# MATHEMATICS

**Paper 9709/12**
**Pure Mathematics 1**

## Key messages

Since the change in syllabus, the previous reports have each highlighted that the question paper contains a statement in the rubric on the front cover that 'no marks will be given for unsupported answers from a calculator.' Although this message has been taken on board by most candidates, there is still a significant minority whose working does not contain enough detail. Clear working must always be shown to justify solutions.

For quadratic equations, for example, it is necessary to show factorisation, use of the quadratic formula or completing the square as stated in the syllabus. Using calculators to solve equations and writing down only the solution is not sufficient for certain marks to be awarded. It is also insufficient to quote only the formula: candidates need to show values substituted into it. Candidates should ensure that factors must always produce the coefficients of the quadratic equation when expanded. This message is particularly relevant to **Questions 5**, **7(a)** and **9(c)**.

## General comments

Nearly all candidates were able to attempt most of the questions and many very good scripts were seen. The requirement that candidates know and can use the algebra, geometry and trigonometry techniques studied at GCSE level (or equivalent) seemed to be evident in the responses from most centres.

## Comments on specific questions

### Question 1

The need to form a quadratic equation, usually in $x$, was clear to most candidates, as was the need to use the discriminant of their quadratic equation. The necessary presentation of the discriminant as a perfect square was sometimes seen, but the correct interpretation of this was rarely seen. This, combined with sign errors, meant a minority of candidates were awarded more than half marks.

### Question 2

Although it was acceptable to attempt the algebraic interpretation of the transformations in any order, or altogether, most candidates presented their interpretations in the order of transformations given in the question. Many favoured using the completed square form of the equation. The reflection proved to be most challenging of the three transformations, but other common errors included squaring $2x$ incorrectly and multiplying only some of the terms by 3. A few candidates who carried out the three transformations correctly did not leave the result in the required form, so could not be awarded the final A1.

### Question 3

The differentiation of this type of function seemed to be well understood by most candidates. The interpretation of the rates of change to come to the realisation that $\frac{\mathrm{d}y}{\mathrm{d}x} = 1$ or a chain rule calculation using $\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{\mathrm{d}y}{\mathrm{d}t}$ was not always evident. The false assumption $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ was followed in many incorrect solutions.

**Question 4**

**(a)** Most candidates did not take account of the fact that the first term should be 5.02 with $n = 20$ (or 5.00 with $n = 21$) and could only be awarded a mark for selection and use of the correct formula.

**(b)** Incorrect interpretation of the number of terms was a common error for most candidates. However, the correct value of the common ratio and the use of the appropriate formula were both evident in most answers.

**Question 5**

A variety of different methods were seen, the most common being to substitute the equation of the line into the equation of the circle and solve the resulting quadratic. The formation of the circle equation from its radius and centre was well understood by most who chose to use this approach. Other methods successfully found the equation of the perpendicular bisector of *AB* then its mid-point and used either the midpoint formula or a vector method to find the coordinates of *B*. Some candidates incorrectly used the coordinates of *A* when forming an equation of the circle. Very few diagrams were seen and, for those could not make a start with any strategy, a diagram might have helped them visualise a suitable method.

**Question 6**

Most candidates were able to identify the two terms either from an expansion or by considering the general term. They usually went on to form an equation using the given condition. Errors were seen in manipulating powers and some candidates omitted the negative solution when they found the square root of $\frac{1}{9}$.

**Question 7**

**(a)** This was a well answered question with both the relationship between tangent and sine and cosine, and the trigonometric identity $\sin^2\theta + \cos^2\theta = 1$, being used accurately to find a quadratic equation in cosine. Many correct answers were seen, although some came from calculator solutions of the quadratic equation. As mentioned in the Key Messages, answers from a calculator must be supported with full working. Most candidates correctly gave answers in the first and fourth quadrants.

**(b)** This part only required use of the relationships used in **part (a)** and some very clear solutions were seen, albeit with a lot of variation in the number of steps used. Others made multiple attempts where, often, initially correct working was spoilt by poor division of fractions. Candidates should be reminded that, in this type of question, they should quote the relationships they are using in each stage.

**Question 8**

**(a)** Most candidates were able to successfully find the length of *AC* and angle *ADC* using combinations of simple trigonometry or the sine or cosine rule and Pythagoras' theorem. These were generally used correctly to find the arc length *AC*. These reports have often commented on the effect of premature rounding of intermediate answers on the final answer; this was particularly evident in both this part and **7(b)**. Use of the arc length formula with degrees or radians as appropriate was usually seen. There were a few instances of Pythagoras' theorem being applied to non-right angle triangles.

**(b)** Again, angles in degrees or radians were usually used with the appropriate sector area formula. Most correct answers involved subtraction of the area of triangle *ADC* from area of sector *ADC,* although a significant minority chose to use the formula of the area of a segment using angle *ADC*. Answers which were prematurely rounded in **part (a)** invariably resulted in an inaccurate answer in this part.

**Question 9**

**(a)** Most candidates presented their answers in a correct form. When $x \leqslant -1$ was presented as an answer no marks could be awarded as this could have been a restatement of the domain.

**(b)** Most candidates reached the square root of a correct expression. Some stated this as ± but few realised that the given domain could only be achieved using the negative root. Candidates should be reminded that examiners must be convinced that square root signs refer to the whole of a fraction where this is appropriate.

**(c)** Finding the correct expressions for fg($x$) and gf($x$) seemed to be understood by the majority of candidates. However, sign errors and algebraic errors when squaring and combining the expressions often prevented candidates reaching the correct quartic equation. It was essential to show a method of solving the quartic equation in $x$ as solving a quadratic in $x^2$. Factorising, completing the square, or using the quadratic formula, gained full credit. Some candidates substituted $x = x^2$, but then did not take the square root of their answers. A minority stated that, given the domain of f($x$), both the solutions needed to be negative.

