MATHEMATICS

Paper 9709/12
Pure Mathematics 1

Key messages

Previous reports have highlighted that the rubric states that all necessary working must be shown. Although this message has been taken on board by many candidates, it is still apparent that equation solvers had been used in some questions and that some definite integrals had been evaluated using calculator functions. Answers that are not supported by clear full working cannot be awarded full marks and in some cases no marks can be gained.

In questions involving calculation with answers required to a given degree of accuracy it is always advisable for candidates to work with at least one more degree of accuracy until presenting their final answer.

The use of the completed square form of a quadratic function to find its maximum, minimum or range was not always seen, often leading to unnecessarily long solutions.

General comments

This paper gave nearly all candidates the opportunity to show their knowledge at a basic A-level standard and also provided question parts which stretched the most able. The questions involving calculus, the binomial expansion and series were particularly well answered.

Most candidates were able to complete the paper in the available time but transcription and reading errors suggest that available checking time was not utilised as well as it might have been.

Comments on specific questions

Question 1

Most candidates integrated the expression successfully. Some substituted the limits the wrong way round. Many used another variable as the upper limit then considered what happened as this variable tended to infinity. However, there was considerable confusion between infinity and zero, with many stating incorrectly that $-2 \div 0 = 0$ for example. Some sign errors were also seen, resulting in $\frac{-2}{3}$ instead of $\frac{2}{3}$ as the final answer.

Question 2

(a) Both parts of this question were the most challenging on the whole paper. In this part only a small minority of candidates stated the correct coordinates in radians. Errors included misinterpreting the period of the graph, e.g. as $\frac{3\pi}{4}$ or $\frac{3\pi}{2}$, or the minimum point, e.g. as –1 instead of –*k*. Also seen were (540, –*k*) and (3π, –1). Some candidates did not give a response or produced some working that did not lead to coordinates.

(b) Many candidates did not state the coordinates, and gave only an equation of the curve. Some reached both a correct final equation and the coordinates of the point, but these were rare. Common errors included applying the translation in the *x*-direction instead of *y*-direction or reflecting the curve in the *y*-axis instead of the *x*-axis.

Question 3

Many candidates produced a fully correct response to this question. The integration of this type of function appeared to be well understood as was the use of the constant of integration. Some arithmetical errors were seen in finding *c* or in substituting $x = 5$ to obtain the value of *a*.

Question 4

(a) Most candidates expanded the bracket to obtain 3 terms and attempted to use the trigonometric identity $\sin^2\theta + \cos^2\theta = 1$ at least once. Some obtained only 2 terms or introduced a sign error when making use of this identity. A completely correct argument was seen in many cases but a surprisingly large number of candidates omitted theta, used a horizontal line that was too short to denote the correct fraction or cancelled terms incorrectly.

(b) The use of the result from **part (a)** was seen in nearly every answer. Some candidates used a calculator to solve the resulting cubic equation in $\tan\theta$ but it was necessary to show steps in a method of obtaining values for $\tan\theta$ to gain the available method mark. The few candidates who considered the solution from $\tan\theta = 0$ usually scored full marks but even some of these candidates omitted the negative possibility when solving $\tan^2\theta - \frac{2}{5}$. Hence few completely correct solutions were seen.

Question 5

The procedure for obtaining the equation of the normal from the equation of the curve was well understood. Many used the chain rule to correctly differentiate the curve equation avoiding sign and numerical errors. A few errors were reported in the use of the *x*-coordinate of the given point to find the gradient of the tangent but many correct gradients were seen which led to correct normal gradients. Most correct equations were then manipulated into the required form to gain the final accuracy mark.

Question 6

The application of the binomial expansion was well understood and many completely correct solutions were seen. Most answers involved completing the expansion up to the point where the two required terms were found rather than using the general term expression. With so few terms in the expansion and no negative terms this proved a very successful route. Although some answers continued in this vein multiplying all terms in the expansion by $(5 - ax)$ most chose to use the required two terms only to find the terms in x^3. The solution of the resulting cubic equation caused few problems and the correct answer usually followed. Those candidates who thought the cube root could be negative were not awarded the final accuracy mark.

Question 7

(a) The form of the curve equation meant examiners saw very few attempts at equating the gradient of the line and curve with nearly all solutions seen involving elimination of *y* (or *x*) to obtain a quadratic equation. Setting the discriminant of this equation to zero was usually seen and led to many correct answers. Algebraic errors and sign errors reinforce the point made earlier that careful checking of solutions can avoid careless errors.

(b) It was generally appreciated that the value of *k* found in **part (a)** should be used to complete a quadratic in *x* or *y* whose solution would lead to the coordinates of *P*. It was expected that the quadratic solution would be clearly shown and where it wasn't a method mark was not awarded. The few candidates who equated the gradients correctly in **part (a)**, obtaining the coordinates of *P* there, only had to quote them here to gain both marks.

