Cambridge International A Level

MATHEMATICS **9709/62**

Paper 6 Probability & Statistics 2 **May/June 2024**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

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Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.

2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.

3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.

4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).

5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.

6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Question	Answer	Marks	Guidance
1(a)	N(145, 145)	**B1**	Stated or implied.
	$\pm\frac{150.5-145}{\sqrt{145}}$ [= ±0.457]	**M1**	Condone incorrect or omitted continuity correction.
	Φ('0.457')	**M1**	For area consistent with their working.
	= 0.676 (3sf)	**A1**	**SC**: Unsupported answer of 0.676 scores **B3**. Unsupported answer of 0.646 or 0.661 scores **B2**. Unsupported answer of 0.6799 scores **B1**.
		4	
1(b)	145 > 15	**B1**	Explicit. λ > 15 B0 if λ = 145 not stated. Accept ⩾ Accept mean for λ.
		1	

Question	Answer	Marks	Guidance
2(a)	[567, 109], 665, 21	**B2**	B1 for each. Allow 021. If more than 2 **answers** given, count first two and ISW.
		2	

Question	Answer	Marks	Guidance
2(b)	Est(μ) = $\frac{610}{30}$ or $\frac{61}{3}$	**B1**	OE or 20.3.
	Est(σ^2) = $\frac{30}{29}\left(\frac{12405}{30}-\left(\frac{610}{30}\right)^2\right)$ or $\frac{1}{29}\left(12405-\frac{610^2}{30}\right)$	**M1**	Use of correct formula.
	= 0.0575 (3sf)	**A1**	Accept $\frac{5}{87}$.
		3	
2(c)	Variance is [unrealistically] small so Henri has [probably] made a mistake/claim is [probably] correct	**B1 FT**	Need both parts. Need 'small' OE, not just < 0.1. FT *their* < 0.1 variance value (not –ve), e.g. 0.0556 (if omit $\frac{30}{29}$). Accept 's.d. = 0.24 is small, so Henri has probably made a mistake'. Note: 'mean is large/small' scores B0, but 'mean large compared to variance so Henri prob made a mistake' scores B1.
		1	

Question	Answer	Marks	Guidance
3	$\frac{18}{50} - z \times \sqrt{\frac{\frac{18}{50}\times(1-\frac{18}{50})}{50}} = 0.244$	**M1**	Use of correct equation.
	z = 1.709 or 1.708	**A1**	Accept 1.71 if nothing better seen.
	ϕ^{-1}('1.709') = 0.956 ; 1 − 2(1 – '0.956') [= 0.912]	**M1**	Attempt area above or below their 1.709 and use correct method to find α.
	α = 91	**A1**	Allow α = 91% 0.91 or 91.2 score A0.
		4	

Question	Answer	Marks	Guidance
4(a)	0.5	**B1**	
		1	
4(b)	$E(X_1 - 2X_2 + 3) = 10 - 20 + 3$ [= −7] or $E(2X_2 - X_1 - 3) = 20 - 10 - 3$ [= 7]	**B1**	Or equivalent using $X_1 - 2X_2 = 10 - 20$ [= –10] or $2X_2 - X_1 = 20 - 10$ [= +10].
	$\text{Var}(X_1 - 2X_2 + 3) = 12 + 2^2 \times 12 + 0$ [= 60]	**B1**	
	$\frac{0-('-7')}{\sqrt{'60'}}$ [= 0.904]	**M1**	Or numerator 3–‘10’ or –3–(‘–10’), but not ‘–3 –10’ (i.e. numerator must be ‘7’ or ‘–7’).
	$1 - \Phi$(‘0.904’)	**M1**	For area consistent with their working.
	= 0.183	**A1**	
		5	

Question	Answer	Marks	Guidance
5(a)	$e^{-3.1}(1 + 3.1 + \frac{3.1^2}{2!} + \frac{3.1^3}{3!})$ or $e^{-3.1}(1 + 3.1 + 4.805 + 4.965)$ or 0.0450 + 0.1397 + 0.2165 + 0.22368	**M1**	Condone one end error. Any λ. Accept fully correct Σ notation. Expression must be seen.
	= 0.625 (3sf)	**A1**	Correct answer with no working scores **SC B1**.
		2	