**Question 10**

**(a)** As the answer was given, most candidates realised it was necessary to show all the steps leading to the result, including the evaluation of $4^{-\frac{1}{2}}$ as $\frac{1}{2}$.

**(b)** This integration process seemed to be well understood and many candidates scored full marks in this part. Errors in dealing with the division by $\frac{1}{2}$ and omission of the constant of integration were occasionally seen.

**(c)** This part was also well answered. Usually, candidates correctly set the given $\frac{\mathrm{d}y}{\mathrm{d}x}$ to 0, with -2 substituted for $k$. Most candidates dealt with the power of $x$ correctly and found the required $x$-coordinate. However, some candidates ended their response after only having evaluated $x$, without also finding the value of the $y$-coordinate.

**(d)** This was another well answered part, with nearly all completely correct answers coming from evaluation of the second derivative at $x = \frac{1}{4}$. A few candidates looked at the gradient change from values of $x$ either side of $x = \frac{1}{4}$ and those who chose a lower value between zero and $\frac{1}{4}$ usually gained both the available marks.

**Question 11**

**(a)** This part was answered well by nearly all candidates. The most common errors included inverting the gradient or swapping the $x$ and $y$ coordinates when using the straight-line equation.

**(b)** Although there were many correctly worked solutions there were also a lot of scripts where candidates gained few marks. The question clearly stated that the region was rotated about the $y$-axis, but a significant number of candidates did not take this into account in their working. Others did not square the two functions (so did not integrate $x^2$) and some integrated a function of $y$ but used limits for $x$. In a number of cases, correct answers were found from a calculator after errors in the working. As mentioned in the Key Messages, it is not possible to gain credit for unsupported answers of this type. It was essential to show all steps, including the integration of both functions, the substitution of the correct limits, and the subtraction of the volume between the curve and the $y$-axis from the volume between the line and the $y$-axis. An alternative method, used successfully by a small number of candidates, involved finding the volume of a frustum of a cone, using the coordinates of the given points, followed by subtracting the volume between the curve and the $y$-axis.

# MATHEMATICS

**Paper 9709/22**
**Pure Mathematics 2**

There were too few candidates for a meaningful report to be produced.

# MATHEMATICS

**Paper 9709/32**
**Pure Mathematics 3**

**Key Messages**

Candidates need to:

- know what is meant by arg($z - z_0$) = $\alpha\pi$, ( $-1 < \alpha \le 1$), and how to draw this on an Argand diagram. Showing equal scales on both axes is essential. See **Question 2**.

- be able to divide a polynomial containing constants by a second polynomial. See **Question 3**.

- be able to take real and imaginary parts of an equation. See **Question 4**.

- know how to integrate $\dfrac{1}{e^{\alpha x}}$, $\sin^2(nx)$ and $\cos^2(nx)$, where $\alpha$ is a real number and $n$ is an integer. See **Question 9**.

- understand that integrating $\dfrac{\alpha x + \beta}{\varepsilon x^2 + \delta}$ may require splitting this into two separate integrals, namely $\dfrac{\alpha x}{\varepsilon x^2 + \delta}$ and $\dfrac{\beta}{\varepsilon x^2 + \delta}$. See **Question 11b**.

**General comments**

The standard of work on this paper was variable. However, regardless of the candidate's ability, this made little difference to the level of presentation and candidates of all ability levels can improve in this area. Several candidates showed multiple different attempts at certain questions, often jumping between these different methods as they worked through their solutions. Candidates are advised to demonstrate a single solution worked clearly and methodically, deleting any early approach should they wish to change their mind. Often candidates did not present their work in this manner which made it challenging to follow. This, coupled with the numerous different methods being tried on a single question, such as in **Question 3** and **Question 11(b)**, meant work was often complicated to mark.

Candidates often gave good responses to **Question 1**, **Question 7(b)**, **Question 7(c), Question 8(a)**, **Question 8(b)**, **Question 10(a)** and **Question 11(a)**. However, **Question 2(a)**, **Question 2(b)**, **Question 3**, **Question 4**, **Question 7(a)**, **Question 9** and **Question 11(b)** proved challenging for many.

**Comments on Specific Questions**

**Question 1**

Most candidates scored full marks, however a few believed $\ln(a \pm b)$ could be replaced by $\ln a \pm \ln b$ and hence made no progress.

**Question 2**

**(a)** Few candidates scored more than a single mark, with the errors being wide ranging. These included having no scale or showing the scale on only one axis; drawing the line $y = 3$ instead of the line $x = 3$; having the line $x = 3$ confined to the first quadrant; drawing full lines instead of half-lines; showing half lines starting from the origin instead of 1 + 2i; showing half lines being symmetrical about the vertical line instead of the horizontal line; drawing lines asymmetrical about the horizontal line; marking lines at an angle of $\frac{1}{3}\pi$ but clearly drawing an angle of $\frac{1}{4}\pi$ or less; showing a single line at $\frac{1}{3}\pi$; and finally the presence of a circle, were all very common.

**(b)** Approximately fewer than five percent of the candidates scored any marks here, since candidates required a correct sketch in **2(a)** before they could make any progress. This work should have been followed by recognising that the point needed was where arg $(z - 1 - 2i) = -\frac{1}{3}\pi$ and the line $x = 3$ met, together with the calculation of the distance of this point below the $x$-axis, namely $2\sqrt{3} - 2$.

