Cambridge Assessment International Education

Cambridge International A Level

MATHEMATICS **9709/33**

Paper 3 Pure Mathematics 3 **May/June 2020**

MARK SCHEME

Maximum Mark: 75

Published

Students did not sit exam papers in the June 2020 series due to the Covid-19 global pandemic.

This mark scheme is published to support teachers and students and should be read together with the question paper. It shows the requirements of the exam. The answer column of the mark scheme shows the proposed basis on which Examiners would award marks for this exam. Where appropriate, this column also provides the most likely acceptable alternative responses expected from students. Examiners usually review the mark scheme after they have seen student responses and update the mark scheme if appropriate. In the June series, Examiners were unable to consider the acceptability of alternative responses, as there were no student responses to consider.

Mark schemes should usually be read together with the Principal Examiner Report for Teachers. However, because students did not sit exam papers, there is no Principal Examiner Report for Teachers for the June 2020 series.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the June 2020 series for most Cambridge IGCSE™ and Cambridge International A & AS Level components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks
1	State or imply non-modular inequality $(2x-1)^2 > 3^2(x+2)^2$, or corresponding quadratic equation, or pair of linear equations	**B1**
	Make reasonable attempt at solving a 3-term quadratic, or solve two linear equations for x	**M1**
	Obtain critical values $x=-7$ and $x=-1$	**A1**
	State final answer $-7 < x < -1$	**A1**
	Alternative method for question 1	
	Obtain critical value $x=-1$ from a graphical method, or by solving a linear equation or linear inequality	**B1**
	Obtain critical value $x=-7$ similarly	**B2**
	State final answer $-7 < x < -1$ [Do not condone $\leqslant$ for $<$ in the final answer.]	**B1**
		4

Question	Answer	Marks
2	Commence integration and reach $a(2-x)e^{-2x}+b\int e^{-2x}\,dx$, or equivalent	**M1***
	Obtain $-\frac{1}{2}(2-x)e^{-2x}-\frac{1}{2}\int e^{-2x}\,dx$, or equivalent	**A1**
	Complete integration and obtain $-\frac{1}{2}(2-x)e^{-2x}+\frac{1}{4}e^{-2x}$, or equivalent	**A1**
	Use limits correctly, having integrated twice	**DM1**
	Obtain answer $\frac{1}{4}\left(3-e^{-2}\right)$, or exact equivalent	**A1**
		5

Question	Answer	Marks
3(a)	Remove logarithms correctly and state $1 + e^{-x} = e^{-2x}$, or equivalent	**B1**
	Show equation is $u^2 + u - 1 = 0$, where $u = e^x$, or equivalent	**B1**
		2
3(b)	Solve a 3-term quadratic for u	**M1**
	Obtain root $\frac{1}{2}\left(-1+\sqrt{5}\right)$, or decimal in [0.61, 0.62]	**A1**
	Use correct method for finding x from a positive root	**M1**
	Obtain answer $x = -0.481$ only	**A1**
		4

Question	Answer	Marks
4(a)	Use the product rule	**M1**
	State or imply derivative of $\tan^{-1}(\frac{1}{2}x)$ is of the form $k/(4+x^2)$, where $k = 2$ or 4, or equivalent	**M1**
	Obtain correct derivative in any form, e.g. $\tan^{-1}\left(\frac{1}{2}x\right)+\frac{2x}{x^2+4}$, or equivalent	**A1**
		3
4(b)	State or imply y-coordinate is $\frac{1}{2}\pi$	**B1**
	Carry out a complete method for finding p, e.g. by obtaining the equation of the tangent and setting $x = 0$, or by equating the gradient at $x = 2$ to $\frac{\frac{1}{2}\pi - p}{2}$	**M1**
	Obtain answer $p = -1$	**A1**
		3

Question	Answer	Marks
5	Use tan $2A$ formula to express RHS in terms of tan θ	**M1**
	Use tan $(A \pm B)$ formula to express LHS in terms of tan θ	**M1**
	Using tan 45° = 1, obtain a correct horizontal equation in any form	**A1**
	Reduce equation to $2\tan^2\theta + \tan\theta - 1 = 0$	**A1**
	Solve a 3-term quadratic and find a value of θ	**M1**
	Obtain answer $\theta = 26.6°$ and no other	**A1**
		6

Question	Answer	Marks
6(a)	Sketch a relevant graph, e.g. $y = x^5$	B1
	Sketch a second relevant graph, e.g. $y = x + 2$ and justify the given statement	B1
		2
6(b)	State a suitable equation, e.g. $x = \dfrac{4x^5 + 2}{5x^4 - 1}$	B1
	Rearrange this as $x^5 = 2 + x$ or commence working *vice versa*	B1
		2
6(c)	Use the iterative formula correctly at least once	M1
	Obtain final answer 1.267	A1
	Show sufficient iterations to 5 d.p. to justify 1.267 to 3 d.p., or show there is a sign change in the interval (1.2665, 1.2675)	A1
		3