Question 8

(a) Many completely correct answers were seen. Use of the *n*th term formula and the use of the formula for the sum of the first *n* terms were well understood by the nearly all candidates. A small number made algebraic errors or used the wrong value of *n* in the formula for the sum of the first *n* terms.

(b) The use of the formula for the sum to infinity was seen in most answers. Nearly all candidates found $S = 48$ and many of these found $S_E = 16$. Even though the even terms were clearly described in the question some candidates used $a = 24$ and/or $r = \frac{1}{2}$ or resorted to halving S.

Question 9

(a) The composite function was usually found as $5((3x - 2)^2 + k) - 1$ although it was not always clear that the k should be multiplied by 5. The usual route taken was to equate this to 39, find the discriminant of the resulting quadratic and set it to zero to find k. Those who realised that the minimum value of $5(3x - 2)^2$ is zero were able to quickly reach the equation $5k - 1 = 39$ and the correct value of k. Although it was stated that the value of k was required, some candidates missed out on marks by assuming k had a range of values.

(b) Here again the composite function $(3(5x - 1)^2 + k$ was usually found correctly but the obvious connection between the squared term's minimum value and the range was often not seen. This had the effect of making successful solutions considerably longer than was needed.

(c) Considerably more completely correct answers were seen to this part than **parts (a)** and **(b)**. Finding the inverse of a linear function was well understood and the formula for $h(x)$ was found by a variety of methods including intuition.

Question 10

(a) Although some candidates chose to find the equation of the circle here and then differentiate the equation to find the tangent gradient, most attempted to find the gradient of the radius between the centre and the given point and use its negative reciprocal to find the tangent gradient. The use of the general equation for a straight line was usually seen presented correctly.

(b) Finding the equation of a circle from the coordinates of its centre and its radius was well understood and many correct equations were seen.

(c) A variety of methods were used in attempts to obtain θ. The easiest of these involved using simple trigonometry and the distance of the centre from the line AB. However, most candidates selected longer methods to find the coordinates of A and B before using trigonometry. As the question stated that angle ACB was θ radians it was expected that the size of the angle would be stated in radians and not degrees. Centres should note that accuracy marks can't be awarded if the final answer is not given to the required degree of accuracy.

(d) The use of the formulae for arc length and sector area were used effectively by those candidates who had found a value of θ. When finding the shaded area, a minority used the segment area formula, $\frac{1}{2}r^2(\theta - \sin\theta)$ but most preferred to use the difference between the areas of sector ABC and triangle ABC.

Question 11

(a) Some candidates chose to write the curve equation as a quadratic in $x^{-\frac{1}{3}}$ and find the minimum of the quadratic using calculus or completion of the square but most chose to differentiate the equation of the curve directly. Those who could manage the negative fractional powers usually found the required coordinates correctly. A clear, correct method was required as incorrect cubing could lead to fortuitously correct coordinates.

(b) Successful candidates realised that the x-coordinates of points A and B were required and recognised the equation as a quadratic in $x^{-\frac{1}{3}}$, or equivalent, and factorised $2a^2 - 3a + 1 = 0$, or equivalent, to find the two values of x. Candidates who did not show the factorisation or used a calculator to solve the equation were not able to score full marks. Although the x-coordinates of A and B were not always found, the requirement to integrate the curve equation was appreciated by the majority of candidates and often carried out successfully. Candidates who obtained an answer of –0.5 usually realised this was an actual area of 0.5.

MATHEMATICS

Paper 9709/22
Pure Mathematics 2

There were too few candidates for a meaningful report to be produced.

MATHEMATICS

Paper 9709/32
Pure Mathematics 3

Key messages

Candidates need to:

- ensure that they are prepared when they enter the examination room, that is they have a pencil, black biro, ruler, protractor and compasses. This particularly applied to **Question 5**.
- undertake quick sketches of the complex number in the Argand diagram, when dealing with the argument of a complex number, in order to be able to determine which of the two trigonometrical answers is the correct one, see **Question 3(a)**.
- understand exactly what they are differentiating with respect to and not just add $\frac{\mathrm{d}y}{\mathrm{d}x}$ in front of every term, see **Question 6(a)**.
- know that opposite sides of a quadrilateral being equal and parallel only proves that it is a parallelogram and additional information regarding one of the angles being a right-angle is necessary to establish the presence of a rectangle, see **Question 9(a)**.

General comments

The standard of work produced for this component was variable. Some candidates seemed to find the earlier short questions as challenging as the later questions, which was certainly not the intention.

Understanding of how to present an accurate Argand diagram remains an issue. It is not sufficient to just draw circles and lines, the actual parameter details need also to be clearly displayed, for example coordinates of the centre of a circle, its radius, etc. To help this, it is recommended that a compass and ruler are used in any constructions.

