CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Level

MARK SCHEME for the May/June 2014 series

9709 MATHEMATICS

9709/33 Paper 3, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the May/June 2014 series for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level components and some Ordinary Level components.



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Mark Scheme Notes

Marks are of the following three types:

- M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B Mark for a correct result or statement independent of method marks.
- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- The symbol
 [↑] implies that the A or B mark indicated is allowed for work correctly following
 on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
 A and B marks are not given for fortuitously "correct" answers or results obtained from
 incorrect working.
- Note: B2 or A2 means that the candidate can earn 2 or 0.
 B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.
- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

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The following abbreviations may be used in a mark scheme or used on the scripts:

AEF	Any Equivalent Form (of answer is equally acceptable)
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD	Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only – often written by a "fortuitous" answer
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
sos	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

- MR −1 A penalty of MR −1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through \nabla" marks. MR is not applied when the candidate misreads his own figures this is regarded as an error in accuracy. An MR −2 penalty may be applied in particular cases if agreed at the coordination meeting.
- PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

	Pa	ge 4	Mark Scheme	Syllabus	Paper		
	,		GCE A LEVEL – May/June 2014	9709	33		
1	Use	law of the	logarithm of a quotient or product or $2 = \log_{10} 100$		M1		
			thms and obtain $x + 9 = 100x$, or equivalent		A1		
	Obta	ain answer	$x = \frac{1}{11}$		A1	3	
2	State a correct unsimplified version of the x or x^2 or x^3 term						
	State correct first two terms $1 - x$				A1		
	Obta	ain the nex	t two terms $2x^2 - \frac{14}{3}x^3$		A1 + A1	4	
	[Symbolic binomial coefficients, e.g. $\binom{-\frac{1}{3}}{3}$ are not sufficient for the M mark.]						
3	(i)	Use tan(A	$\pm B$) formula and obtain an equation in $\tan x$		M1		
		Using tan	$60^{\circ} = \sqrt{3}$, obtain a horizontal equation in $\tan x$ in any corre	ct form	A1		
			e equation to the given form		A1	3	
	(ii)	Solve the	given quadratic for tan x		M1		
	()	Obtain a c	correct answer, e.g. $x = 21.6^{\circ}$		A1		
			second answer, e.g. $x = 128.4^{\circ}$, and no others aswers outside the given interval. Treat answers in radians a 24).]	s a misread	A1	3	
4	(i)	Consider	sign of $x - 10/(e^{2x} - 1)$ at $x = 1$ and $x = 2$		M1		
		Complete	the argument correctly with correct calculated values		A1	2	
	(ii)	State or in	$\text{mply } \alpha = \frac{1}{2} \ln(1 + 10/\alpha)$		B1		
		Rearrange	e this as $\alpha = 10/(e^{2\alpha} - 1)$ or work <i>vice versa</i>		B1	2	
	(iii)	Use the it	erative formula correctly at least once		M1		
			aal answer 1.14		A1		
			ficient iterations to 4 d.p. to justify 1.14 to 2 d.p., or show the rval (1.135, 1.145)	nere is a sign chan	ge A1	3	
5	Sepa	arate varial	oles correctly and attempt integration of at least one side		B1		
	•		the form $a\sqrt{(2x+1)}$		M1		
	Express $1/(\cos^2\theta)$ as $\sec^2\theta$				B1		
	Obtain term of the form $k \tan \theta$			M1			
			stant, or use limits $x = 0$, $\theta = \frac{1}{4}\pi$ in a solution with terms a_{λ}	$\sqrt{(2x+1)}$ and k tan			
	$ak \neq 0$			M1			
	Obta	ain correct	solution in any form, e.g. $\sqrt{(2x+1)} = \frac{1}{2} \tan \theta + \frac{1}{2}$		A1		
	Rea	rrange and	obtain $x = \frac{1}{8} (\tan \theta + 1)^2 - \frac{1}{2}$, or equivalent		A1	7	