**Question 3**

This was another question where candidates experienced problems. Again, the reasons for this were wide ranging.

Candidates found it challenging to deal with the unknown constants in the polynomial when attempting long division, hence they rarely had a linear expression to equate to the given remainder. Some tried to subtract the given remainder from the polynomial and equate their remainder to zero, which is a perfectly acceptable approach, but fails to help with this long division issue. Many other candidates erroneously believed that the divisor times the remainder was equal to the polynomial.

Another common approach was to find a root of the divisor and then equate the polynomial to the remainder at that root. Again, an acceptable approach, but in this case the root was complex and when substituted it required taking real and imaginary parts. This is something which candidates found difficult, as seen by their work in **Question 4**. However, the algebra involved in evaluating this complex root up to the power 4, before substitution into the equation, was prone to numerous errors.

Others incorrectly believed that this required root came from setting the remainder to zero, namely $x = -\frac{2}{3}$.

The easiest approach, and one which most successful candidates chose, was to equate the polynomial to the divisor times a quotient $(Ax^2 + Bx + C)$ plus $3x + 2$. Equating the various powers of $x$ then leads to five simple equations which are easily solved, even easier for those who sensibly commenced with $A = 2$.

**Question 4**

Unfortunately, this was the third question in a row which candidates found challenging. Here, this was mainly due to sign errors, omitting constants, $x$'s or $y$'s, or incorrect multiplication of simple terms.

Few candidates reached a correct simplified equation. Many finished with an imaginary part consisting of just a single variable, which made the final part of the question much easier. However, there were several candidates who succeeded in reaching the correct equations and then stopped, not realising that it was necessary to equate both the real parts and the imaginary parts to zero to make further progress.

**Question 5**

**(a)** Many candidates performed well on this question, showing clear details of their cancellation of the $(2t + 1)$ term. However, there were still too many who, when differentiating $te^{2t}$, only differentiated the $e^{2t}$ part or did not recognise the need to use the product rule.

**(b)** This part required candidates to substitute $t = -1$ to find the coordinates of $x$ and $y$, together with the negative inversion of the given answer in **5(a)**. Unfortunately, the gradient of the normal too often remained as $e^{-2t}$ and candidates believed that the answer given should be used in the working to find the answer given, often resulting in candidates showing 0 = 0. Candidates should be aware that the answer given should never be used in trying to obtain that result, it is simply there to show the candidate the form that the examiner wishes the final answer to be presented in.

**Question 6**

**(a)** Candidates were expected to undertake the expansion of $R\cos(\theta - \alpha)$ and equate terms, leading to $R\cos\alpha = 12$ and $R\sin\alpha = 5$, before using Pythagoras and simple trigonometry to find $R$ and $\alpha$.

Whilst there is nothing wrong in quoting formulae, if this is the approach that candidates wish to adopt, then they need to know how to apply them correctly and not finish with $\tan\alpha = \frac{12}{5}$, or with $\cos\alpha = 12$ and $\sin\alpha = 5$, as many candidates did. In this question $R$ is an exact value, however seeing statements such as $R = \dfrac{12}{\cos(0.395)} = 13$ did not necessarily convince that exact mathematics was being used. The exact answer should be gained from $R^2 = (R\sin\alpha)^2 + (R\cos\alpha)^2 = 5^2 + 12^2$, resulting in $R = 13$.

**(b)** Most candidates scored the mark for $\cos^{-1}\left(\dfrac{6}{R}\right)$ but then often continued to use $\theta$ instead of $2x$.

Candidates using $2x$ were unable to obtain more than the method mark unless $\alpha$ obtained in **6(a)** was correct. Many candidates obtaining the correct answer of 0.743 were unsure as to whether there was another answer in the interval, and if there was, that it needed the use of $2\pi - \cos^{-1}\left(\dfrac{6}{R}\right)$ to generate it.

**Question 7**

**(a)** Candidates either had little trouble in scoring full marks or were unsure how to start the question.

For those that made progress, a factor of 3 was usually associated with the correct term, however some of the expressions for the area of major sector had $\pi$ instead of $2\pi$. Others dropped the value of $\frac{1}{2}$ in their working or omitted it from some of their statements regarding areas. Often the area of the triangle was quoted correctly as $r^2\cos\left(\dfrac{x}{2}\right)\sin\left(\dfrac{x}{2}\right)$, but candidates continued with this throughout, despite the answer given informing them that they should have used $\frac{1}{2}r^2\sin x$ from the start or introduce it from using the double angle formula.

**(b)** A few candidates omitted this section, but in most cases excellent solutions were produced.

**(c)** This good work was continued here, with most candidates scoring full marks. The only blemishes were a few candidates who, for example, believed that they had shown convergence to two decimal places when they reached 2.1865, 2.1831.

**Question 8**

**(a)** Candidates were well trained in the product rule and usually had little trouble in securing the first three marks, however many then began to use decimals despite the question requesting exact coordinates. Unfortunately, many who did stay with exact notation then muddled their final line by expressing the $y$ coordinate as $e^{-1} - \frac{1}{3}$ instead of $-\dfrac{1}{3e}$, or an equivalent correct form.

**(b)** This was another question where the technique, namely integration by parts, was accurately applied, with most candidates securing the first four marks. However, only a few candidates realised that area must be positive, which meant candidates should have been dealing with the modulus of their integral. The fact that many had $\frac{1}{64}\ln\left(\frac{1}{2}\right)$ together with $-\frac{15}{256}$ meant candidates did not always recognise that this answer was negative, which was not the case for those who happened to have converted to $-\frac{1}{64}\ln 2$.