Comments on specific questions

Question 1

The majority of candidates found this question straightforward. Most candidates used the method of long division to find the quotient and remainder, although there were a few who used the expanding brackets and comparing coefficients approach. The main errors were sign errors or careless arithmetic ones. Candidates should take care in setting out their long division method, ensuring like terms are clearly vertically beneath each other so as to avoid confusion when subtracting.

Question 2

(a) Many candidates gained full marks on this question, but a significant number found re-writing $\sqrt{4-x}$ in the correct form challenging and so were not awarded the first two marks. Many carried out the expansion for $4\left(1-\frac{x}{4}\right)^{\frac{1}{2}}$ or for $\frac{1}{2}\left(1-\frac{x}{4}\right)^{\frac{1}{2}}$. While some candidates attempted to expand $(2x-5)$, most gained the mark for multiplying their expansion by $(2x-5)$ to obtain two terms in x^2.

(b) This part was found to be challenging with relatively few candidates gaining the mark. Many did not attempt the question and others gave answers such as $\frac{2}{5} < x < 4$.

Question 3

This question proved challenging to most candidates, particularly **part (b).**

(a) Those candidates who expanded z^2 in Cartesian form tended to be much more successful finding r and θ than those who found r and θ for z and then attempted to use these to find them for z^2. In general, those who expanded first found the correct value for r and, apart from occasional sign errors, found the correct value for θ also. Those who found r and θ for z first often found an incorrect value for θ, namely $-\frac{1}{6}\pi$ rather than $\frac{5}{6}\pi$, and then doubled this to find the value for θ of $-\frac{1}{3}\pi$, which may have appeared correct, but has been found from an incorrect method and therefore scored no marks.

(b) Rather a large number of candidates did not attempt this part. Of those who did, many only considered the statement about the modulus being 12 and did not consider that $z^2\omega$ is real, meaning only one mark was available. Those who considered both bits of information were able to gain two marks. Candidates were expected to consider that if $z^2\omega$ is real, then α + their θ must equal either 0, or $\pm\,\pi$.

Question 4

Candidates found this question difficult although many gained the first two marks by using the laws of logs to re-write the given information in the form $\ln p - \ln q = a$ and $\ln p + 2\ln q = b$. Those candidates who then proceeded to find values for $\ln p$ and $\ln q$ generally managed to complete the question correctly. This was the most common method for approaching this question although some candidates did use the alternative method.

Question 5

(a) Candidates should remember that on a sketch of an Argand diagram it is important to either show a scale on both axes or to label points and distances clearly. Most candidates were able to correctly show a circle with centre (4, 2) and a radius of 3. There were fewer candidates who managed to show the correct straight line $\text{Re}(z) = 5$ and therefore could not gain the mark for shading the correct region.

(b) Many candidates did not attempt this part. However, there were candidates who had shaded the correct region in **part (a)**, knew where the greatest value of arg z occurred in **part (b)**, and performed correct Pythagoras and trigonometry to gain full marks in this part. Those candidates who had shown a correct diagram in **part (a)** but considered the opposite region were still able to gain a special case mark here for finding the greatest value of arg z for points in their region.

Question 6

(a) Most candidates gained full marks for this question. Marks were not awarded because of incorrect notation, in particular using $\frac{dy}{dx}$ rather than $\frac{d}{dx}$. Others were not awarded marks due to not equating to $2x$ or 0 consistently.

(b) Many candidates found this part challenging. Candidates often recognised the need to equate the numerator to 0 and so could state an equation in terms of x or y only, but then struggled to manipulate the algebra and so could not gain full marks. A sizable number of students obtained the correct quadratic equation after substituting but then simply stated ‘no real solutions’ without either finding solutions or working out the discriminant and showing that it was < 0. Some candidates equated the denominator to 0 and therefore gained no marks.

Question 7

(a) The majority of candidates had no problems with using the product rule correctly, setting their derivative equal to zero, and rearranging to gain the given equation. Occasionally candidates did not spot the need to use the product rule, or after they had differentiated correctly then did not know how to proceed to obtain the equation given in the question.

(b) In most cases excellent solutions were produced to this part. Occasionally candidates substituted 0.4 and 0.5 into the right side of the equation given in **part (b)** but did not know how to proceed from this to show the verification.

(c) This part was again completed successfully by the vast majority of candidates. Occasionally candidates did not show convergence to two decimal places successfully but still assumed that α was equal to 0.47 to two decimal places.

Question 8

(a) Candidates generally either gained full marks for this part or 0 marks. A few made errors in working out the angle and a few also worked in degrees rather than radians. However, a number of candidates struggled with this question and just worked out R and α using the values given in the question, namely 3 and $2\sqrt{2}$. Most candidates who managed to solve this question did know to give the exact value of R and the value of α to three decimal places.

