Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

FURTHER MATHEMATICS **9231/23**

Paper 2 **May/June 2016**

3 hours

Additional Materials: List of Formulae (MF10)

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READ THESE INSTRUCTIONS FIRST

An answer booklet is provided inside this question paper. You should follow the instructions on the front cover of the answer booklet. If you need additional answer paper ask the invigilator for a continuation booklet.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
Where a numerical value is necessary, take the acceleration due to gravity to be $10\,\text{m}\,\text{s}^{-2}$.
The use of a calculator is expected, where appropriate.
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **5** printed pages, **3** blank pages and **1** insert.

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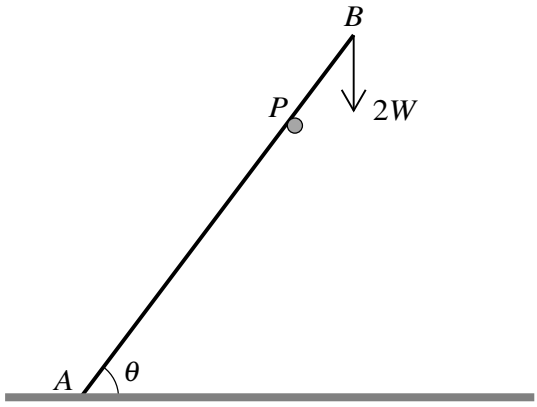
1 A particle P is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. The particle moves in complete vertical circles with centre O. The tension in the string when P is at its lowest point is twice the tension in the string when P is at its highest point. Find, in terms of a and g, the greatest speed of P during the motion. [6]

2



A uniform rod AB of length $2a$ and weight W rests with its lower end A in contact with a rough horizontal surface. The rod rests on a smooth peg at the point P. A particle of weight $2W$ is suspended from the end B of the rod. The system is in limiting equilibrium with the angle between the rod and the horizontal surface equal to θ, where $\tan\theta = \frac{4}{3}$ (see diagram). The coefficient of friction between the rod and the surface is $\frac{2}{3}$. Find the distance AP. [8]

3 A particle P of mass 0.2 kg is moving in a circle of radius 0.5 m. At time t s its velocity is $(t^2 + ct + d)\,\mathrm{m\,s^{-1}}$, where c and d are constants. The magnitude of the resultant force acting on P when $t = 3$ is $\frac{2}{5}\sqrt{(17)}$ N. The transverse component of the acceleration of P when $t = 3$ is $2\,\mathrm{m\,s^{-2}}$. Find the values of c and d. [8]

4 An object consists of a uniform rod AB, of mass $4m$ and length $6a$, and a uniform disc, of mass m and radius $2a$ and with centre O. The end B of the rod is rigidly attached to a point on the circumference of the disc, so that ABO is a straight line. The object is free to rotate in a vertical plane about a fixed smooth horizontal axis through A, perpendicular to the plane of the disc. Show that the moment of inertia of the object about this axis is $114ma^2$. [4]

The object is held with AB at an angle of 60° with the downward vertical at A, and released from rest. Find, in terms of a and g, the speed of the point B when AB is vertical. [5]

5 Three small smooth spheres A, B and C, of masses $5m$, m and km respectively, are at rest on a smooth horizontal surface. The spheres lie in a straight line with B between A and C. Sphere A is projected directly towards B with speed u. The coefficient of restitution between any pair of the spheres A, B and C is $\frac{4}{5}$. Show that the speed of A after the first collision is $\frac{7}{10}u$ and find the speed of B. [3]

Given that the speeds of A and C are equal after the second collision,

(i) show that the value of k is $\frac{20}{7}$, [4]

(ii) find the percentage loss in the kinetic energy of the system as a result of the two collisions. [5]

6 Pia is practising for a high jump competition. In order to qualify for the competition, she needs to succeed in jumping a certain height. At each attempt the probability that she succeeds is $\frac{1}{4}$. The number of attempts that Pia makes, up to and including her first success, is denoted by the random variable N. State the mean and variance of N. [2]

Find the probability that Pia

(i) succeeds in jumping the qualifying height in at most 4 attempts, [2]

(ii) takes more than 6 attempts to succeed in jumping the qualifying height. [1]

7 The lifetime, in months, of a particular type of light bulb is a random variable T and its probability density function, f, is given by

$$\mathrm{f}(t) = \begin{cases} \lambda \mathrm{e}^{-\lambda t} & t \geqslant 0, \\ 0 & \text{otherwise,} \end{cases}$$

where λ is a positive constant. The manager of a shopping mall uses this type of light bulb and he notes that, out of 1000 new light bulbs, 280 were still working after 12 months.

