MATHEMATICS

Paper 9709/12
Pure Mathematics

Key messages

Centres would be well advised to ensure students are familiar with the advice for candidates in the syllabus. Particularly relevant to this paper are:

- answers to problems involving trigonometry and calculus which can be obtained directly from some types of calculator will gain no credit unless accompanied by clear working.
- numerical answers required to a given accuracy can rarely be reached if answers to intermediate parts are not used to a greater degree of accuracy.

In some questions, even though it may not be specifically required, a sketch graph can be most helpful.

General comments

When a result is given, the working leading to that result must be clear, unambiguous and correct for all marks to be awarded.

Although a false zero solution can be obtained and then discarded in some algebraic solutions, this is rare and generally, where there is a zero solution as in **4(ii)**, zero solutions are valid.

Comments on specific questions

Question 1

Those candidates who selected the appropriate term from the expansion were able to quickly reach the required value. Dealing with the negative term provided a challenge for some candidates.

Answer: $p = 6$

Question 2

The correct application of integration to obtain the Cartesian equation was often seen. When a constant of integration was used the correct value of k was usually reached.

Answer: $k = \frac{2}{3}$

Question 3

Most solutions began with attempts to find angle CBA or angle CBD. The best answers used non-rounded values for these angles to find the areas of the sector BCYD and the triangle BCD. Candidates demonstrated good knowledge of the sector area formula and the different triangle area formulae. Some used the formula for the area of a segment. Most used the formula appropriate to the units of their angle CBA or CBD and realised that in order to reach a correct solution intermediate values of angles and areas needed to be expressed to more than three significant figures.

Answer: 118.0

Question 4

(i) The differentiation required to find both derivatives was well understood by most candidates and many found both results correctly.

(ii) The method of setting the first derivative to zero was almost always seen and there were many completely correct methods used to find both the values of x. It was expected that the second derivative would be used to identify the turning points but a few chose to use the change in sign of the gradient. This was acceptable when clearly explained but not when using the point where $x = \frac{1}{2}$.

Answers: **(i)** $\frac{dy}{dx} = -2(2x-1)^{-2} + 2$, $\frac{d^2y}{dx^2} = 8(2x-1)^{-3}$ **(ii)** $x = 0$ (Maximum), $x = 1$ (Minimum)

Question 5

(i) The need to use the scalar product and set it equal to zero appeared to be understood very well by the majority of candidates who generally went on to solve the resulting quadratic correctly.

(ii) As with part **(i)** the requirement to use the scalar product was evident in most answers and many wholly correct methods were seen. When they did not lead to a correct answer this was usually the result of a rounding error.

Answers: **(i)** $q = 2$ and $q = -\frac{11}{3}$ **(ii)** 130.7°

Question 6

(i) The S_n formula for a geometric progression was almost always correctly quoted and used with $r = 2$. The candidates who used $S_n = \frac{p(2^n - 1)}{(2-1)}$ were invariably more successful at obtaining the correct result without error than those who used the alternative $S_n = \frac{p(1-2^n)}{(1-2)}$.

(ii) The n^{th} term and S_n formulae were correctly quoted and used by the majority of candidates to form equations in n and p. Solving the two equations produced some straightforward algebraic methods and some very drawn out methods but many led to the correct values of n and p.

Answer: **(ii)** $n = 42$, $p = 8$

Question 7

(a) Candidates were familiar with solving a trigonometric quadratic but the use of 2θ, rather than the more familiar θ, appeared to cause some confusion. Most candidates were able to set up a quadratic in cos 2θ, but a significant proportion then arrived at $\cos\theta = -\frac{1}{3}$ rather than $\cos 2\theta = -\frac{1}{3}$. Some candidates gave only one solution in the range $0^\circ \le \theta \le 180^\circ$, rather than two solutions. Once again it should be noted that candidates need to avoid rounding prematurely. There is a need to work with values to a greater degree of accuracy than required in the final answer.

(b) Almost all candidates were able to find the correct value of a. Finding the value of b was completed less successfully. Candidates are expected to be able to work with exact values and those who used $\sqrt{3}$ in finding b were able to find the exact value as required. Those candidates who assumed that $\tan\left(\frac{b\pi}{6}\right) = b\tan\left(\frac{\pi}{6}\right)$ were unable to find the correct value of b.

Answers: **(a)** 54.7° and 125.3° **(b)** $a = \sqrt{3}$, $b = 2$

Question 8

(i) This part proved to be a straightforward two marks for most candidates

(ii) It was expected that part **(i)** would be used to find the value of x at the minimum point and use this as the value of k. Many candidates did this but some only quoted $x = 2$ which gained no credit.

(iii) The method for obtaining the inverse of a quadratic function using the completed square form was well understood and applied. Only the stronger candidates made the connection between the domain of f(x) and the range of $f^{-1}(x)$ and realised that the negative square root alternative should be used. Stating $k = 1$ at the start of this question part should have led to the implication that the range of f was now $f(x)<4$ which was the required domain for $f^{-1}(x)$.

(iv) A correct expression for the combined function was reached by nearly all candidates who attempted this part. Some candidates used either the domain or range of f(x) to deduce the upper value of the range of gf but the positive nature of gf was not seen by many and the lower limit was usually missing.

Answers: **(i)** $(x-2)^2+3$ **(ii)** Largest k is 2 **(iii)** $f^{-1}(x) = 2-\sqrt{(x-3)}$, $x > 4$

(iv) $gf(x) = \dfrac{2}{((x-2)^2+2)}$, $0 < gf(x) < \dfrac{2}{3}$

Question 9

In both parts answers unsupported by detailed working gained very little or no credit

(i) This part was accessible to the majority of candidates and many gave fully correct solutions. Almost all candidates appreciated the need to integrate πy^2 and those who did generally scored full marks for this part.

(ii) Candidates were generally aware of the need to find $\dfrac{dy}{dx}$, but met with varying degrees of success. The most common source of error was the application of the chain rule. The majority of candidates were able to find the coordinates of P and most of them were able to find the gradient of a perpendicular. Finding the equation of a straight line through a given point with a given gradient was generally well understood.

Answers: **(i)** $\dfrac{117\pi}{4}$ **(ii)** $\dfrac{78}{11}$

Question 10

(i) The elimination of y from the two equations was favoured by most candidates**.** Either accurate squaring of both sides of the resulting equation or obtaining a quadratic in $x^{\frac{1}{2}}$ usually led to the required coordinates. Correct application of the distance formula was seen in nearly all attempts.