(b) This question was challenging for the majority, the most candidates either not finding all the solutions or including other solutions in the interval. However, most candidates did gain some marks and even if they gained 0 marks in **part (a)** they were able to link the equation in **part (b)** to their solution from **part (a)**.

Question 9

(a) A minority of candidates scored full marks in this part. Many solutions showed that the shape has two pairs of parallel sides, and just came to the conclusion that the shape must therefore be a rectangle. Another common misconception was to show that the shape has two pairs of equal sides and again come to the conclusion that the shape must therefore be a rectangle. Both of these solutions should have gone on to show that sides are perpendicular in order to fully show that *OABC* is a rectangle. They were many alternative methods that could be used here and it was occasionally seen that pairs of opposite sides are equal and that the diagonals of *OABC* are equal, hence showing that *OABC* is a rectangle.

(b) It should be noted in this part that candidates are instructed to use a scalar product to find the acute angle here. Therefore, any attempts not using the scalar product, for example Pythagoras or trigonometry, scored zero marks. This part in general was completed successfully by candidates, although there were a number of solutions which found the angle between a diagonal and a side of the rectangle, rather than the angle between the diagonals of the rectangle. It would have helped these candidates to sketch a diagram of the rectangle, to label the diagonals with vectors, and then use the scalar product of these vectors. Occasionally candidates attempted to find the angle between two sides of the rectangle and did not seem concerned when their answer was 90°.

Question 10

(a) This question was generally well done with many candidates gaining full marks. Relatively few candidates used an incorrect form of the partial fractions. However, often the constants were not found using the most efficient method.

(b) Most candidates gained some marks in this question by integrating the individual terms correctly. Many did not gain more than 3 marks as they struggled when substituting the integration limits and the subsequent simplifications required. Some candidates did not leave their solution in the form required. Very few candidates substituted a value for *a*, but those that did were able to gain the first 3 marks.

Question 11

A few candidates did not attempt this question. More candidates correctly separated variables and gained the first mark, but then did not know how to proceed. Other candidates recognise the need on the left side of the equation to integrate by parts and usually did so successfully, apart from some sign errors. The right side of the equation caused more problems, with several candidates not realising the need to use a double angle formula to express this side as something which could then be written as $\sec^2\theta$, hence producing a relatively standard integral. Other candidates correctly used the double angle formula but muddled a sign, and hence were never able to remove the constant terms and so could make no further progress.

Unfortunately, too often candidates converted their $\frac{1}{2\cos^2\theta}$ into $2\sec^2\theta$. However, candidates who had integrated both sides of the equation to obtain expressions of the correct form could go on to gain marks for correctly substituting in the initial conditions for the differential equation and to obtaining a value for tan θ when $y = 1$. Candidates should note that when a question asks for an exact value, a decimal answer will not gain full marks.

MATHEMATICS

Paper 9709/42
Mechanics

Key messages

- Candidates are reminded that information given in the stem of a question applies to the whole question unless further conditions are stated subsequently.
- Non-exact numerical answers are required correct to three significant figures as stated on the front of the question paper. Candidates would be advised to carry out all working to at least four significant figures if a final answer is required to three significant figures.

General comments

Many candidates were suitably prepared for the demand of this paper, with many questions attempted well.

Candidates at all levels were able to show their knowledge of the subject. **Questions 3**, **4** and **5(a)** were found to be the most accessible questions whilst **Questions 2(b)**, **6(b)**, **7(b)** and **7(c)** proved to be the most challenging.

In questions such as **Question 6** and **Question 7**, where the sine of an angle is given, it is not necessary to evaluate the angle and doing so may lead to approximations which could affect the accuracy of final answers.

Comments on specific questions

Question 1

(a) The majority of candidates correctly found the distance of the particle from *O* after 20 seconds. Then they were able to use the idea that the gradient of a displacement-time graph gives speed or velocity. However, a significant number of candidates gave the answer as $3.5\,\text{m s}^{-1}$ with no indication of direction as required by the request for the velocity of the particle.

(b) Candidates needed to state or indicate on the graph the speed of the particle is $5\,\text{m s}^{-1}$ during the first 10 seconds of motion. The velocity-time graph should be a series of four horizontal lines which indicate that the velocity during each section of motion is constant, which only a minority of candidates drew.

Question 2

(a) The request in this question was for a speed, but a significant number of candidates gave a negative answer having used downwards as the positive direction. This gave $5 = u + 10 \times 2$, which is correct and leads to $u = -15$, but the final answer should be $15\,\text{m s}^{-1}$. Those who used upwards as the positive direction were usually successful. The main errors arose from misunderstanding the relative signs of the final speed and the acceleration due to gravity, so it was common to see $5 = u + (-10) \times 2$.