**Question 9**

This question proved extremely difficult for most. Despite much algebra and trigonometry being presented, few marks were scored.

Separation of variables was no problem, however the conversion of $\frac{1}{e^{3y}}$ and $\sin^2(2x)$ into something that they could integrate, namely $e^{-3y}$ and $\frac{1}{2}(1 - \cos(4x))$, was a challenge for most. Some candidates had incorrect signs in the trigonometry formula and candidates should note that these are available in the formula book.

**Question 10**

**(a)** Many candidates scored full marks, however there were some basic misconceptions seen, such as believing $\sqrt{14}\sqrt{14}$ to be $2\sqrt{14}$ and not knowing what an obtuse angle was.

**(b)** Most candidates knew how to obtain the direction vector together with either of the two points, but often arithmetical or miscopying errors were seen. Sometimes, the direction vector and the point were interchanged within the vector equation of the line formula.

However, the most common error was the incorrect notation on the left-hand side of the equation. The left-hand side should be an underlined $\underline{r}$ i.e. not left blank, expressed in words or just denoted as *l*.

**(c)** Again, most candidates knew what to do for this section but had arithmetical or miscopying errors. Vector questions need great care with the arithmetic, checking as one proceeds, otherwise the loss of marks can be considerable. When solving the two line equations, both parameters should be found and used to establish the intersection point, as different points reveal an error in the solution of equations part of the question. However, it does not preclude there being errors when establishing the line equations, so constant checks are essential.

**Question 11**

**(a)** Many candidates scored full marks with little difficulty. However, several incorrect formulations of the partial fractions were often seen, with $\frac{A}{1+x}+\frac{B}{4+x^2}$ and $\frac{Ax+B}{1+x}+\frac{C}{4+x^2}$ being the most common.

**(b)** Virtually all candidates established the correct integral for $\frac{A}{1+x}$, however the integral of $\frac{Bx+C}{4+x^2}$ proved challenging for most. This integral should have been subdivided into $\frac{Bx}{4+x^2}$ and $\frac{C}{4+x^2}$, as opposed to being treated as $(Bx + C)$ multiplied by the integral of $\frac{1}{4+x^2}$. Following the subdivision, the first integral is easily undertaken by setting $z = 2x$, whilst the latter is available in the formula book. Finally, to achieve the last accuracy mark, it was necessary to see some detailed ln work and not simply go from $3\ln 3 + \ln 8 - \ln 2 - \frac{1}{8}\pi$ to the answer given.

# MATHEMATICS

**Paper 9709/42**
**Mechanics**

## Key messages

When answering questions involving any system of forces, a well annotated force diagram could help candidates to make sure that they include all relevant terms when forming either an equilibrium situation, or a Newton's law equation. Such a diagram would have been particularly useful in **Questions 4(c)**, **5** and **6**.

Non-exact numerical answers are required correct to three significant figures as stated on the question paper. Candidates would be advised to carry out all working to at least four significant figures if a final answer is required to three significant figures.

## General comments

The paper was well attempted and candidates at all levels were able to show their knowledge of the subject. **Questions 1**, **3(a)** and **6(a)** were found to be the most accessible questions, whilst **Questions 5**, **6(b)** and **7(c)** proved to be the most challenging.

## Comments on specific questions

### Question 1

**(a)** This question was answered well by most candidates using one of two methods. The most efficient method being to use $Pt = W$, giving $4\,500 \times t = 600 \times 15$ and leading to a time of 2 seconds. However, the most popular method seen was to use $P = Fv$ to obtain a speed of $7.5\,\text{m}\,\text{s}^{-1}$ and then using this speed to get the required time.

**(b)** There were many good answers seen for this part, with most finding an acceleration of $-3\,\text{m}\,\text{s}^{-2}$ from the use of Newton's second law and then using this in the constant acceleration formula $v = u + at$. The two most common errors seen were to use the distance from the first part of the motion, and to have an equation such as $0 = 7.5 + 3t$ that would lead to a negative time.

### Question 2

**(a)** This question was answered well by the majority of candidates. Very occasionally, an acceleration of $10\,\text{m}\,\text{s}^{-2}$ rather than $-10\,\text{m}\,\text{s}^{-2}$ was used.

**(b)** Only a minority of candidates achieved a fully correct answer. Most answers did have a distance travelled for *P* and *Q* in terms of *t* using $s = ut + \frac{1}{2}at^2$. The method then required the use of the fact that the total distance travelled by *P* and *Q* is 18 m to form an equation in *t*, and then use this *t* value to find the required distance. However, it was more often seen for candidates to equate the distance for *P* and the distance for *Q* without any use of the fact that the particles were initially 18 m apart.

**Question 3**

The concept of variable acceleration being related to calculus rather than constant acceleration was well understood and this question was well answered by many candidates.

**(a)** Many candidates integrated correctly and evaluated the resulting expression at $t = 9$.

**(b)** Again, many candidates integrated correctly here and equated this to the expression found in **part (a)**. The majority then completed the question by solving the equation for $t$. However, some candidates found difficulty in dealing with the fractional indices when solving. Another common error seen was for the expression for displacement to be equated to 72 m, which was the speed found in **part (a)**.

**Question 4**

**(a)** No resistance on the truck and the truck moving with constant speed should have indicated that the tension in the coupling was zero. A common incorrect answer often seen was 0.2 N.

**(b)** This request was often correct from the use of $P = Fv$.

**(c)** Many good answers were seen, with the use of two of the three possible Newton's second law equations and a calculation to find the driving force. However, a significant number of candidates seemed confused about the driving force, using it with both the equations for the locomotive and the truck. Another common error was to include the tension in the coupling when attempting the equation for the whole system.