(ii) Most successful solutions involved setting the gradient of the curve equal to the gradient of AB although a few attempts involved finding the equation of the tangent parallel to AB and then its single point of intersection with the curve.

(iii) Those candidates who had found the point T in part **(ii)** were usually able to find the equation of the line through T perpendicular to AB and solve this with the given equation of AB. Some candidates did not realise that $y = x + 3$ was the equation of AB and found the equation from their coordinates of A and B found in part **(i)**

Answers: **(i)** $8\sqrt{2}$ (or equivalent) **(ii)** (4,8) **(iii)** $(\dfrac{9}{2}, \dfrac{15}{2})$

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Key messages

Candidates should always ensure that they are working to the required accuracy throughout the question in order to give their final answer to 3 significant figures or the accuracy otherwise specified. A check should also be made to ensure that the question has been fully answered and that any answer is given in the required form. In questions where a candidate is expected to show a certain result, it is essential that each step of the process is shown.

General comments

As the number of candidates taking the paper was small compared with other sessions, it is difficult to make generalisations on the overall performance and common errors. The report has been written to indicate the intended method for each question.

Comments on specific questions

Question 1

It was intended that candidates make use of the identity $\sec^2\theta = 1 + \tan^2\theta$ in order to obtain a quadratic equation in $\tan\theta$. This quadratic equation could then be factorised in order to obtain 2 solutions.

Answer: 71.6°, 153.4°

Question 2

A value of $x = -\frac{1}{2}$ could be obtained from either obtaining 2 linear equations with the signs of 2x being different in each case, of by forming a quadratic equation by squaring each side of the given equation. It was intended that this value of $x = -\frac{1}{2}$ be substituted into the given expression $|4x - 3| - |6x|$ and then the expression evaluated. Many candidates squared the given expression in an incorrect attempt to deal with the modulus signs.

Answer: 2

Question 3

It was essential that the given equation be written in the form $\ln y = \ln A + px + p$ to start the solution, realising that the gradient of the given straight line graph was equal to *p.* The gradient of 0.75 could be worked out from the given points on the graph. A substitution of one of the sets of coordinates into $\ln y = \ln A + px + p$, giving either $2.835 = \ln A + 0.75(1) + 0.75$ or $6.585 = \ln A + 0.75(6) + 0.75$, and evaluation would then give a value for *A*. It was important that candidates made correct substitutions into the straight line equation.

It was also acceptable to produce 2 simultaneous equations from substitution into the straight line equation $\ln y = \ln A + px + p$ and then solve for p and for A.

Answer: $p = 0.75$, $A = 3.80$

Question 4

(i) An attempt at algebraic long division, showing all the relevant steps in the process, was expected.

(ii) By making use of the work in part **(i)**, it could be deduced that $4x^3 + 8x^2 + 11x + 4$ could be written as $(2x-1)(2x^2+3x+4)$. Calculation of the discriminant of the quadratic expression provided the evidence that the given equation had only one root. Attempts to solve the equation $(2x^2+3x+4)=0$ using the quadratic formula were also acceptable. It was essential that candidates offered a conclusion after either using the discriminant or the quadratic formula.

Question 5

(i) To obtain full marks in this part, it was essential that the steps $10(4x+1) = e^{2x}$ and $\ln(40x+10) = 2x$ be shown in order to achieve the given result.

(ii) Candidates needed to ensure that they had written down sufficient iterations to the required level of accuracy. All too often, a candidate obtained the final iteration required on their calculator, but omitted to write it down. A check that each iteration is written down and that the final answer is given to the required level of accuracy prevents the needless loss of accuracy marks.

(iii) It was intended that the answer obtained in part **(ii)** be substituted into the gradient function, obtained by differentiation of a quotient.

Answer: **(ii)** 2.316 **(iii)** 16.1

Question 6

(a) Candidates were expected to deduce that the integrals required were of logarithmic form from the result they were ultimately expected to show. Again, as this was a question where a result was to be shown, each step of the process needed to be shown. The 4 required steps were the integrals in logarithmic form, application of the limits, use of the addition/subtraction rules for logarithms and use of the power rule for logarithms. The last three steps did not have to be done in this order.

(b) An expansion of the given brackets and simplification using $\sin 2x = 2\sin x \cos x$, needed to be performed before using the double angle formula for $2\cos^2 x$. This resulted in an integrand of $\cos 2x + 1 + 4\cos x$ and each term being in a form which could be integrated.

Answer: **(b)** $x + \frac{1}{2}\sin 2x + 4\sin x + c$

Question 7

(i) It was expected that candidates would find $\frac{dx}{dt}$, $\frac{dy}{dt}$ and hence $\frac{dy}{dx}$ which could then be equated to 2 and then manipulated to obtain the given result. Again, as a specific result needed to be shown, it was essential that each of the above steps be seen.

(ii) Even if the required result has not been shown in part **(i)**, candidates could still use this result in part **(ii)**. Often candidates do not do this. It was essential that the angles should be in radians and that the identity $R\sin(2t-\alpha) \equiv R\sin 2t\cos\alpha - R\cos 2t\sin\alpha$ be made use of. Once a value for the parameter t was found it was essential to then continue to calculate the x and y coordinate to the given level of accuracy.

Answer: **(ii)** $(0.361, 3.57)$

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Paper 9709/32
Pure Mathematics

Key messages

- Candidates need to know what is meant by an exact answer.
- Candidates should be able to apply the chain rule in differentiation.
- Candidates need to understand the difference between significant figures and decimal places.
- Candidates should be confident knowing when to use brackets.

General comments

The standard of work on this paper varied considerably, although all questions were accessible to the strongest candidates. Some candidates found certain questions extremely difficult and made little progress. To ensure they are fully prepared for the exam, candidates are advised to study the complete syllabus, and to make use of the available past question papers and mark schemes.

The questions or parts of questions that were generally done well were **Question 1(i) (**the laws of logarithms), **Question 2(i)** (numerical iteration), **Question 4** (integration by parts), **Question 8(i)** (partial fractions), **Question 8(ii)** (binomial expansions), **Question 9(i)** (angle between planes). Those that were done less well were **Question 1(ii)** (solution of quadratic equation), **Question 3(ii)** (solution of trigonometrical equation), **Question 5** (implicit differentiation), **Question 6** (solution of differential equation), **Question 7(a)** (solution of complex quadratic equation), **Question 7(b)** (sketch in Argand diagram), **Question 9(ii)** (line of intersection of planes), **Question 10(i)** (integration using substitution) and **Question 10(ii)** (chain rule).