(b) This part was answered less successfully, with a significant number of candidates misinterpreting the question as asking for the distance from the initial projection point to the point where the speed of the particle is $10\,\text{m s}^{-1}$. The method required is to double the distance from the point where the speed of the particle is $10\,\text{m s}^{-1}$ to the highest point. This distance could be found by using $0^2 = 10^2 + 2 \times (-10) \times s$ to find the distance directly. Alternatively, it could be found by finding the difference of the distance from the point where the speed of the particle is 15 m s^{-1} to the highest point, and the distance from the initial projection point to the point where the speed of the particle is 10 m s^{-1}.

Question 3

This question was answered well by many candidates. The majority of candidates could resolve in two directions correctly, using $F = \mu R$ at some stage to find μ. The main error seen was to omit the weight component when resolving parallel to the inclined plane.

Question 4

Many candidates gained the majority of the marks available for this question. Most responses achieved the first three marks, although the layout was often quite poor. Those who knew how to solve for F and θ were usually successful. However, the errors seen were mainly at the stage when candidates attempted to solve for F and θ due to poor algebraic manipulation.

Question 5

The concept of variable acceleration being related to calculus rather than constant acceleration was well understood, so this question was a good source of marks for most candidates.

(a) Many candidates integrated well and used the boundary values of $t = 0$ and $t = \frac{1}{2}$ correctly.

(b) Most candidates scored the first two marks for finding the positive value of t at which the acceleration is zero. When finding the total distance, many did not use the fact that the velocity is given in the question as negative in the region $t > \frac{1}{2}$ and evaluated the integral from **part (a)** using limits of 0 and 3, which gives displacement. Because of the change of sign of velocity, the integral needed to be evaluated twice, using limits 0 and $\frac{1}{2}$ for one and limits $\frac{1}{2}$ and 3 for the other, dealing appropriately with the negative sign in the second evaluation of the integral.

Question 6

Candidates should be aware that information given in the stem applies to the whole question unless other conditions are stated in later parts of the question.

(a) This was a typical connected particles question. To answer the request, two Newton's second law equations are needed from the three possible equations. There were a significant number of good responses seen. The main errors came from omitting the acceleration due to gravity in the weight components, from not including either the resistances or weight components or from including a tension in the system equation.

(b) Very few fully correct answers were seen here. The most common error was to concentrate solely on using an energy equation for the car but omitting a term representing the work done by the force in the tow-bar. Some responses did not include the work done by the driving force when looking at the system equation. Another common error was not to include the work done against the car by the 800 N resistance force.

Question 7

This question proved to be quite demanding in parts.

(a) This was generally well answered, with most candidates finding the acceleration as $a = 6$ and then using constant acceleration equations to verify their results.

(b) There were not many fully correct answers seen to this part. Most candidates were able to show that the acceleration on the section BC was 4 m s^{-2}. However, very few candidates were able to cope with the time lag between the particles when dealing with their motion on *BC* up to collision. Some continued to use $a = 6$ and others were still using the distance 0.75 m, which had no relevance to the motion on *BC*.

(c) Very few fully correct answers were seen here. Errors were usually made when finding the speeds of the particles as they were about to collide, with many incorrectly using the speeds of the particles at *B*. Only a minority of candidates were able to find the correct distance travelled between *B* and the point of impact of the particles. Despite these errors, candidates who had made a reasonable attempt to find the speed of the combined particle after collision and the distance remaining to be travelled on *BC* were still able to score the method marks available in this part of the question.

MATHEMATICS

Paper 9709/52
Probability & Statistics 1

Key messages

Candidates should be aware of the need to communicate their method clearly. Simply stating values often does not provide sufficient evidence of the calculation undertaken, especially when there are errors earlier in the solution. The use of algebra to communicate processes is anticipated at this level and enables candidates to review their method effectively. When errors are corrected, candidates would be well advised to cross through and replace the term. It is extremely difficult to interpret accurately terms that are overwritten.

Candidates should state non-exact answers to three significant figures. Exact answers must be stated exactly. Candidates should have a clear understanding of how significant figures work for decimal values less than 1. It is important that candidates realise the need to work to at least four significant figures throughout to justify a final value to three significant figures. Many candidates rounded prematurely in normal approximation questions, which caused them to identify incorrect values from the normal distribution tables. It is an inefficient use of time to convert an exact fractional value to an inexact decimal equivalent, as there is no requirement for probabilities to be stated as a decimal.

The interpretation of success criteria is an essential skill for this component. Candidates would be well advised to include this within their preparation.

General comments

Although many well-structured responses were seen, some candidates made it difficult to follow their solutions by not using the response space in a clear manner. The best solutions often included some simple notation to clarify the process that was being used.

The use of simple sketches and diagrams can help to clarify both context and information provided. These were often seen in successful solutions. There was a significant improvement in the quality of histograms, with few freehand lines being observed.

Sufficient time seems to have been available for candidates to complete all the work they were able to, although some candidates may not have managed their time effectively. A few candidates did not appear to have prepared well for some topics, in particular when more than one technique was required within a solution. Many good solutions were seen for **Questions 3** and **4**. The context in **Questions 5** and **6** was found to be challenging for many.