**Question 5**

Many candidates confused tension and thrust in this question.

**(a)** Only a significant minority of candidates appreciated that the force in the struts were thrusts with a direction towards point $D$. A significant number of candidates had the force in the struts as a tension, which did lead to a negative force. As the request was for the magnitude of the force, the majority of answers seen were given as positive. Only a minority of candidates gave a negative answer.

**(b)** Again, the majority who made a valid attempt at this question had the force in the strut $BD$ as a tension. However, this did not have any effect on the solution by resolving forces vertically and horizontally and solving for $F$. A common error seen was to obtain a negative tension from the vertical equation and making this positive before substituting into the horizontal equation. Also, $F$ was sometimes assumed to be acting horizontally to the left. This resulted in a negative value which was not always made positive, given that the question stated that $F$ was the magnitude of the horizontal force applied.

**Question 6**

**(a) (i)** This question was well answered by the majority of candidates. Common errors included the omission of a weight component, no use of the given acceleration, or not resolving the tension in the rope.

**(ii)** Many good attempts to evaluate the coefficient of friction were seen. The main error, as is common in these types of tasks, was to omit the component of the tension when resolving perpendicular to the plane to find the normal reaction, so getting $R = 2g\cos 30$.

**(b)** The approach usually seen was to find the maximum possible friction of $0.8 \times (2g\cos 30 - 15\sin 20)$ and compare this with the friction required to stop the block moving up the plane of $15\cos 20 - 2g\sin 30$. This shows that the friction needed to stop the block moving up the plane is less than the maximum friction and hence the block does not move. Even though many candidates have correct values to compare, the incorrect conclusion often stated was that the block moved down the plane. This conclusion was not appreciating that this would have to arise from the friction acting up the plane, and so a comparison with $2g\sin 30 - 15\cos 20$ would be required. In this scenario this value is negative.

**Question 7**

Many candidates appeared to find this a demanding question in parts.

**(a)** A significant number of candidates scored well on this part. The method did require the speed of particle *P* when it arrived at *B*, to be found using energy, and then to use the principle of conservation of momentum to show the speed of *Q* to be $10\,\text{m}\,\text{s}^{-1}$ after the collision. The main error seen by examiners was to use constant acceleration to find the speed of particle *P* when it arrived at the point *B*.

**(b)** Most candidates used momentum correctly to find the speed of the combined particles after particle *Q* collided with particle *R*. Then candidates used one of two different methods to complete the request, both with equal success. Some used energy to find the vertical height of point F above point C, before finding the required angle from use of trigonometry. Others used constant acceleration to find the acceleration on the slope before then equating this to $g\sin\theta$ to find the required angle.

**(c)** Very few candidates scored full marks on this part. The most commonly seen approach was to work with time. Firstly finding the time for particle *Q* to reach the point *C* (0.7 s). Then finding the time for the combined particle to travel up the slope and return to the point *C* (0.8 s). Then determining that the total time that particle *P* has been in motion since it collided with particle *Q* is 1.5 s. Using this total time, the distance of between the particle *P* and the combined particle can be found to be 1 metre. Finally, the remaining time until *P* collides with the combined particle can be found as *P* moves 4*t* metres and the combined particle moves 2*t* metres, so the total distance moved is 6*t* metres, which equates to 1 metre. This gives an additional time of $\frac{1}{6}$ s, leading to a distance of $\frac{20}{3}$ metres from *B*. The most common error seen was to omit the time that the combined particle takes to come back down the slope.

# MATHEMATICS

**Paper 9709/52**
**Probability & Statistics 1**

## Key messages

Candidates should be aware of the need to communicate their method clearly. Simply stating values often does not provide sufficient evidence of the calculation undertaken, especially when working is not fully correct earlier in their solution. The use of algebra to communicate processes is anticipated at this level, as this enables candidates to review their method effectively and is an essential tool when showing given statements are true. When errors are corrected, candidates would be well advised to clearly cross these through and replace the original working. Where a candidate's workings are overwritten, it is often difficult to interpret these accurately.

As stated in the paper instructions, candidates should state only non-exact answers to three significant figures. There should be a clear understanding of how significant figures work for decimal values less than 1. It is important that candidates realise the need to work to at least four significant figures throughout to justify a value correct to three significant figures. Many candidates rounded prematurely in normal approximation questions which produced inaccurate values from the tables and lost accuracy in their solutions. It is not an efficient use of time to convert an exact fractional value to an inexact decimal equivalent, as there is no requirement for probabilities to be stated as a decimal.

The interpretation of success criteria is an essential skill for this component. This is particularly the case where the answer should be presented with reference to the context of the question. Candidates would be well advised to include a focus on this within their preparation.

## General comments

Many well-structured responses were seen, however some candidates made it difficult to follow their thinking within their solution by not using the response space in a clear manner. The best solutions often included some simple notation to clarify the processes which were being used.

The use of simple sketches and diagrams can help to clarify both context and information provided. These were often seen in successful solutions. Candidates should be aware that cumulative frequency graphs are constructed with a curve, and that these need to be accurately drawn. It was pleasing to see an improvement in the labelling of the statistical diagrams from the previous series.

Sufficient time seems to have been available for candidates to complete all the work they were able to, although some candidates may not have managed their time effectively. The majority of candidates performed well across a range of topics. However, a few candidates did find questions more challenging when more than one technique was required within a solution. Many good solutions were seen for **Questions 1, 4** and **6**. The context in **Questions 5** and **7** was found to be challenging for many.