In general the presentation of the work was good. Candidates should bear in mind that scripts will be scanned for marking and they should use a **black** biro, reasonable sized lettering and symbols and present their work clearly. Ink should be avoided since this tends to be absorbed by the paper and is often visible in a scan of the reverse side.

It was pleasing to see that candidates were aware of the need to show sufficient working in their solutions. Previous reports mentioned this in the context of solving a quadratic equation and substituting limits into an integral. Some important points for candidates to understand are that they must use the method asked for in the question **and** note carefully what form of the final answer is acceptable (**Questions 1(ii)**, **2(i)**, **3(i)**, **10(i)**, and **10(ii)**). These points will be discussed in detail below.

Where answers are given after the comments on individual questions, it should be understood that the form given is not necessarily the only ‘correct answer’.

Comments on specific questions

Question 1

(i) This question was done well. However, a few candidates had + 4x, whilst others believed that they were working with natural logarithms, hence e^2 was often seen.

Answer: $x^2 - 4x - 100 = 0$

(ii) Candidates starting with the incorrect quadratic equation were still able to gain the method mark, provided they solved using the quadratic formula approach. However those candidates who showed no working and went straight to their calculator were unable to score anything. Although most candidates started from the correct quadratic equation, approximately half of these did not gain both marks since they did not reject the negative root.

Answer: 12.2

Question 2

(i) Nearly all candidates scored full marks, the exceptions usually being those who did not calculate α to the required number of decimal places.

Answer: 1.3195

(ii) Again most candidates gained both marks, although a few candidates did not use consistent symbols throughout their equations, whilst others gave a decimal answer rather than an exact answer.

Answer: $\alpha = \sqrt[5]{4}$

Question 3

(i) Most candidates were able to correctly expand $\sin(\theta + 45°)$, but $\cos(\theta + 60°)$ proved more difficult. Often sign errors were present, as was the presence of 45° instead of 60°. Incorrect exact values of trigonometrical ratios were rare, however many arithmetical errors were seen, so that not many candidates acquired the correct expression for $\tan\theta$ even though un-simplified forms were accepted.

Answer: $\tan\theta = (2\sqrt{2} - 1)/(1 - \sqrt{6})$ or equivalent

(ii) Many candidates did not gain full marks due to incorrectly opting for $(360° - \theta^*)$ instead of $(180° + \theta^*)$ as their second answer.

Answer: 128.4° 308.4°

Question 4

This was a high scoring question with most candidates gaining 4 or 5 marks. Even the weaker candidates usually managed to undertake the integration by parts correctly, although many of these candidates then did not consider the remaining integrand as just a power of x term. A few candidates did not realise that the form of the answer given meant that converting to decimals was not appropriate.

Question 5

Nearly all candidates found this to be a difficult question. Unless they realised that either implicit differentiation or the derivative of $\sin^{-1}(\tan x)$ involving the chain rule was required, no progress was possible. However whilst many candidates did reach this stage, the conversion to the given answer as a function of x proved too difficult for most. Candidates did not realise that it was necessary to introduce Pythagoras and then the given relation to reach a term involving $1 - \tan^2 x$ from which the answer given is relatively easy to establish.

Question 6

Most candidates separated the variables correctly, but then often experienced trouble within their integration. It was common to see the minus sign omitted from one or both of the resulting integrals, as well as the integer 2 from the denominator of y^{-2} often appearing instead in its numerator. Even those candidates with the correct integrals often dropped negative signs when substituting the given conditions. Many candidates were unable to apply correct algebraic manipulation, with errors often seen in moving e^{-1} and $1/e$ within the equation resulting from the second boundary condition, with the inversion of some of the coefficients of these terms as well. Finally even some of those with the two correct simple equations did not solve accurately for k and the arbitrary constant.

Answer: $y = e^{x/2}$

Question 7

(a) Unfortunately candidates who tried to use Real and Imaginary Parts to solve this equation quickly found out that the resulting equations were not easily solvable without the use of a calculator. The correct approach was to use the quadratic formula, and if the arithmetic was undertaken correctly the calculation should have required the square root of a negative integer. However, many made arithmetical errors during their calculation and finished with something far more complicated. The correct two solutions were $((4 + 3i) \pm 3i)/(2 + 2i)$ which then required the multiplication of both the numerator and the denominator by $(2 - 2i)$.

NB Throughout the whole process all the multiplications of the complex numbers must be shown in detail, hence the statement within the question, 'showing all your working'. The reason for this is that today nearly all very basic calculators will undertake the task of multiplying complex numbers and if such detail was not demanded the question would simply become a button pressing exercise.

Answer: $1 - \mathrm{I}$; $5/2 + (1/2)i$

(b) Other than plotting the point u correctly, few candidates made any real progress. A few of the errors were the arg inequality being from the origin instead of from the point u, the horizontal bisector being at $z = 2i$ instead of at $z = i$, and a vertical bisector instead of a horizontal bisector. Even when all the correct lines were displayed candidates often incorrectly shaded the additional region up to $z = 2i$ or omitted the triangular region below the real axis.

Question 8

(i) Candidates who realised from the start that a 3 term expression was required usually worked accurately to score full marks, as did many candidates who instead considered a two term expression, with one of the terms being an improper fraction of linear polynomials which was then subdivided. However, the former method is the one to be encouraged as in the subdivision of the improper fraction approach it is easy to incorrectly set these two partial fractions to the original expression, instead of to that with the already determined partial fraction removed.

Answer: $2 - 4/(2 + x) + 6/(3 - 2x)$

(ii) This question also produced some sound mathematics, where even candidates who had just a couple of incorrect partial fractions from **(i)** were able to recover to gain 3 marks in this part. The removal of the constant terms from both partial fractions was usually undertaken correctly, as were the expansions. Unfortunately, miscopying of a sign or coefficient was seen too often. Candidates need to take extreme care when using an expression derived in an earlier part of the question.