Comments on specific questions

Question 1

(a) The majority of candidates interpreted the context appropriately and calculated the probability without replacing the red marble. An unexpected number of candidates truncated their probability without providing a more accurate answer, which is always penalised. A small number of solutions simply stated the probability that a red marble was chosen.

(b) Most candidates identified that a conditional probability needed to be calculated and used the anticipated approach. The best solutions calculated the probabilities required for the numerator and denominator of the conditional probability formula separately, clearly identifying the conditions being applied and showing the unsimplified calculations. Stronger candidates recognised that the value from **part (a)** was combined with the numerator to form the denominator. The weakest solutions simply calculated P(Blue, Red).

A small number of candidates did not justify their values with supporting calculations and still arrived at the anticipated answer. Candidates should be aware that unsupported answers do not gain credit in most situations.

Question 2

(a) The context was identified as a binomial distribution by almost all candidates with at least one binomial term attempted. The most efficient method was to calculate 1 – P(8, 9, 10). The best solutions stated the unsimplified calculation and then evaluated accurately without recording the probabilities of individual outcomes. It was disappointing that a number of candidates omitted necessary brackets in their expressions, which is not acceptable at this level.

An unexpected number of candidates used the less efficient, but more obvious, approach of simply adding the probabilities of the required outcomes. The weakest solutions often had a single binomial term stated, often P(8), or used a success probability of 0.8.

As has been highlighted in previous reports, many candidates find interpreting the success criteria challenging. Many weaker candidates assumed that scoring 8 goals was a required outcome.

(b) The majority of candidates identified that the geometric distribution was appropriate for the context. Good solutions used the efficient approach of finding the complement to P(not scoring on 4 attempts), with a clear calculation stated and then evaluated. The alternative approach of summing the probabilities of scoring on the first, second, third or fourth attempt was usually successful. Again, candidates did not always interpret the success criteria accurately and included the fifth attempt in their solution. Candidates would be well advised to practice interpreting probability success criteria, as there is a consistency in wording used to identify which values are required.

A very small number of candidates attempted to use the binomial distribution, but with little success.

(c) The context of this question was found challenging to interpret by many candidates. The best solutions often had a simple representation as to when the goals could be scored, and then calculated the probabilities for each outcome. Stronger solutions identified that the third goal was scored on the seventh attempt, so that combinations could be used to determine how many different ways the other two goals could be scored in the first six attempts. A significant number of candidates simply calculated the probability of three goals being scored with no restrictions applied. Some candidates did not read the question with sufficient care and continued to use 0.7 as the success probability.

Question 3

(a) A significant improvement in the quality of histograms was noted this session. The majority of solutions included clear calculations of the frequency densities, often efficiently placed by the given data table. Almost all candidates used the correct class boundaries. Appropriate scales were used for almost all histograms to enable accurate plotting. Candidates should anticipate that at least half the grid will be required for each axis. It was encouraging that extremely few freehand lines were seen. At this level, candidates should use a ruler to draw lines as statistical diagrams should be an accurate visual representation of the given data. The most common error was failing to label the axes fully, as the omission of ‘time’ or ‘minutes’ on the data axes were noted frequently. The weakest candidates usually constructed a frequency graph, but often gained credit for having the correct class boundaries for the columns.

(b) Many good solutions were seen. The best of these often identified the mid-values by the data table and stated a clear unsimplified calculation for the estimated mean which was then directly evaluated. Weaker solutions divided by the number of classes rather than the total number of readings. The weakest solutions simply found the mean of the mid-values.

(c) Although a significant number of incorrect solutions were noted, a large number of candidates identified an appropriate lower quartile term, and then summed the frequencies to identify the correct class. A common error was to assume that the positional value of the lower quartile was a time and then identify which class contained this time.

Question 4

The majority of candidates recognised that the use of the normal distribution was required in this question. Candidates should be aware that the substitution of values into the unsimplified normal standardisation formula is expected as supporting work in this syllabus topic. The use of simple sketches of the normal distribution curve was seen in many of the most successful solutions, as this can clarify the probability area required. Candidates need to be confident in their use of resources when 'converting' between probabilities and *z*-values, as a frequent error was to reverse the conversion in a solution.

(a) Many good solutions of this fairly textbook style question were seen. Most candidates substituted the values accurately into the normal standardisation formula before evaluating, used the tables appropriately to find the probability and found the correct probability area. Weaker solutions found the complementary value.

As also highlighted in previous reports, a significant number of candidates did not meet the demand of the question and determine the number of bags they would expect from the random sample. Candidates should be aware that a single integer value should be stated as a conclusion to their work in this situation. It is good practice to read the question again after completing a solution to ensure that all the demands of the question have been met.