Question	Answer	Marks	Guidance
5(b)	$[\lambda] = 5.5$	**B1**	SOI
	$1 - e^{-5.5}(1 + 5.5 + \frac{5.5^2}{2!} + \frac{5.5^3}{3!} + \frac{5.5^4}{4!})$ or $1 - e^{-5.5}(1 + 5.5 + 15.125 + 27.7292 + 38.1276)$ or $1 - e^{-5.5}(0.004087 + 0.0224772 + 0.061812 + 0.113323 + 0.155819)$	**M1**	Condone one end error. Any λ. Accept fully correct Σ notation. Expression must be seen.
	= 0.642 or 0.643 (3sf)	**A1**	Correct answer with no working scores **SC B1 B1**.
		3	
5(c)	$[P(X = 3) \times P(Y = 2) =] = e^{-3.1} \times \frac{3.1^3}{3!} \times e^{-2.4} \times \frac{2.4^2}{2!}$ or 0.223676 × 0.261267 [= 0.05844]	**M1**	Find P(3 in first half AND 2 in second half). Must see expression.
	[P(total 5) =] $e^{-5.5} \times \frac{5.5^5}{5!}$ or 0.17140	**M1**	Use of 5.5 to find P(5).
	P(P(exactly 3 in 1st half given total 5) = $\frac{\text{P(exactly 3 in 1st half and total 5)}}{\text{P(total 5)}}$	**M1**	Attempt at conditional probability; numerator = *their* 0.05844 and denominator = P(total 5) Note: $(\frac{3.1^3}{3!} \times \frac{2.4^2}{2!}) \div (\frac{5.5^5}{5!})$ scores M1 M1 M1.
	$[= \frac{'0.05844'}{'0.17140'}] = 0.341$ (3sf)	**A1**	
		4	

Question	Answer	Marks	Guidance
6(a)	H_0: Population mean mass = 510 g H_1: Population mean mass < 510 g	**B1**	Allow 'μ' but not just 'mean'.
	$\pm\dfrac{508-510}{10\div\sqrt{120}}$	**M1**	Standardising must have $\sqrt{120}$.
	$= \pm -2.191$ or -2.190	**A1**	
	$-2.191 < -1.96$ or $2.191 > 1.96$ Area comparison: 0.0143 or $0.0142 < 0.025$	**M1**	OE For valid comparison. Inequality sign the wrong way round scores M1 A0.
	[Reject H_0] There is sufficient evidence to suggest that the [mean] mass has decreased	**A1FT**	OE In context (must be 'decreased' OE, not 'changed'); not definite. No contradictions. Condone 'there is sufficient evidence to support the inspector's claim'. NB: Accept alternative method using critical value (= 508.21) and comparison with 508. Condone 509.79 compared with 510. Two tail test scores maximum B0 M1 A1 M1 A0; must have comparison with 0.0125 or 2.24/2.241.
		5	

Question	Answer	Marks	Guidance
6(b)	$\frac{\text{cv}-510}{10\div\sqrt{120}}=-1.96$	M1	Standardising to find critical value (must use 510 and $10\div\sqrt{120}$). Accept ± 1.96.
	cv = 508.21	A1	Accept 3 sf if nothing better seen. Note: cv could be found in **(a)**.
	$z=\pm\frac{508.21-506}{10\div\sqrt{120}}$ [= 2.421]	M1	Standardising with their 508.21 and 506 (must use $10\div\sqrt{120}$).
	$P(\overline{X}>508.21 \mid \mu=506)=1-\Phi(\text{'}2.421\text{'})$	M1	For area consistent with their working.
	= 0.0077 to 0.0080 (2sf)	A1	Note: $\frac{510-506}{10\div\sqrt{120}}$ scores max M0 A0 M1 M1 A0.
		5	