Answer: $2 + (7/3)x + (7/18)x^2$

Question 9

(i) Many fully correct solutions were seen. Nearly all candidates commenced with the correct normal vectors, with any errors restricted to an arithmetical error in the scalar product or modulus, or choosing the incorrect angle at the end.

Answer: 56.9° or 0.994 radians

(ii) Many successful different methods were seen to this question, although the vector product approach was probably the most common. Unfortunately arithmetical errors were often present within this calculation. Some candidates scored the 3 marks for the vector product but had little understanding of actually what they were doing. This was in evidence when candidates opted to take this vector to be a point on the line or to use the normal vector to one of the planes as a point through which the line passes.

Answer: $\mathbf{r} = \mathbf{i} + \mathbf{j} + 4\mathbf{k} + \alpha(\mathbf{i} - 3\mathbf{j} - 7\mathbf{k})$ or equivalent

Question 10

(i) Several candidates incorrectly tried to undertake the integration with a mixture of u and x still within their integrand. Many candidates knew exactly what was required to evaluate the integral but made several careless errors. One such error was that of a sign in the derivative of cos x which, if it was the only error present, prevented the awarding of 2 marks. The other error was the omission of the bracket when introducing Pythagoras. This resulted in no additional marks being gained since the integrand was incorrect and not of the form required, hence the comment earlier in the report that candidates must be able to introduce and remove brackets correctly.

Answer: 8/21

Question 10

(ii) This was another calculus question in which candidates knew what to do but made many slips. Not introducing the chain rule in at least one of the two derivatives resulted in no marks being gained. Some candidates managed to introduce it successfully once and hence gained a couple of marks. Those who were successful on both occasions were able to score half marks. The remainder of the question required the combination of $\cos^{3/2} x$ with $\cos^{-1/2} x$ to produce $\cos^2 x$ and $\sin^4 x$ and $\sin^{-2} x$ to produce $\sin^2 x$. Poor algebra often meant that final equations were not of an acceptable form. Finally, there were cases seen where a candidate with the correct trigonometrical equation, for example $\tan \theta = \sqrt{6}$, then presented their answer as 1.18, that is to 3 significant figures instead to the required form of 3 decimal places, hence the early comment concerning decimal places and significant figures.

Answer: 1.183

MATHEMATICS

Paper 9709/42
Mechanics

Key messages

- Non-exact numerical answers are required correct to three significant figures as stated on the question paper and cases where this was not adhered to were seen in **Questions 1**, **3**, **4** and **7**. Candidates would be advised to carry out all working to at least 4sf if a final answer is required to 3sf.
- When answering questions involving any system of forces, a well annotated force diagram can help candidates make sure that they include all relevant terms when forming an equilibrium situation or a Newton's Law equation or a work/energy equation. Such a diagram was particularly useful on this paper in **Question 1** and both parts of **Question 4**.
- In questions such as **Question 6** in this paper, where acceleration is given as a function of time, it is important to realise that calculus must be used and that it is not possible to apply the equations of constant acceleration.

General comments

The paper was generally very well done by many candidates although as usual a wide range of marks was seen.

The presentation of the work was good in most cases and as the papers are now scanned, it is important that candidates write answers clearly using black pen.

In **Question 7**, the angle α was given exactly as $\sin \alpha = 0.8$ There is no need to evaluate the angle in this case and problems such as this can often lead to exact answers. so any approximation of the angle can lead to a loss of accuracy.

The examination allowed candidates at all levels to show their knowledge of the subject, whilst differentiating well between even the stronger candidates. **Questions 2** and **3** were found to be the easiest questions whilst **Questions 4** and **7** proved to be the most challenging.

One of the rubrics on this paper is to take $g = 10$ and it has been noted that virtually all candidates are now following this instruction. In fact in some cases, such as in **Question 2(i)** in this paper, it is impossible to achieve a correct given answer unless this value is used.

Comments on specific questions

Question 1

It would be sensible to draw a force diagram for this question showing the given tension in the string, the weight of the ring, the friction force acting vertically downwards and the reaction force acting directly to the left. The equilibrium equation in the vertical includes the weight, friction and the vertical component of the tension, namely 2.5 sin 15. Resolving forces horizontally gives the value of the normal reaction, R, in terms of the horizontal component of the tension. Replacing the friction term by $F = \mu R$ enables the required value of the coefficient of friction to be found. As the ring is on the point of moving upwards it must be realised that friction is acting vertically downwards and that the normal reaction acts directly to the left. A number of candidates assumed that the reaction also included a component of the weight when in fact the value of the reaction $R = 2.5 \cos 15$. Some candidates also made approximations in their calculations and did not gain accuracy marks.

Answer: $\mu = 0.144$

Question 2

(i) Since the answer is given in this question, care must be taken to show all working clearly. Most candidates used the constant acceleration formulae to approach this problem. It is a good idea to state the values of the variables which will be used, namely $u = 30$, $v = 0$ and $a = –g = –10$ before using the equations. By substituting these values into the equation $v^2 = u^2 + 2as$, the required given answer can be achieved. Some candidates wrongly chose to use $v = 30$, $u = 0$ and $a = g = 10$ which produced the given answer but with incorrect working and hence lost marks. It is also possible to achieve the answer by using energy principles. Another method seen was to find the time taken for the particle to reach the highest point and use this value of t in the equation $s = ut + ½at^2$ to find the required maximum height.

Answer: The maximum height is 45 m (given)

(ii) There are two approaches to this question, depending on the order in which the required time and speed are found. If the time is found first then use of the constant acceleration formula $s = ut + ½at^2$ with $s = 33.75$, $u = 30$ and $a = –g = –10$ leads to a quadratic equation in t which has 2 solutions, one corresponding to the upward motion and the other to the downward motion. The smaller of the two values should be taken for the required time. Once this time t has been found then use of the equation $v = u + at$ enables the required speed to be found. An alternative approach is to find the speed first by using $v^2 = u^2 + 2as$ with $u = 30$, $a = -g$ and $s = 33.75$. Once the speed has been found, use of the equation $v = u + at$ will give the value of the required time, t. Errors that were seen here generally occurred through use of $u = 0$ and hence an incorrect quadratic equation being found for t. Overall most candidates made a good attempt at this question.