## Comments on specific questions

### Question 1

Many candidates used the area around the data table both to calculate the cumulative frequency and to state the mid-class values. In these cases, it is important that these calculations are presented in a manner so that the working is allocated to the relevant part.

**(a)** Almost all candidates calculated the cumulative frequency accurately. The majority of solutions used the vertical axis for the cumulative frequency, which is the normal convention. Most candidates plotted the required points on the class upper boundary, although a very small number used a continuity correction. This suggests that only integer value times had been recorded, which was not implied by the data table or the question. It was also noted that some candidates plotted at the mid-class or class lower boundary values.

A common misunderstanding was to assume that the number of hours recorded started at 0, so the cumulative frequency graph commenced at $(0, 0)$ and not $(30, 0)$ as indicated by the data.

Candidates should be aware that a curve is required for a cumulative frequency graph, as the use of line segments creates a cumulative frequency polygon which is not acceptable.

It was encouraging to see many graphs with correctly labelled axes, although a few candidates used scales which made it difficult to plot points to the required degree of accuracy.

Weaker candidates often plotted a frequency graph or bar chart.

**(b)** The majority of candidates interpreted the question correctly and calculated 70% of the number of years. It was pleasing to see that most candidates provided evidence on their graph of where they were obtaining their value. Good solutions included ruled lines perpendicular to the axes to enable the values to be read accurately. A surprising number of candidates did not read values from their graphs to the expected accuracy. Candidates who had used more challenging scales in **1(a)** often struggled to interpret their graph accurately.

**(c)** Many good solutions to this part were seen. The best of these initially stated the mid-class values, provided a clear calculation for the estimated mean, and then evaluated an exact answer. As stated in the Key messages, candidates should be aware that there is no requirement to convert an exact answer to a three significant figure. Some candidates chose to present their solutions within a table structure. While this is clear, it provides more detail than is strictly required compared to calculating individual values. Common errors included using either the class upper boundary or the class width in the mean calculation. A small number of candidates applied a continuity correction to the mid-class value.

**Question 2**

Several solutions included a tree diagram in **2(a)** to clarify the possible outcomes of the coins. This was often helpful in **2(b)**.

**(a)** Candidates were required to provide evidence that the given statement was true. The majority of candidates were able to find a calculation that gave a result of 0.225 but did not provide a justification for their expression. Good solutions often identified the possible scenarios that produced exactly one head, then stated the probability calculation for each scenario prior to showing that these summed to 0.225. The best solutions clearly identified which coin was biased, rather than assuming that the probabilities alone were sufficient.

An alternative combinations approach was successfully utilised by a small number of candidates, which was sufficient justification for the different scenarios where the head was on a fair coin.

**(b)** Almost all candidates determined that $\mathrm{P}(X=2)+\mathrm{P}(X=3)=0.650$ and this was utilised to find their second probability. The most successful solutions considered $\mathrm{P}(X=3)$, which was a similar calculation to **part (a)**. Candidates who first found $\mathrm{P}(X=2)$ often omitted other possible scenarios. Candidates who had drawn a tree diagram in **part (a)** were able to identify the required scenarios efficiently. Weaker candidates simply stated that each probability was 0.325. Several solutions were seen where the total probability was not equal to 1, which should be a prompt that an error has been made.

**(c)** Most candidates who completed the probability distribution table attempted to find the variance. The best solutions included a clear, unevaluated expression using the standard variance formula, before stating the answer to three significant figures. Weaker solutions failed to subtract the square of the given expected value.

A few candidates recalculated the given expected value, although not always accurately.

**Question 3**

Almost all candidates identified that the binomial approximation was the appropriate approach for **part (a),** and most identified that the geometric approximation was required in **parts (b)** and **(c)**.

**(a)** This standard binomial approximation question was answered well by most candidates. Almost all stated at least one correct binomial term, but a common error was to include 17 residents as meeting the stated success criteria. Candidates are well advised to consider the difference of the success criteria ‘at least 17’, which includes 17, and ‘more than 17’, which does not.

A significant number of solutions were inaccurate due to a premature approximation of values in the calculation. As highlighted in the Key messages, candidates should be aware that at least four significant figures must be used to obtain an answer that will be accurate to three significant figures. Good practice is to state the unevaluated expression, then use the calculator efficiently to provide an answer to at least four significant figures, before finally rounding to three significant figures. There is no expectation of intermediate arithmetical steps to be shown at this level.

**(b)** Most solutions used the geometric approximation. Good solutions evaluated the solution exactly and did not round the answer to three significant figures. As highlighted in the Key messages, candidates should be aware of the syllabus requirement leave exact values unrounded. Weaker solutions failed to interpret the success criteria accurately, often finding the probability for the sixth person not being in favour.

**(c)** Most candidates found this part challenging, with a significant number of candidates omitting it altogether. The success criteria required both the seventh person to not be in favour, as well as exactly one of the previous six people not to be in favour. The most successful solutions listed the six possible scenarios and calculated the probabilities of each before summing. More able candidates recognised that combinations could be used efficiently for this.

The most common misconception was to assume that multiplying the probabilities for five residents in favour and two residents not in favour was all that was required.

**Question 4**

Many good solutions were seen for this probability question. The best included a tree diagram which clarified the information provided. Candidates should be aware that a visual representation of probability data and the related scenarios often leads to efficient solutions. Good solutions then identified the relevant branch and formed a linear equation which was solved accurately. Weaker solutions often used the probability complement for one of the criteria. Again, some candidates unnecessarily converted their exact fractional probability to a rounded three significant figure answer.

A small number of candidates misinterpreted the success criteria, assuming that Aran was not wearing a hat, or not wearing a scarf or not wearing both of them, and then solved the more complex equation.