Question	Answer	Marks	Guidance
7(a)	$k\int_0^{\pi}(1+\cos x)\mathrm{d}x=1$	M1	Attempt integrate f(x) with correct limits and equate to 1.
	$k\left[x+\sin x\right]_0^{\pi}=1$	A1	Correct integration.
	[e.g. $k(\pi+\sin\pi-(0+0))=1$], $k\pi=1$, $k=\frac{1}{\pi}$	A1	AG Some evidence of substitution of limits, i.e. at least one interim step (e.g. $k\pi = 1$) as minimum requirement. Convincingly obtained; no errors seen.
		3	

Question	Answer	Marks	Guidance
7(b)	$\frac{1}{\pi}\left[x+\sin x\right]_0^{0.83}$ or $\frac{1}{\pi}(0.83 + \sin 0.83)$ | $\frac{1}{\pi}\left[x+\sin x\right]_0^{0.84}$ or $\frac{1}{\pi}(0.84 + \sin 0.84)$	**M1**	Substitute correct limits into their integral. OR_1: integrate 0 to 0.83 and 0.84 to π. OR_2: use g(*m*) = *m* + sin *m* – (π/2) and find g(0.83) and g(0.84). OR_3: use h(*m*) = *m* + sin *m* and find h(0.83) and h(0.84). Both attempted.
	= 0.499 (3 sf) | = 0.504 (3 sf)	**A1**	OR_1: 0.499 and 0.496. OR_2: g(0.83) = –0.00286/7 and g(0.84) = 0.0138/9. OR_3: h(0.83) = 1.57 and h(0.84) = 1.58 or 1.59. Both correct.
	'0.499' < 0.5 < '0.504' hence 0.83 < median < 0.84 Equivalent to –0.000912 < 0 < 0.00441 hence 0.83 < median < 0.84	**A1FT**	FT their areas; dep 0.5 is between their areas OE. OR_1: 0.499 < 0.5 and 0.496 < 0.5, so 0.83 < *m* < 0.84. OR_2: g(0.83) > 0 g(0.84) < 0 OE, so 0.83 < m < 0.84. OR_3: h(0.83) < $\frac{\pi}{2}$ h(0.84) > $\frac{\pi}{2}$, so 0.83 < *m* < 0.84. Both statements needed. Note: A score of M1 A0 A1FT is possible.
			If 0 scored, **SC:** $\frac{1}{\pi}(m + \sin m) = 0.5$ **B1** and *m* = 0.831 to 0.832, so 0.83< *m* < 0.84 **B1**.
		3	

Question	Answer	Marks	Guidance
7(c)	$\frac{1}{\pi}\int_0^{\pi}(x + x\cos x)\mathrm{d}x$	**M1***	Attempt integrate $x\mathrm{f}(x)$. Ignore limits.
	$= \frac{1}{\pi}\left(\left[\frac{x^2}{2}\right]_0^{\pi} + [x\sin x]_0^{\pi} - \int_0^{\pi}(\sin x\mathrm{d}x)\right)$	**DM1**	OE Attempt to integrate (using 'parts') with correct limits, reaching an expression of the form $ax^2 + uv - \int v\,\mathrm{d}u$. OR using parts to integrate $x(1+\cos x)$ reaching an expression of the form $uv - \int v\,\mathrm{d}u$ i.e. $\frac{1}{\pi}(x^2 + x\sin x - \int(x + \sin x)\,\mathrm{d}x))$
	$= \frac{1}{\pi}\left(\frac{x^2}{2} + x\sin x + \cos x\right)$ or e.g. $\frac{\pi}{2} + \frac{1}{\pi}(0 - [-\cos x]_0^{\pi})$ or e.g. $\frac{\pi}{2} + \frac{1}{\pi}((-1 - 1))$	**A1**	Integration fully correct.
	$= \frac{\pi}{2} - \frac{2}{\pi}$	**A1**	OE ISW after correct exact value seen. **SC1**: Unsupported answer of $\frac{\pi}{2} - \frac{2}{\pi}$ scores **B3**. **SC2**: Unsupported answer of 0.934 scores **B2**.
		4	