Answers: The time taken to reach 33.75 m for the first time is 1.5 s. The speed of the particle is then 15 ms$^{–1}$

Question 3

Most candidates made a very good attempt at this question. The majority of candidates resolved forces vertically and horizontally. When resolving horizontally this produced a two term equation for $F \cos α$. Resolving vertically produced a two term equation for $F \sin α$. The values of F and α were found either by squaring and adding the two expressions, giving a value for F, or by using trigonometry, finding tan α and hence α itself. The main source of error seen in this question was using the wrong sign when resolving, but the majority of candidates produced excellent solutions

Answers: $F = 31.5$ (to 3sf) $α = 73.2$ (to 1dp)

Question 4

(i) In this question it is first necessary to use the given information in the relationship $P = Fv$ to determine the driving force, F, acting on the car. Since the car and trailer are travelling at a constant speed, the total resistance forces acting on the system of car and trailer must exactly balance this driving force. Applying this information enables the resistance force R N acting on the car to be found. Most candidates found this correctly. An error sometimes seen was to think that the resistance R itself balanced the driving force leading to an incorrect answer. Overall most candidates found this result correctly.

Answer: $R = 220$

(ii) In this question there are several solution methods available. The acceleration, a, can be found directly by applying Newton's second law to the system of car and trailer since this will not involve the tension, T, in the rod. Many candidates followed this method. Errors included missing out some resistance terms or wrongly including the tension in the rod. Once the acceleration is found, Newton's second law must be applied either to the car or to the trailer in order to find the required tension. An alternative method is to apply Newton's second law to both the car and to the trailer and solve the resulting simultaneous equations for a and T. Both methods were seen. This was found by candidates to be one of the more difficult questions, mainly due to including extra terms in their equations or forgetting to include relevant terms. Some candidates lost marks due to prematurely approximating their value of a and this also affected their resulting calculations for T.

Answers: The acceleration $a = 1/9$ ms$^{–2}$ = 0.111 ms$^{–2}$ (to 3sf) The tension in the rod $T = 113$ N (to 3sf)

Question 5

(i) In this question the most straightforward method of finding the distance travelled in the first 8 seconds is to evaluate the area bounded by the three lines of the graph and the horizontal t-axis. From the given information about the motion in the first 3 seconds, the speed at $t = 3$ can be found. Once this value is known, the area can be obtained by adding together the areas of the triangle based on $t = 0$ to $t = 3$, the trapezium based on $t = 3$ to $t = 5$ and the triangle based on $t = 5$ to $t = 8$. This value gives the distance travelled in the first 8 seconds. Almost all candidates made a good attempt at this question. Any mistakes seen were mainly from numerical errors when adding the areas or for misquoting formulae for areas.

Answer: The distance travelled in the first 8 seconds is 40 m

(ii) This part of the question requires an understanding of displacement and that the given value of 32 m enables the area enclosed between the lines of the graph and the t-axis from $t = 8$ to $t = 16$ to be determined. There were a number of candidates who believed that this area was 32 m when in fact the area should be taken as 40 m – 32 m. Once this area is known, it can be equated to an expression for the area of the triangle with base $t = 8$ to $t = 16$ and height V. This equation can be solved to give the required value of V. A number of candidates successfully reached this stage but gave their final answer as $V = 2$ when in fact it is clear from the diagram that V must be negative. Some tried to estimate the value of t at which the velocity reached a value of V but this is not necessary since the solution is independent of this value of t.

Answer: $V = –2$

Question 6

(i) In this question the acceleration is given as a function of time and so calculus must be used to determine velocity and it must be remembered that the equations for constant acceleration cannot be used. As there is a given answer, care must be taken to show all working. It is necessary first to find the velocity of the particle since the question refers to the time at which the particle is at instantaneous rest. Integration of the given expression for acceleration gives the velocity as a function of time, using the given conditions to find the constant of integration. The expression for velocity should now be set to zero and solved to give the value of t when the particle is at instantaneous rest. This t value is now used in the given expression for a to determine the required value of acceleration. Most candidates made good attempts at this question. Some did not show very much working when solving $v = 0$ and some struggled to manipulate the equation, which involved fractional powers of t.

Answers: The particle is at instantaneous rest at $t = 4$ and the acceleration is then $16 ms^{-2}$ (given)

(ii) In this part of the question the displacement of the particle at time $t = 5$ is required. This involves integrating the expression found for v in **6(i)** and using the initial conditions to find the constant of integration. Substituting the value $t = 5$ into this expression leads to the required result. In fact the answer is negative and some candidates performed the integration and substitution correctly but gave their final answer as positive. It must be remembered that the sign is important when dealing with displacement and the negative sign means that at $t = 5$ the particle is positioned on the negative side of the origin.

Answer: The displacement at $t = 5$ is –9.05 m (to 3sf)

Question 7

(i) In order to determine the work done by the friction force between points P and Q, it is necessary to find the reaction force on the particle as it moves from P to Q. This is achieved by resolving forces perpendicular to the plane of motion and then using the relationship $F = \mu R$. Once the friction force has been found, use of the definition 'Work done = Force × Distance moved in the direction of the force' will determine the required result. Most candidates made good attempts at this question. Some errors were seen when candidates also included the effects of the weight of the particle, which was equivalent to wrongly including the potential energy into their calculations. When using the definition of work done, some candidates multiplied their friction force by 8 cos α rather than by 8 which is the distance moved in the direction of the frictional force.

Answer: The work done by the friction force = 6 J

(ii) As the motion between *P* and *Q* is one of constant acceleration, it is possible to approach this problem either by using the constant acceleration formulae or by using energy methods. If constant acceleration is used then Newton's second law must be applied to the particle in the direction of motion parallel to the plane. The forces acting on the particle are the component of the weight acting down the plane and the friction force which was found in **7(i)** which also acts down the plane against the direction of motion. Combining these forces and equating to 'mass × acceleration' will give the acceleration of the particle. The value of this acceleration, $a = -11$, along with $u = 15$ and $s = 8$ can be used in the constant acceleration formulae to give the required value of v, the speed of the particle at *Q*. Some candidates forgot that both forces are acting against the motion, which produces a deceleration. In some cases the acceleration was wrongly taken either as $a = g$ or $a = -g$.

For those applying the work-energy equation, it was necessary to consider 4 terms: the initial kinetic energy; the final kinetic energy; the gain in potential energy; the work done against friction. Combining these terms produces the required value of v, the speed of the particle at *Q*. Some errors that were seen when this method was used were terms being missed out or all terms included but with incorrect signs. In some cases the friction force was used rather than the WD against friction as one term in the energy equation, which gave a dimensionally incorrect term in the equation.

