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Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

1774762721

MATHEMATICS **9709/23**

Paper 2 Pure Mathematics 2 **October/November 2024**

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

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1 The variables x and y satisfy the equation $a^{2y} = e^{3x+k}$, where a and k are constants. The graph of y against x is a straight line.

(a) Use logarithms to show that the gradient of the straight line is $\frac{3}{2\ln a}$. [1]

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(b) Given that the straight line passes through the points $(0.4, 0.95)$ and $(3.3, 3.80)$, find the values of a and k. [4]

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2 Solve the inequality $|x-7| > 4x+3$. [4]

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3 The function f is defined by $\mathrm{f}(x) = \tan^2\left(\frac{1}{2}x\right)$ for $0 \leqslant x < \pi$.

(a) Find the exact value of $\mathrm{f}'\left(\frac{2}{3}\pi\right)$. [3]

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(b) Find the exact value of $\int_0^{\frac{1}{2}\pi} \left(\mathrm{f}(x)+\sin x\right)\mathrm{d}x$. [4]

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4 The polynomial $\mathrm{p}(x)$ is defined by

$$\mathrm{p}(x) = ax^3 - ax^2 - 15x + 18,$$

where a is a constant. It is given that $(x+2)$ is a factor of $\mathrm{p}(x)$.

(a) Find the value of a. [2]

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(b) Hence factorise $\mathrm{p}(x)$ completely. [3]

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(c) Solve the equation $\mathrm{p}(\mathrm{cosec}^2\theta) = 0$ for $-90° < \theta < 90°$. [3]

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5 It is given that $\int_a^{a^3} \frac{10}{2x+1}\,\mathrm{d}x = 7$, where a is a constant greater than 1.

(a) Show that $a = \sqrt[3]{0.5\mathrm{e}^{1.4}(2a+1) - 0.5}$. [5]

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(b) Use an iterative formula, based on the equation in part **(a)**, to find the value of a correct to 3 significant figures. Use an initial value of 2 and give the result of each iteration to 5 significant figures. [3]

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6 A curve has parametric equations

$$x = \frac{e^{2t} - 2}{e^{2t} + 1}, \quad y = e^{3t} + 1.$$

(a) Find an expression for $\frac{dy}{dx}$ in terms of t. [4]

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(b) Find the exact gradient of the curve at the point where the curve crosses the y-axis. [3]

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7 **(a)** Prove that $\cos(\theta+30°)\cos(\theta+60°) \equiv \frac{1}{4}\sqrt{3} - \frac{1}{2}\sin 2\theta$. [4]

(b) Solve the equation $5\cos(2\alpha+30°)\cos(2\alpha+60°) = 1$ for $0° < \alpha < 90°$. [4]

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(c) Show that the exact value of $\cos 20° \cos 50° + \cos 40° \cos 70°$ is $\frac{1}{2}\sqrt{3}$. [3]

Additional page

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