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6		Obtain correct derivative of RHS in any form Obtain correct derivative of LHS in any form			B1 B1	
	Set	$\frac{\mathrm{d}y}{\mathrm{d}x}$ equal t	o zero and obtain a horizontal equation		M1	
	Obt	tain a corre	ct equation, e.g. $x^2 + y^2 = 1$, from correct work		A1	
			in the curve equation, or otherwise, obtain an equation in x	e^2 or y^2	M1	
		$tain x = \frac{1}{2}\sqrt{x}$	3		A1	
	Obt	$tain y = \frac{1}{2}$			A1	7
7	(a)	EITHER:	Multiply numerator and denominator by $1-4i$, or equival	ent, and use $i^2 = -$		
			Simplify numerator to $-17 - 17i$, or denominator to 17 Obtain final answer $-1 - i$		A1 A1	
		OR:	Using $i^2 = -1$, obtain two equations in x and y, and solve	for x or for y	M1	
		0111	Obtain $x = -1$ or $y = -1$, or equivalent	101 11 01 101)	A1	
			Obtain final answer $-1 - i$		A1	3
	(b)		v a point representing 2 + i in relatively correct position		B1	
			v a circle with centre 2 + i and radius 1 v the perpendicular bisector of the line segment joining i and	1.2	B1√ B1	
			the correct region	u <i>2</i>	B1	4
		(ii) State requi	or imply that the angle between the tangents from the o	rigin to the circle	is M1	
		_	in answer 0.927 radians (or 53.1°)		A1	2
8	(i)	Use a cor	rect method for finding a constant		M1	
			the of $A = 3$, $B = 3$, $C = 0$		A1	
			second value chird value		A1 A1	4
	(ii)	Integrate	and obtain term $-3\ln(2-x)$		B1 √	
		Integrate	and obtain term of the form $k \ln(2 + x^2)$		M1	
		Obtain ter	$rm \frac{3}{2} ln(2+x^2)$		A1√	
			e limits correctly in an integral of the form $a \ln(2-x) + b \ln(x)$	$(2+x^2)$, where ab	≠ 0 M1	
		Obtain gi	ven answer after full and correct working		A1	5
9	(i)		e for x and dx throughout using $u = \sin x$ and $du = \cos x dx$, o	r equivalent	M1	
			tegrand e^{2u}		A1	
			definite integral $\frac{1}{2}e^{2u}$		A1	
			is $u = 0$, $u = 1$ correctly, or equivalent		M1	_
		Obtain an	swer $\frac{1}{2}(e^2-1)$, or exact equivalent		A1	5

Mark Scheme

Syllabus

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(ii)		rule or product rule		M1	
		rrect terms of the derivative in any form, e.g. $2\cos x e^{2\sin x}$	$\cos x - e^{2\sin x} \sin x$	A1 + A1	
	•	erivative to zero and obtain a quadratic equation in $\sin x$		M1	
		-term quadratic and obtain a value of x		M1	
	Obtain an	swer 0.896		A1	6
10 (i)	Express g	general point of <i>l</i> in component form, e.g. $(1+3\lambda, 2-2\lambda, -1+2\lambda, 2-2\lambda)$	-2λ)	B1	
	Substitute	e in given equation of p and solve for λ		M1	
	Obtain fir	nal answer $-\frac{1}{2}\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$, or equivalent, from $\lambda = -\frac{1}{2}$		A1	3
(ii)	State or in	mply a vector normal to the plane, e.g. $2\mathbf{i} + 3\mathbf{j} - 5\mathbf{k}$		B1	
	Using the	e correct process, evaluate the scalar product of a directio	n vector for l and	l a	
	normal fo			M1	
		correct process for the moduli, divide the scalar product by	by the product of t		
		and find the inverse sine or cosine of the result swer 23.2° (or 0.404 radians)		M1	4
	Obtain an	swer 25.2 (or 0.404 radians)		A1	4
(iii)	EITHER:	State $2a + 3b - 5c = 0$ or $3a - 2b + 2c = 0$		B1	
()		Obtain two relevant equations and solve for one ratio, e.g.	a:b	M1	
		Obtain $a : b : c = 4 : 19 : 13$, or equivalent		A1	
		Substitute coordinates of a relevant point in $4x + 19y + 13$.	z = d, and evaluate		
	OD1	Obtain answer $4x + 19y + 13z = 29$, or equivalent		A1	
	<i>OR</i> 1:	Attempt to calculate vector product of relev	ant vectors, e	.g. M1	
		$(2\mathbf{i} + 3\mathbf{j} - 5\mathbf{k}) \times (3\mathbf{i} - 2\mathbf{j} + 2\mathbf{k})$ Obtain two correct components of the product		A1	
		Obtain correct product, e.g. $-4i - 19j - 13k$		A1	
		Substitute coordinates of a relevant point in $4x + 19y + 13$.	z = d	M1	
		Obtain answer $4x + 19y + 13z = 29$, or equivalent		A1	
	OR2:	Attempt to form a 2-parameter equation with relevant vector		M1	
		State a correct equation, e.g. $\mathbf{r} = \mathbf{i} + 2\mathbf{j} - \mathbf{k} + \lambda(2\mathbf{i} + 3\mathbf{j} - 5\mathbf{k})$	$) + \mu(3\mathbf{i} - 2\mathbf{j} + 2\mathbf{k})$	A1	
		State 3 equations in x , y , z , λ and μ		A1	
		Eliminate λ and μ		M1	
	OD1	Obtain answer $4x + 19y + 13z = 29$, or equivalent	C 1.4 :	A1	
	OR3:	Using a relevant point and relevant direction vectors, equation for the plane	form a determina	ant M1	
		$\begin{vmatrix} x-1 & y-2 & z+1 \end{vmatrix}$			
		State a correct equation, e.g. $\begin{vmatrix} 2 & 3 & -5 \end{vmatrix} = 0$		A1	
		State a correct equation, e.g. $\begin{vmatrix} x-1 & y-2 & z+1 \\ 2 & 3 & -5 \\ 3 & -2 & 2 \end{vmatrix} = 0$			
		Attempt to expand the determinant		M1	
		Obtain correct values of two cofactors		A1	
		Obtain answer $4x + 19y + 13z = 29$, or equivalent		A1	5