(i) Find an estimate for the value of λ. [3]

(ii) Hence estimate the mean of T. [1]

An improved version of the light bulb is produced. The lifetime of the improved version also has a negative exponential distribution, with a mean 25% larger than the original mean.

(iii) What percentage of the improved version of the light bulb fail within the first 12 months? [4]

8 The coach of a national athletics team carried out an investigation into the effect of high altitude training on the times of 400-metre runners. The times, in seconds, were recorded before and after a six-week period of high altitude training, for a random sample of 8 athletes. The results are given in the following table.

Athlete	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*
Before	52.3	56.2	54.3	49.2	48.4	50.1	46.8	51.1
After	51.6	53.2	54.9	49.0	48.1	49.2	46.8	49.6

Stating any assumption that you make, test, at the 2.5% significance level, whether there is an improvement in athletes' times after high altitude training. [8]

9 A company produces four different flavours of ice cream: mint, strawberry, chocolate and fudge. Customers were asked which of the four flavours they preferred. The responses from a random sample of 80 male customers and 70 female customers are given in the following table.

	Mint	Strawberry	Chocolate	Fudge
Male	25	12	30	13
Female	16	22	25	7

Test, at the 10% significance level, whether there is a difference between ice cream flavour preferences of male and female customers. [9]

[Turn over

10 A Physics teacher takes a random sample of size 5 from his students. Their marks in a written test (x) and a practical test (y) are given in the following table.

Student	*A*	*B*	*C*	*D*	*E*
x	11	14	16	12	15
y	9	12	14	13	15

(i) Find the product moment correlation coefficient. [3]

(ii) Test, at the 5% significance level, whether there is evidence of positive correlation. [4]

The teacher takes a second random sample, this time of size 7, and the marks for this sample are summarised as follows.

$$\Sigma x = 96 \quad \Sigma x^2 = 1366 \quad \Sigma y = 86 \quad \Sigma y^2 = 1096 \quad \Sigma xy = 1220$$

(iii) Use the combined sample of size 12 to find the equation of the regression line of y on x, giving your answer in the form $y = px + q$, where p and q are constants. [4]

One particular student scored 13 in the written test, but was absent for the practical test.

(iv) Estimate this student's mark in the practical test and comment on the reliability of your estimate. [2]

11 Answer only **one** of the following two alternatives.

EITHER

The fixed points A and B are such that $AB = 3.2$ m and A is vertically above B. One end of a light elastic string, of natural length 0.8 m and modulus of elasticity 8 N, is attached to a particle P of mass 0.2 kg. The other end of the string is attached to A. One end of a second light elastic string, of natural length 1.2 m and modulus of elasticity 4 N, is attached to P and the other end is attached to B.

(i) Find the length of AP when the particle is in equilibrium. [4]

The particle is now pulled vertically down from its equilibrium position and released from rest at the point where $AP = 1.65$ m.

(ii) Show that P performs simple harmonic motion and find the period of the motion. [6]

(iii) Find the speed of P when it is at the mid-point of AB. Give your answer correct to 3 significant figures. [4]

OR

The annual salaries of workers at two factories, A and B, are to be compared. The salaries, in tens of thousands of dollars, at A and B, are denoted by x and y respectively. For a random sample of 40 workers in factory A and a random sample of 50 workers in factory B the results are as follows.

$$\Sigma x = 256.0 \qquad \Sigma x^2 = 1910.8 \qquad \Sigma y = 382.9 \qquad \Sigma y^2 = 3148.8$$

The population mean salaries for A and B are denoted by μ_A and μ_B respectively. The population variances for salaries at A and B cannot be assumed to be equal. Test, at the 1% significance level, whether μ_B is greater than μ_A. [10]

The width of an α% confidence interval for $\mu_B - \mu_A$ is found to be 1.82. Find the value of α. [4]

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