**Question 5**

Candidates found this probability question challenging and few made an attempt. Many candidates were able to identify the probability of event *A* as $\frac{1}{2}$, but made little progress in determining the probability of event *B*. The most successful solutions used a simple diagram of the four boxes and listed the possible scenarios that could be obtained, before identifying those that fulfilled the given criteria. A few failed to realise that the yellow and green marbles could simply swap places and the red marble would still fulfil the criteria. Less successful approaches were to attempt to use permutations to calculate possible outcomes, or to assume that the green and yellow marbles were always in boxes *M* and *N*.

Where candidates had found probabilities for events *A* and *B*, most used the relationship $\mathrm{P}(A)\times\mathrm{P}(B)=\mathrm{P}(A\cap B)$ to determine independence and interpreted their findings appropriately. A very small number of candidates used the conditional probability approach and also interpreted their data appropriately.

**Question 6**

This was a relatively standard normal approximation question. It was encouraging to see sketches of the normal distribution being used to identify the appropriate probability area curve in many solutions. The number of candidates who did not attempt **6(c)** was higher than anticipated.

**(a)** Almost all candidates were able to find the anticipated probability. Good solutions clearly stated the normal standardisation formula before evaluating to find the *z*-value and then stating the appropriate probability accurately. A small number of solutions assumed that time was not continuous and used a continuity correction. An unexpectedly high number of solutions did not include an unevaluated normal standardisation formula with values substituted, which is essential working for this component.

A few candidates found the complementary probability, which may have been avoided by using a simple sketch of the normal distribution curve.

**(b)** Most candidates used the normal standardisation formula with both of the times given, and accurately found the probability that a single cyclist met the criteria. As the upper criteria was the complement to **6(a)**, it was anticipated that candidates would use their previous work here. A significant number of candidates did not make any further progress and did not find the probability that all four cyclists fulfilled the criteria. The best solutions used a four or more significant figure probability in the calculation and then rounded the final answer to three significant figures. A common error was to approximate prematurely when finding the probability for a single cyclist and then using the three significant figure value, resulting in an inaccurate solution.

**(c)** An unexpected number of candidates omitted this question, although it involved a fairly standard process. Most candidates were able to state the required *z*-values for the given probabilities. The use of critical values is expected in this component. A common error was not to state the critical value linked to a probability of 10%, but instead to find a value using an alternative approach. The best solutions then formed the two simultaneous equations and showed a clear algebraic method to find the values of the mean and standard deviation. Any correct method for solving simultaneous equations was acceptable, but if a calculator is used, then there is an expectation that the two equations will be in the same form.

The most common error was to link 36 minutes with 0.739, an error which the use of a sketch of the normal distribution curve could have helped avoid. A small number of solutions included a time continuity correction.

**Question 7**

Most candidates used an appropriate combinations approach to this question. Solutions with simple ‘diagrams’ illustrating possible scenarios were often more successful.

**(a)** The criteria used in this part was found challenging by some, but many good solutions were seen. The first approach was to consider the arrangements of the letters with the Es together and the Ds removed. The best solutions used a simple diagram to illustrate the scenario and found the number of arrangements for these seven letters, then considered how the Ds would be inserted and multiplied by the correct factor. The most able candidates used ${}^6C_2$ at this stage. A common error was not dividing by 2 as the Ds could not be identified.

The second approach was considering the total number of arrangements that all the letters could make with the Es together and then subtracting the number of arrangements with the Es and the Ds both being grouped together. The best solutions clearly identified the scenario being considered and then used the separate answers to calculate the required value. Again, a common error with this method was not dividing the first term by 2 as the Ds could not be identified.

**(b)** Most candidates did not appreciate that a different approach was required for this part as a probability was requested, so the repeated letters should be considered identifiable to ensure that the total number of possible outcomes could be obtained.

The most successful solutions illustrated the possible scenarios that fulfilled the given condition and then calculated the number of arrangements for each scenario. The values were then summed to produce the numerator of the probability. The denominator required the calculation of the total possible number of arrangements and, because a probability was required, repeats were not considered.

The majority of candidates did assume that the letters were not identifiable and used a similar method but divided through consistently by 2! for the Ds and 3! for the Es and obtained the same final answer.

A small number of candidates assumed that the condition in **7(a)** also applied. Candidates should be aware that the question would be structured on the paper so that any requirement for both parts would be stated before the **part (a)** information.

**(c)** This question had a more standard success criteria and most candidates were able to make some progress to a solution. The best solutions listed the possible scenarios in a systematic manner, then calculated the number of selections which fulfilled each scenario before showing a clear addition. Many solutions omitted one or more possible scenarios and some solutions included scenarios with three Ds, which is not possible from the word DELIVERED. A common misunderstanding was that the Ds and Es needed to be included in the calculation of the selections, and so multiplication by additional combinational terms was seen frequently.

Some more able candidates recognised that, as all possible scenarios involved DE, they were effectively looking at selections from the remaining seven letters. Therefore, as only the E was repeated, the scenarios consisted of assuming zero, one or two Es are selected, with the remaining letters picked from the letters DLIVR.

# MATHEMATICS

**Paper 9709/62**
**Probability & Statistics 2**

## Key messages

- Candidates are reminded of the need to present work clearly with just one solution offered.
- Full working must be clearly shown. A final answer with no justification is not sufficient.
- If questions are continued on additional pages, the question number must be clearly identified.
- It is important that candidates refer to the context given in the question when requested to do so, as general statements (such as those found in the course textbook) will not be sufficient.
- Candidates should appreciate the difference between rounding to three significant figures and three decimal places, as the latter will not always give the accuracy required on this paper.