(b) Again, the best solutions often had a simple sketch to help identify the probability area that is being considered, which enabled the appropriate *z*-value to be found. Many solutions had the required normal standardisation formula, but it was often equated to a probability rather than a *z*-value and so could gain little credit. Many good responses had a clear algebraic solution of the equation formed. Weaker candidates often had solutions that resulted in a negative standard deviation. Where they realised that this was not possible, the negative sign was omitted from the solution, but either no supporting evidence was given to justify this or their error in the initial equation was not resolved, so no credit could be awarded for the anticipated value.

(c) This was a very standard normal approximation for a binomial distribution question. Good solutions showed clear working for the mean and variance values, appropriate substitution into the normal standardisation formula with a continuity correction and then using an efficient method to find the required probability. Common errors were to identify the variance as the standard deviation, to use the upper rather than the lower boundary for the continuity correction or to find the complement of the required area. A few students attempted to use the binomial distribution, which could gain no credit as it does not follow the instruction in the question.

A small number of candidates used the mean and standard deviation from previous parts of the question so could make limited progress.

Question 5

(a) This question was found challenging by many candidates. Good candidates identified the possible scenarios which gave a score of 2, linked the probabilities to the correct game and then reached the required total. Weaker solutions simply stated the calculation required but failed to show why it produced $P(X = 2)$. The weakest solutions did not obtain 0.114 as the final probability. Good candidates recognised that an arrangement skill was within the question and used ${}^3C_1$, for example, to communicate the number of possible ways WLL could be obtained.

(b) The probability distribution table was successfully completed by many. Good candidates found their final probability using the property that the probabilities sum to 1. $P(0) = 0.08$ was a surprisingly frequent error. Good solutions included clear calculations for the probabilities. **Part (a)** was anticipated to guide candidates to realise that ordering events was important, but many simply used the probability for any one way the score could be obtained.

(c) Calculating the variance from data in a probability distribution table is quite standard, and even candidates who were unsuccessful showed some basic understanding of the requirements. As in previous sessions, if the probabilities do not sum to 1 in the table, the method for $E(X)$ is not rewarded but it will be condoned when $Var(X)$ is calculated. The best solutions stated a single unsimplified expression for the variance, which was then evaluated. Most solutions calculated $E(X)$

initially and then used the result to complete the task. A number of solutions were spoiled because of rounding values too early. It is good practice to avoid rounding until the final answer.

Question 6

Solutions that were successful often had a visual representation of the conditions given in the question. These solutions often included more text identifying the intention of the candidate, which did seem to allow checking to occur effectively.

(a) There were many successful solutions which identified all the possible scenarios, used combinations appropriately to determine the outcome for each scenario and summed accurately. Common errors were to include either (5 men, 0 women) or (1 man, 4 women) as possible outcomes. Some candidates added rather than multiplied when calculating the outcome of a scenario. It was unclear as to why $\times\ {}^5C_0$ was seen in solutions of some good candidates. Weaker candidates simply identified a single possible scenario in the solution.

(b) Many candidates were successful here. Good solutions clearly identified the possible scenarios and clearly stated the calculation before evaluating. A common error was not realising that the men and women could swap positions. A small number of candidates rounded the exact answer to three significant figures. Candidates should be aware that rounding should only be applied for non-exact answers.

(c) This question was found challenging by many. Several different approaches could be used, and these were completed accurately in approximately equal numbers. A simple diagram to visualise the approach was a common feature in the best solutions. A common error was to ignore that Olly and Petra could be arranged in two ways, with $\times$ 2, or similar being omitted. Weaker solutions often multiplied by additional terms, which may have been due to misunderstanding how the front row could have been arranged.

MATHEMATICS

Paper 9709/62
Probability & Statistics 2

Key messages

- Candidates need to work to the required level of accuracy. It is important that accuracy is not lost due to rounding answers to three decimal places rather than three significant figures.
- Answers, when required, should be given in context rather than quoting general textbook conditions or definitions.
- In all questions, sufficient and clear working must be shown to justify answers.
- All working should be completed in the appropriate question space of the answer booklet. If answers need to be continued on the additional page, these must be clearly labelled with the correct question number.

General comments

This was a reasonably well attempted paper. Candidates were able to demonstrate their knowledge of the topics tested and many good scripts were seen.

Question 1(a), **2(a)**, **3(a)**, **4(a)**, **5(a)** and **5(b)** were particularly well attempted, whilst question **5(c)** in particular, and for some candidates **Question 6(c)** and **7(b)**, proved challenging. As has been noted in the past, questions requiring a justification or an explanation (such as **Question 1(b)**, **2(b)** and **3(b)**) cause problems for some candidates.

Timing did not appear to be a problem for candidates and on the whole presentation was acceptable and adequate working was usually shown. However, there were some occasions where candidates did not fully justify their answers.

Comments on individual questions follow, but it should be noted that there were many good, fully correct solutions offered as well as the common errors highlighted below.