Question	Answer	Marks
7(a)	State or imply the form $\dfrac{A}{2x-1}+\dfrac{B}{2x+1}$ and use a relevant method to find A or B	**M1**
	Obtain $A=1$, $B=-1$	**A1**
		2
7(b)	Square the result of part (**a**) and substitute the fractions of part (**a**)	**M1**
	Obtain the given answer correctly	**A1**
		2
7(c)	Integrate and obtain $-\dfrac{1}{2(2x-1)}-\dfrac{1}{2}\ln(2x-1)+\dfrac{1}{2}\ln(2x+1)-\dfrac{1}{2(2x+1)}$, or equivalent	**B3, 2, 1, 0**
	Substitute limits correctly	**M1**
	Obtain the given answer correctly	**A1**
		5

Question	Answer	Marks
8(a)	State or imply $\overrightarrow{AB}$ or $\overrightarrow{AD}$ in component form	**B1**
	Use a correct method for finding the position vector of *C*	**M1**
	Obtain answer $4\mathbf{i}+3\mathbf{j}+4\mathbf{k}$, or equivalent	**A1**
	Using the correct process for the moduli, compare lengths of a pair of adjacent sides, e.g. *AB* and *AD*	**M1**
	Show that *ABCD* has a pair of unequal adjacent sides	**A1**
	Alternative method for question 8(a)	
	State or imply $\overrightarrow{AB}$ or $\overrightarrow{AD}$ in component form	**B1**
	Use a correct method for finding the position vector of *C*	**M1**
	Obtain answer $4\mathbf{i}+3\mathbf{j}+4\mathbf{k}$, or equivalent	**A1**
	Use the correct process to calculate the scalar product of $\overrightarrow{AC}$ and $\overrightarrow{BD}$, or equivalent	**M1**
	Show that the diagonals of *ABCD* are not perpendicular	**A1**
		5
8(b)	Use the correct process to calculate the scalar product of a pair of relevant vectors, e.g. $\overrightarrow{AB}$ and $\overrightarrow{AD}$	**M1**
	Using the correct process for the moduli, divide the scalar product by the product of the moduli of the two vectors and evaluate the inverse cosine of the result	**M1**
	Obtain answer 100.3°	**A1**
		3

Question	Answer	Marks
8(c)	Use a correct method to calculate the area, e.g. calculate *AB.AC* sin *BAD*	**M1**
	Obtain answer 11.0 (**FT** on angle BAD)	**A1 FT**
		2

Question	Answer	Marks
9(a)	Eliminate u or w and obtain an equation w or u	**M1**
	Obtain a quadratic in u or w, e.g. $u^2 - 2iu - 6 = 0$ or $w^2 + 2iw - 6 = 0$	**A1**
	Solve a 3-term quadratic for u or for w	**M1**
	Obtain answer $u = \sqrt{5} + \mathrm{i}$, $w = \sqrt{5} - \mathrm{i}$	**A1**
	Obtain answer $u = -\sqrt{5} + \mathrm{i}$, $w = -\sqrt{5} - \mathrm{i}$	**A1**
		5
9(b)	Show the point representing $2 + 2\mathrm{i}$	**B1**
	Show a circle with centre $2 + 2\mathrm{i}$ and radius 2 (**FT** is on the position of $2 + 2\mathrm{i}$)	**B1 FT**
	Show half-line from origin at 45° to the positive x-axis	**B1**
	Show line for Re $z = 3$	**B1**
	Shade the correct region	**B1**
		5

Question	Answer	Marks
10(a)	State or imply $\frac{\mathrm{d}V}{\mathrm{d}t} = -k\sqrt{h}$	**B1**
	State or imply $\frac{\mathrm{d}V}{\mathrm{d}h} = 2\pi rh - \pi h^2$, or equivalent	**B1**
	Use $\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{\mathrm{d}V}{\mathrm{d}h}.\frac{\mathrm{d}h}{\mathrm{d}t}$	**M1**
	Obtain the **given answer** correctly	**A1**
		4
10(b)	Separate variables and attempt integration of at least one side	**M1**
	Obtain terms $\frac{4}{3}rh^{\frac{3}{2}} - \frac{2}{5}h^{\frac{5}{2}}$ and $-Bt$	**A3, 2, 1, 0**
	Use $t = 0$, $h = r$ to find a constant of integration c	**M1**
	Use $t = 14$, $h = 0$ to find B	**M1**
	Obtain correct c and B, e.g. $c = \frac{14}{15}r^{\frac{5}{2}}$, $B = \frac{1}{15}r^{\frac{5}{2}}$	**A1**
	Obtain final answer $t = 14 - 20\left(\frac{h}{r}\right)^{\frac{3}{2}} + 6\left(\frac{h}{r}\right)^{\frac{5}{2}}$, or equivalent	**A1**
		8