## General comments

This was a reasonably well attempted paper. There were some very good and well-presented scripts, but equally there were some candidates who seemed to not be fully prepared for the demands of the paper. Questions that were well attempted were **Questions 1**, **2(b)(i)**, **2(c)**, **3(b)** and **5(a),** whilst **Questions 5(b)**, **6(b)(c)** and **(d)** and **2(b)(ii)** were found to be more demanding. On the whole, adequate working was given, but there were occasions where candidates did not fully justify their answers.

Comments on individual questions follow, but it should be noted that there were many good, fully correct solutions offered as well as the common errors highlighted below.

## Comments on specific questions

### Question 1

**(a)** This was a good, standard question and was well attempted. Most candidates found the correct $z$-value and substituted it into an equation of the correct form. However, some candidates set up an incorrect confidence interval using 49 rather than $\frac{49}{140}$ .

**(b)** Some candidates realised that reducing the percentage of the confidence interval was required here, though some thought it should increase. Others did not realise that the question required use of Anita's survey results, so changing the number of students in the sample was not an option here.

### Question 2

**(a)** Many candidates found two correct assumptions, but as highlighted in the Key messages, these were not always put into the context of the question. Others were confused by what the question was asking and gave answers which were not assumptions, such as 'mean and variance must be equal'.

**(b) (i)** This question was well attempted with most candidates showing the required working and reaching the correct answer. It is important that the Poisson expression is clearly written out i.e. all relevant working is shown. As highlighted in the Key messages, some candidates gave an answer of 0.087, possibly confusing three significant figures with three decimal places.

**(ii)** A variety of errors were seen here. Two Poisson probabilities were required, which needed to be multiplied together and then doubled. Very few candidates realised the need to double and gave a final answer of 0.111 . Other candidates found the two probabilities but added them and often end errors were seen when calculating 'at least two'.

**(c)** This part was well attempted. Common errors seen were standardising without a continuity correction, standardising with an incorrect one, or choosing the wrong probability area. Many candidates gained full marks.

**Question 3**

**(a)** In the past, questions like this have not been well attempted, so it was pleasing to note here that more candidates were able to use the symmetry of the diagram to work out the required probability. It is important that candidates have an understanding of probability density functions and their graphs rather than just relying on formulae to find probabilities.

**(b) (i)** This part was particularly well attempted, and most candidates were able to show by integration that *k* was $\frac{10}{81}$. Few errors were seen here, though weaker candidates occasionally did not clearly equate their integral to 1, and errors attempting to integrate f($x$) were seen.

**(ii)** Many candidates calculated E($X$) rather than realising that, from given information, it must equal $\frac{3}{2}$. Some candidates merely calculated E($X^2$) and left their answer as $\frac{18}{7}$, and errors attempting to integrate $x^2$f($x$) were seen, but overall, this part was well attempted.

**Question 4**

**(a)** It was pleasing to note that this part, requiring the probability of a Type I error, was well attempted. Many candidates successfully calculated the correct Poisson probability and fully justified their answer. Some candidates used an incorrect $\lambda$, or incorrectly included P($X$ = 3) in their expression, but the most common error was to calculate 1 – P($X$ = 0,1,2) rather than P($X$ = 0,1,2). Occasionally full working was omitted, which as highlighted in the Key messages, is not acceptable. It was expected here that the full Poisson expression was clearly shown, and not just a final answer given. Occasionally a final answer of 0.077 was seen, which once again shows a possible confusion between three significant figures (as required) and three decimal places.

**(b)** Again, it was pleasing to note that many candidates correctly found the probability of a Type II error in this context, as questions on Type I and Type II errors have not always been well answered in the past. As in **part (a)**, end errors and lack of working were noted. The most common error was to calculate P(0,1,2) rather than 1 – P(0,1,2). Again, a final answer of 0.063 was occasionally seen with no previous working to justify three significant figures of accuracy.

**Question 5**

**(a)** This part was well attempted. Errors included incorrect values for the variance (commonly using $3.6^2 + 2 \times 3.7^2$ rather than $3.6^2 + 4 \times 3.7^2$ or subtracting the two variances rather than adding) and finding an incorrect probability area (>0.5 rather than <0.5).

**(b)** This was a more unusual question of its type and was a challenge for many candidates. Candidates had difficulty with how to deal with the 60 per cent and the 40 per cent of the boxes containing different packets. It was necessary to deal with the large packets and find the probability of the total mass being greater than 4080 separately from the small packets and their probability of the total mass being greater than 4080. These two probabilities with the relative proportions of 60 per cent and 40 per cent were then to be combined for the final probability. Few candidates scored full marks.

**Question 6**

**(a)** Most candidates answered this question well, showing understanding of the difference between one and two tailed tests.

**(b)** Many candidates did not give a valid comparison here, which was required to explain the conclusion drawn. The conclusion should have been in context and candidates did not always give their conclusion in this form.

**(c)** Many candidates calculated $\Phi(2.14)$ rather than $1 - \Phi(2.14)$, whereas others left their answer as 0.0162 or did not find the correct inequality. Some candidates did not know how to approach the question.

**(d)** The most common error here was inconsistent use of signs. Many candidates standardised correctly, though some omitted $\sqrt{100}$. Many realised that this standardisation should be equated to the appropriate *z*-value, but used +1.645 when –1.645 was required. The sign needed to be consistent with the standardisation attempt. Other errors included using 1.96 or sometimes not equating to a *z*-value at all. After reaching 25.4, many candidates thought a region was required rather than a value for *m*.