Comments on specific questions

Question 1

(a) This was a well attempted question with many candidates successfully finding the unbiased estimates of the population mean and variance of X. Very few candidates mistakenly found the biased estimate for the variance and confusion between alternative formulae for the unbiased variance was not as common as has been noted in the past. However, a common error was to give an answer of 0.038, rather than giving this to three significant figures (0.0384) or better. Candidates may have confused three decimal places with three significant figures or may not have appreciated that the zero after the decimal point is not a significant figure.

(b) This part was not as well attempted. Many candidates gave answers referring to the size of the sample (even though the size was given) or gave irrelevant comments about either the mean and variance or the underlying distribution.

Question 2

(a) Finding the required confidence interval was well attempted. Errors included use of an incorrect *z*-value or a missing *z*-value in an otherwise correct formula. Some candidates centred their interval on 78 rather than $\frac{78}{250}$. Most candidates gave their answer in the required form as an interval.

(b) This part was not as well attempted. Candidates were required to clearly state the fact that 0.4 (or 40%) did not lie within the calculated confidence interval, thus leading to the conclusion that the claim was unlikely to be true. Statements such as 'it' does not lie in the confidence interval were not clear enough; a comparison with 0.4 or 40% was required.

Question 3

(a) Candidates made a good attempt at this part. Most used the correct approximating distribution, though some candidates used the given Binomial distribution rather than using an approximating distribution. A common error was to include an extra term, P(3), in the Poisson expression. Candidates should note that their Poisson expression must be clearly seen as it is an integral part of the working. Some candidates, as in **Question 1(a)**, did not show their solution to three significant figures and gave an inaccurate answer of 0.023.

(b) Candidates needed to use the context of the question and give the values of n and np in order to justify their answer, i.e., clearly stating that here $n = 6000$, therefore $n > 50$ and $np = 0.6$, therefore $np < 5$. It was important in this part that the answer was related to the question, so merely stating $n > 50$ and $np < 5$ was not sufficient.

Question 4

(a) Candidates made a good attempt at this part with a large number of candidates finding correct values for the mean and variance of $X - Y$ and then standardising correctly. The main error noted was to find the wrong probability area (greater than 0.5 rather than less than 0.5). As in earlier questions, answers accurate to three significant figures were not always shown.

(b) This part was also reasonably well attempted, though errors when finding the variance were more common than in **part (a)**, i.e., many candidates used 0.8 and 0.85 rather than 0.8^2 and 0.85^2 in their variance calculation. Errors were also seen when finding the probability area.

Question 5

(a) This part was very well attempted, with few errors made.

(b) Again, this part was very well attempted. Occasional errors included use of an incorrect value for λ (for example 0.25 rather than 5×0.25), an incorrect inclusion of an extra term in the Poisson expression and in the accuracy of the final value.

(c) In general, this part was not well attempted. Candidates needed to consider the 3 cases of 'no boys late and 1 girl late', 'no boys late and 2 girls late' or '1 boy late and 2 girls late', then, after finding these probabilities, they needed to be added together. Candidates who realised this were usually successful in reaching the correct final answer, but many candidates did not use a correct approach.

(d) The significance test here was not particularly well done. Many candidates calculated the probability of exactly 4 students being late rather than the probability of greater than or equal to 4, or used a Normal distribution rather than Poisson. Errors also included incorrect (or missing) hypotheses. For questions such as this, conclusions should be written in context and include a level of uncertainty in the language used.

Question 6

(a) It was pleasing to note that, overall, better attempts were made on this question than has been the case with similar questions in the past. There was still some evidence of candidates wanting to use set methods involving integration to find probabilities rather than looking at the symmetry and properties of the probability density function.

(b) On this part, many candidates attempted to integrate f(x), but errors were made in pairing up limits with the corresponding probability value. Commonly, an integration attempt with limits of 2 and 5 was incorrectly equated to 1 rather than $\frac{117}{256}$.

(c) The domain of g(x) needed to be found first on this part. Candidates did not always manage to do this successfully, and even some of those who did find $-1 \leq x \leq 2$ restricted the domain to $0 \leq x \leq 2$, possibly thinking that x had to be non-negative as the question stated g(x) had to be non-negative. Many candidates stated incorrect limits but gained marks for knowing to integrate x^2 g(x) and then subtract the value of the square of their mean. Many did not use symmetry to find E(X) but spent time on calculating it.

Question 7

(a) There were a variety of answers seen here, but 0.05 and 0.99 were the most commonly seen incorrect values. Some candidates gave a description of a Type I error rather than, as requested, stating the value of its probability; it is important that candidates read the question carefully.

(b) This question was not well attempted. Many candidates could not identify the rejection region and therefore standardised with incorrect values. Other errors included standardising without $\sqrt{300}$ and finding an incorrect probability area (> 0.5 rather than < 0.5).