}^4 \to \mathbb{R}^4$ is represented by the matrix **A**, where

$$\mathbf{A} = \begin{pmatrix} 1 & 3 & 5 & 7 \\ 2 & 8 & 7 & 9 \\ 3 & 13 & 9 & 11 \\ 6 & 24 & 21 & 27 \end{pmatrix}.$$

Find

(i) the rank of **A**, [3]

(ii) a basis for the range space of T, [1]

(iii) a basis for the null space of T. [4]

6 Find the general solution of the differential equation

$$\frac{d^2x}{dt^2} + 7\frac{dx}{dt} + 10x = 116\sin 2t. \qquad [8]$$

State an approximate solution for large positive values of t. [1]

7 The curve C has equation $y = e^{-2x}$. Find, giving your answers correct to 3 significant figures,

(i) the mean value of $\frac{dy}{dx}$ over the interval $0 \leqslant x \leqslant 2$, [2]

(ii) the coordinates of the centroid of the region bounded by C, $x = 0$, $x = 2$ and $y = 0$. [9]

8 A curve C has equation $x^2 + 4xy - y^2 + 20 = 0$. Show that, at stationary points on C, $x = -2y$. [3]

Find the coordinates of the stationary points on C, and determine their nature by considering the value of $\frac{d^2y}{dx^2}$ at the stationary points. [8]

9 Evaluate $\int_0^{\frac{1}{2}\pi} x\sin x\,dx$. [2]

Given that $I_n = \int_0^{\frac{1}{2}\pi} x^n \sin x\,dx$, prove that, for $n > 1$,

$$I_n = n\left(\tfrac{1}{2}\pi\right)^{n-1} - n(n-1)I_{n-2}. \qquad [4]$$

By first using the substitution $x = \cos^{-1} u$, find the value of

$$\int_0^1 \left(\cos^{-1} u\right)^3 du,$$

giving your answer in an exact form. [5]

10 Let $z = \cos\theta + i\sin\theta$. Show that

$$z^n + \frac{1}{z^n} = 2\cos n\theta \qquad \text{and} \qquad z^n - \frac{1}{z^n} = 2i\sin n\theta. \qquad [2]$$

By considering $\left(z - \frac{1}{z}\right)^4 \left(z + \frac{1}{z}\right)^2$, show that

$$\sin^4\theta\cos^2\theta = \tfrac{1}{32}(\cos 6\theta - 2\cos 4\theta - \cos 2\theta + 2). \qquad [7]$$

Hence find the exact value of $\int_0^{\frac{1}{4}\pi} \sin^4\theta\cos^2\theta\,d\theta$. [3]

[Question 11 is printed on the next page.]

11 Answer only **one** of the following two alternatives.

EITHER

The lines l_1 and l_2 have equations

$$\mathbf{r} = 6\mathbf{i} - 3\mathbf{j} + s(3\mathbf{i} - 4\mathbf{j} - 2\mathbf{k}) \quad \text{and} \quad \mathbf{r} = 2\mathbf{i} - \mathbf{j} - 4\mathbf{k} + t(\mathbf{i} - 3\mathbf{j} - \mathbf{k})$$

respectively. The point P on l_1 and the point Q on l_2 are such that PQ is perpendicular to both l_1 and l_2. Show that the position vector of P is $3\mathbf{i} + \mathbf{j} + 2\mathbf{k}$ and find the position vector of Q. [7]

Find, in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$, an equation of the plane Π which passes through P and is perpendicular to l_1. [3]

The plane Π meets the plane $\mathbf{r} = p\mathbf{i} + q\mathbf{j}$ in the line l_3. Find a vector equation of l_3. [4]

OR

A curve C has parametric equations

$$x = 1 - 3t^2, \quad y = t(1 - 3t^2), \quad \text{for } 0 \leqslant t \leqslant \frac{1}{\sqrt{3}}.$$

Show that $\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = \left(1 + 9t^2\right)^2$. [2]

Hence find

(i) the arc length of C, [2]

(ii) the surface area generated when C is rotated through 2π radians about the x-axis. [3]

Use the fact that $t = \frac{y}{x}$ to find a cartesian equation of C. Hence show that the polar equation of C is $r = \sec\theta(1 - 3\tan^2\theta)$, and state the domain of θ. [4]

Find the area of the region enclosed between C and the initial line. [3]