Answer: The speed of the particle at *Q* is 7 ms^{-1}

(iii) Again in this part there are two possible approaches. One method is to consider the motion from *Q* to *R*. Alternatively the problem can be approached by looking at the total motion from *P* to *R*. As this motion takes place on a curved surface, the equations of constant acceleration do not apply and so energy methods must be used. If the motion from *Q* to *R* is considered, the energy equation takes the form, 'KE lost = PE gained'. This will give the extra height of *R* above *Q* and so the height of *Q* above *P* must be added to this in order to find *h*. Most candidates who used this applied it correctly but some forgot to add the extra height. For those who considered the total motion from *P* to *R*, the work-energy equation takes the form 'KE lost = WD against *F* + PE gained'. When this is solved, *h* is found directly. Some candidates using this approach forgot to include the work done against friction.

Answer: $h = 8.85$

MATHEMATICS

Paper 9709/52
Mechanics

General comments

Candidates found this paper slightly easier than the one set in March last year. This was partly due to the centre of mass question set last year where very few marks were scored.

Most candidates' work was neat and well presented.

Candidates should be reminded that an answer should be given to 3 significant figures unless otherwise stated in the question. This means that they should work to at least 4 significant figures. A few candidates gave answers to 2 significant figures instead of 3 significant figures as requested.

Most candidates now use $g = 10$ as requested on the question paper.

The easier questions proved to be **1**, **3(i)**, **3(ii)**, **7(i)**, **7(ii)** and **7(iii)**.

The harder questions proved to be **2(ii)**, **5(i)**, **5(ii)** and **6(ii)**.

Comments on specific questions

Question 1

This question proved to be straight forward and many candidates scored good marks. Quite often the only mark lost was for the direction. –53.4° was not acceptable. It should have been 53.4° below the horizontal.

Answers: 34.9 m s^{-1} 53.4° to the horizontal

Question 2

(i) Some candidates used $3^2 + 2^2 + 1^2$ instead of $3^3 + 2^3 + 1^3$. A volume was required rather than an area. To solve the problem it was necessary to take moments about the base of the largest cube.

(ii) A similar error was made in this part of the question, with areas being used instead of volumes.

Answers: **(i)** 2.17 m **(ii)** 13:1 or 1:13

Question 3

(i) This part of the question was generally well done.

(ii) Most candidates compared their trajectory equation with the one given on the formula sheet, and reached the required answers. An alternative correct method some candidates used was writing the two equations $V\cos\theta = 4$ and $V\sin\theta = 6$, where θ and V are the angle of projection and the initial speed, and solving for V and θ.

Answers: **(i)** $y = \frac{3}{2}x - \frac{5}{16}x^2$ **(ii)** θ = 56.3° V = 7.21 m s^{-1}

Question 4

Many candidates were able to use the correct method but quite a number made mistakes in their calculations. A number of steps were needed to get to the required answer. The first step was to resolve vertically to find the tension. Hooke's Law then gave the extension in the string, namely 0.3 m. The length of the string was then found to be 1.1 m. A number of candidates used 1.1 m for the radius instead of 1.1 sin60°. The final step was to use Newton's Second Law horizontally.

Answer: 4.06 m s^{-1}

Question 5

(i) A number of candidates did not realise that the greatest speed occurred at the equilibrium position. The first step required was to use Hooke's Law to find the extension at the equilibrium position and then to set up a 3 term energy equation.

(ii) In this part of the question candidates needed to realise that the greatest distance would be reached when the velocity was zero.

Answers: **(i)** 2.18 m s^{-1} **(ii)** 0.864 m

Question 6

(i) A number of candidates used 2r rather than r for the radius of the hole. This part of the question could be solved by taking moments about the centre of the base of the original cylinder.

(ii) This part of the question proved to be too challenging for many candidates. In fact it proved to be the hardest question on the paper. To solve this part of the question it was necessary to take moments about the point of contact in order to find the value of P. By resolving horizontally and vertically F = P and R = W. The final step was to use F = μR.

Answers: **(i)** r/12 **(ii)** 1/240 or 0.00417

Question 7

This question was generally well done and so was a good source of marks for many candidates.

(i) A few candidates used $t = 4$ instead of $t = 8$ and then attempted to manipulate their expression to get 0.24 kg.

(ii) Most candidates used Newton's Second Law to get the required equation. They then went on to integrate and use the correct limits to find the speed of projection.

(iii) Most candidates realised that they needed to integrate again to find a distance. The final step was to apply the correct limits.

Answers: **(i)** 0.24 kg **(ii)** $dv/dt = 0.25t - 2$, 6 m s^{-1} **(iii)** 10.7 m

MATHEMATICS

Paper 9709/62
Probability and Statistics

Key messages

Candidates must show sufficient method to justify their conclusions. Failure to communicate intended processes may produce uncertainty about the final answer.

Candidates are reminded that only non-exact answers should be rounded to 3SF and that greater accuracy needs to be used within calculations, otherwise premature approximation will affect their final answer.

Candidates should be aware that stem-and-leaf diagrams are visual representation of statistical data and need to be constructed with accuracy.

General comments

Although the majority of candidates presented their solutions in a logical manner, there was sometimes a lack of structure which made determining the final answer difficult for examiners. In **Question 2**, this was possibly the cause for the number of solutions that did not provide the required information.

Many good solutions were seen for **Questions 1**, **4** and **5**. The context in **Question 7** was found challenging by many. Sufficient time seems to have been available for candidates to complete all the work they were able to, although a few candidates did not appear to have knowledge of all topic areas of the syllabus.

Comments on specific questions

Question 1

This was a standard probability question, which was attempted by almost all candidates

(i) Good solutions often included a tree diagram to visually interpret the information provided for greater clarity. Although arithmetical errors were noted in some solution, most solutions clearly stated the required calculation.

(ii) The majority of solutions used the standard conditional probability formula, and evaluated the required values either separately, or within the formula. Few solutions used the complementary properties of probability to evaluate the denominator value for ‘not wearing red socks’. Many solutions stated the accurate fraction and then converted to a 3SF decimal. This final step was not necessary.

Answers: **(i)** 0.248 **(ii)** $\frac{17}{47}$

Question 2

The majority of candidates found this coded data question challenging. Many solutions were not clearly presented, with transfer errors noted in a number of answers.

(i) The best solutions stated the variance formula for coded data before substituting the stated values in. This approached allowed clear algebraic rearrangements to be made, leading to the correct final answer. A number of solutions used the substitution $x = x - c$. If a substitution approach is used, candidates should be advised to utilise an alternative unknown value for clarity. Weaker candidates substituted c = 50 from **(ii)**, which could not lead to an acceptable solution. A few candidates attempted to expand their expressions within the formula and made little progress to the solution.

(ii) Most candidates were able to use their value from **(i)** to calculate an appropriate mean for the data.

Answers: **(i)** 328 **(ii)** 58.2

Question 3

This standard normal distribution question was attempted by almost all candidates. A number of final solutions were inaccurate because of premature approximation within calculations or incorrect rounding.

(i) Almost all candidates recognised that time was continuous and so no continuity correction was required within the normal approximation formula. The best solutions often contained a sketch of a normal distribution curve to clarify the context and ensured that the appropriate probability area was stated.

(ii) Almost all solutions used the normal approximation formula. However, a significant number of candidates did not use the normal distribution tables accurately, with the weakest solutions treating the given probability as a *z*-value and stating a probability value from the tables. The algebraic manipulation of the expression was usually clearly stated.

Answers: **(i)** 0.252 **(ii)** 135 *or* 134.6 *or* 134.55

Question 4

Candidates were generally more successful with part **(i)** than with part **(ii)**.

(i) Many good solutions were seen, with almost all solutions containing a probability distribution table in terms of *k* with at least one term correct. Almost all candidates used the fact that the sum of the probabilities is 1, and calculated a value for their *k*. A few candidates simply generated a probability distribution table with a value for *k* substituted.

(ii) Good solutions stated clearly the numerical expressions generated by their probability distribution table in **(i)** before evaluation. Some poor arithmetic was noted, with the variance calculation including the error that $(-1)^2 = -1$. Common errors in the variance calculation were to either omit to subtract the mean or to square the mean value before subtraction. Premature approximation and poor rounding was noted in this part.

Answers: **(i)** $\frac{1}{15}$ **(ii)** $\mathrm{E}(X) = \frac{7}{3}, \mathrm{Var}(X) = \frac{52}{45}$

Question 5

The majority of candidates provided reasonable solutions to this question, with accuracy being a significant discriminator.

(i) Almost all candidates produced a back-to-back stem-and-leaf diagram. The best solutions grouped the raw data before producing the ordered diagram. This approach would help eliminate ordering errors and enable the required accuracy of alignment to be achieved more easily. Although most solutions did include a key, they often only referred to one side of the diagram, and the omission of units is not condoned unless included in the titles. A few candidates did not follow the instructions as to where the teams needed to be placed.

(ii) Most candidates appeared to use their stem-and-leaf diagram from part **(ii)** to find the median and interquartile range, although a few candidates reordered the Dolphins data at the start of this question part. The median was stated accurately by most, with appropriate indication of how to determine the middle term shown. There was less consistency for the Interquartile Range, with common errors being 82–65 = 17 or 75–65 = 10. Very few candidates stated the median and IQR for the incorrect team.

Answer: **(ii)** median = 72, IQR = 15

Question 6

Most solutions recognised that this was binomial distribution and used the appropriate formulae and approximations.

(i) The best solutions cleared stated the outcomes that would satisfy the condition, and provided a clear unsimplified expression using their calculators efficiently to achieve an accurate 3 SF result. Weaker solutions often evaluated each term and, because of premature approximation, achieved an inaccurate final answer. Candidates should be able to interpret conditions such as 'more than 3' to indicate that 3 is not included. This has been highlighted in previous sessions.

(ii) Many good solutions were seen. These clearly stated the initial exponential equation, and then showed an appropriate method of solution. The use of logarithmic properties is acceptable and an efficient method. Candidates may be well advised to confirm that their final answer does fulfil the stated condition.

(iii) The best solutions clearly identified that a normal approximation was appropriate, justified the appropriateness of the decision, and identified clearly the calculations for mean and variance before using the normal distribution formula. Most candidates recognised that, as the data was discrete, a continuity correct was appropriate. A number of solutions used 38.5 and it was unclear whether this was due to confusion applying the correction, or to not interpreting the question correctly. An unexpected error was the number of solutions seen which used the calculated value from **(i)** in this part.

Answers: **(i)** 0.117 **(ii)** 7 **(iii)** 0.173

Question 7

Most solutions recognised that this was a permutation and combination style question, as indicated by the key words 'arrangements' and 'selections'. Solutions which presented their working in a clear manner were often more successful.

(i) This question part was accessible to almost all candidates. Good solutions stated the unsimplified expression before stating an answer. As this value is exact, candidates should be reminded that it should not be rounded to 3SF. An unexpected error was to remove the effect of only one of the repeated letters.

(ii) Good solutions often included a simple 'diagram' to visualise the condition. A number of solutions failed to divide by 2! to remove the effect of the 2 Ms in the letter group.

(iii) Again, the best solutions included a simple visual representation of the condition that is applied. This identified that there were 7 different possible positions for the Ms and that there were 7 other letters to position. Appropriate calculations were then stated and evaluated. A number of solutions did not remove the effect of the 3 As in the letter group, or divided by 2! as in the previous part.

(iv) Some candidates did not respond to the key word 'selections' and provided solutions indicative of arrangements. Again the best solutions provided a clear visual representation of the condition, with weaker responses often just listing possible solutions rather than identifying that ${}^4C_1$ would provide the potential total.

(v) The most common approach to this question was to consider the number of ways that the selections could be made with either 1 or 2 Ms and then reaching a total. The best solutions again provided a visual representation of the conditions that were being considered within each section of work, and then clearly stated the final sum. Weaker solutions simply stated calculations with no clarification of what was being evaluated. A common error was to consider that the Ms were identifiable and multiply the final answer by 2. A significant number of candidates omitted MAA, and achieved a final answer of 15.

Answers: **(i)** 30240 **(ii)** 360 **(iii)** 5880 **(iv)** 4 **(v)** 16

MATHEMATICS

Paper 9709/72
Probability and Statistics

Key messages

Candidates need to be able to correctly round answers to 3.s.f. and to be aware that premature approximation to 3.s.f during working could lead to inaccuracy in the final answer.

In a 'show that' question, all steps in the calculation need to be shown. If essential stages of the working out are missing, full marks will not be obtained (see **Question 4** below).

All the steps required for a Hypothesis test should be fully presented; it is important that the conclusion to the test is fully justified (see comments below on **Question 3** and **Question 6**).

General comments

On this paper, candidates were largely able to demonstrate and apply their knowledge in the situations presented. There was a complete range of scripts. In general, candidates scored well on **Questions 4(i)**, **5(i)** and **5(ii)**, whilst **Questions 1(ii)**, **3** and **5(iii)** proved particularly demanding.

Candidates generally kept to the required level of accuracy and timing did not appear to be a problem.

Detailed comments on individual questions follow. Whilst the comments indicate particular errors and misconceptions, it should be noted that there were also some good and complete answers.

Comments on specific questions

Question 1

Part **(i)** was well attempted; most candidates knew how to construct a confidence interval, with the correct z value, and gave their answer in the appropriate form as an interval.

Part **(ii)** was not as well attempted. It was necessary to apply the Central Limit Theorem in part **(i)** as the population was not given to be Normally distributed. It was important here that candidates' responses clearly referred to the *population* not being given to be Normally distributed. Answers such as 'the distribution was not Normal' were not clear enough and did not score. Even candidates who mentioned the distribution of the plums needed to make it clear that it was the population distribution that they were considering. Some candidates incorrectly referred to the sample and many commented on the value of n.

Answers: **(i)** 60.1 to 64.5 **(ii)** Yes because the population was not given to be Normal.

Question 2

This was a reasonably well attempted question and a good number of candidates scored full marks. The main error seen was an incorrect calculation for Var(X-$3Y$); for example, using 3 instead of 3^2 or subtracting their two values instead of adding. Weaker responses were often unable to make the correct approach to the solution and did not attempt to use (X-$3Y$).

Answer: 0.508

Question 3

This question proved to be very demanding for many candidates. It was common that candidates either did not state their hypotheses or gave incorrect ones, and use of N(32,32) was not often seen, with many candidates using N(21,21) instead. Candidates who did use a Normal distribution often forgot the continuity correction or used an incorrect one. It is important that the comparison of the test z value with the critical z value (or equivalent area comparison) is clearly shown in order to justify the conclusion to the test. If the comparison was not seen (either as an inequality or on a fully labelled diagram) candidates were unable to score marks for their conclusion as it wasn't justified. The final conclusion should be written as a non-definite statement.

Answer: No evidence that there are fewer accidents at B than at A

Question 4

Part **(i)** was well attempted, with many candidates following the correct method to find estimates for both the population mean and the population variance. As the question required it to be show that the unbiased estimate of the population variance was 490 to 3.s.f; candidates needed to show 4.s.f in their calculation to verify that this did round to 490 to 3.s.f as requested. Candidates who did not show the 4.s.f answer did not gain the final answer mark.

Part **(ii)** was less well attempted. Some candidates used 0.9377 in their calculation, failing to realise that a z value was required. The most common error was a sign error when standardising and equating to –1.536; many candidates omitted the minus sign. The final answer for n needed to be a whole number which most candidates recognised.

Answer: **(i)** 8.4; 489.8 = 490 (3sf) **(ii)** 100

Question 5

Parts **(i)** and **(ii)** of this question were well attempted but part **(iii)** proved to be more demanding.

In part **(i)** most candidates used the correct value for λ. The most common error noted was to calculate 1 – P(0,1,2) rather than 1 – P(0,1), and some candidates failed to subtract from 1.

Again in part **(ii)** candidates mostly found the correct value for λ. Calculation of $P(X < 5)$, i.e. $P(X \leq 4)$ was required, and there was an occasional misinterpretation of the question here, with candidates calculating $P(X \leq 5)$.

There were candidates who were unable to find the correct method for part **(iii)**. Trial and improvement was used by some candidates but this was rarely successful as 3.s.f accuracy was required. Candidates who were able to state the correct inequality (or equation) usually realised that the solution required use of logarithms, and went on to successfully find the required value of t, though some candidates didn't use $\lambda = 1.8t$ and only calculated λ rather that t. Correct answers in hours, hours and minutes, or just minutes were all acceptable.

Answers: **(i)** 0.537 **(ii)** 0.928 **(iii)** 2.56 hours

Question 6

Part **(i)** was correctly answered by many candidates. Errors in part **(ii)** included incorrectly stating that the probability was 0.25 or 0.5 rather than 0.05. When asked for an explanation 'in the context of the question' it is important that the answer is not solely a text book definition, so in part **(ii)** reference should have been made to the mean time or equivalent; answers given here were not always clear enough.

Many candidates omitted to state a necessary assumption in **(iii)**, or suggested an incorrect one. Hypotheses were not always given, or incorrect ones were stated. Calculation of the test value for z was reasonably well attempted, but the comparison between this and the critical z value (or equivalent comparison of areas) needed to be clearly stated as an inequality or on a fully labelled diagram to justify the conclusion to the test. Errors here included not showing a clear comparison, using an incorrect value for z (often using the 1 tail value instead of the 2 tail value even when a 2 tail test had been stated) and invalid comparisons of a z value with an area. The final conclusion to the test should be in context and not definite.

Answers: **(i)** Test is for 'difference' **(ii)** 0.05 Conclude that the mean time is different when actually it is not **(iii)** Assume $\sigma = 6.4$ No evidence that the mean time is different

Question 7

A good number of candidates, in part **(i)**, correctly attempted to integrate f(x) using the correct limits of $\pi/6$ and $\pi/4$. Errors included incorrect limits and, whilst the answer could be obtained by integrating between 0 and $\pi/6$, this method needed to be completed by calculating 1 minus the integral; candidates using this method often omitted this step. The integral of cos$x$ was generally known and used.

Part **(ii)** was also well attempted, with many candidates attempting to integrate f(x) and equating to 0.5. Errors included the omission of $\sqrt{2}$ and giving the final answer incorrectly as 20.705 (from the use of degrees rather than radians).

Part **(iii)** required xf(x) to be integrated, which required integration by parts. Many candidates did not recognise the integration technique required. Those who attempted the integral correctly often reached the correct answer, but errors included the omission of $\sqrt{2}$ again as well as sign errors.

Answers: **(i)** 0.293 **(ii)** 0.361 **(iii)** 0.371