CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the May/June 2014 series

9709 MATHEMATICS

9709/13 Paper 1, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the May/June 2014 series for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level components and some Ordinary Level components.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1	$\left(x^2-\frac{2}{x}\right)^5$		
	Term in x is $10 \times (x^2)^2 \times \left(\frac{-2}{x}\right)^3$	B1 B1	B1 10 or ${}^5C_2$ or ${}^5C_3$, B1 $\left(\frac{-2}{x}\right)^3$
	Coefficient $= -80(x)$	B1 [3]	co Must be identified
2	36, 32, **...**		
	(i) $r = \frac{8}{9}$ $S_\infty = (\textit{their } a) \div (1 - \textit{their } r)$	M1	Method for r and S_∞ ok. ($\lvert r \rvert < 1$)
	$S_\infty = 36 \div \frac{1}{9} = 324$	A1 [2]	co
	(ii) $d = -4$	B1	co
	$0 = \frac{n}{2}(72 + (n-1)(-4))$	M1	S_n formula ok and a value for d $\left(\neq \frac{8}{9}\right)$
	$\rightarrow n = 19$	A1 [3]	Condone $n = 0$ but no other soln
3	**(i)** $s = r\theta$	M1	Used with major or minor arc
	Angle of major arc $= 2\pi - 2.2 = (4.083)$	B1	Could be gained in **(ii)**.
	Perimeter $= 12 + 24.5 = 36.5$ or $12\pi - 1.2$ (or full circle − minor arc B1)	A1 [3]	co
	(ii) Area of major sector $= \frac{1}{2}r^2\theta = (73.49)$	M1	Used with major / minor sector.
	Area of triangle $= \frac{1}{2}.6^2 \sin 2.2 = (14.55)$	M1	Correct formula or method. $(2\pi - 2.2) / \sin 2.2$ gets M1M1
	Ratio = 5.05 : 1 (Allow 5.03 → 5.06)	A1 [3]	co
4	$\frac{\tan x + 1}{\sin x \tan x + \cos x} \equiv \sin x + \cos x$		
	(i) LHS $\frac{\left(\frac{s}{c}\right)+1}{\left(\frac{s^2}{c}+c\right)} = \frac{s+c}{s^2+c^2}$	M1 M1	Use of t = s / c twice. Correct algebra and use of $s^2 + c^2 = 1$
	= RHS	A1 [3]	AG all ok
	(ii) $s + c = 3s - 2c$		
	$\rightarrow \tan x = \frac{3}{2}$ Allow $\cos^2 = \frac{4}{13}$, $\sin^2 = \frac{9}{13}$	M1	Uses (i) and $t = \frac{s}{c}$ $t = \frac{2}{3}$ or 0 is M0
	$\rightarrow x = 0.983$ and 4.12 or 4.13	A1 A1√ [3]	co. √ 1st + π, providing no excess solns in range. Allow 0.313π, 1.31π

5	$f(x) = \frac{15}{2x+3}$		
	(i) $f'(x) = \frac{-15}{(2x+3)^2} \times 2$	B1 B1	Without the "×2". For "×2" (indep of 1st B1).
	$(\ )^2$ always +ve $\rightarrow$ $f'(x) < 0$ (No turning points) – therefore an inverse	B1√ [3]	√ providing $(\ )^2$ in $f'(x)$. 1–1 insuff.
	(ii) $y = \frac{15}{2x+3} \rightarrow 2x + 3 = \frac{15}{y}$	M1	Order of ops – allow sign error
	$\rightarrow x = \frac{\frac{15}{y} - 3}{2} \rightarrow \frac{15-3x}{2x}$	A1	co as function of x. Allow $y = \ldots$
	(Range) $0 \leqslant f^{-1}(x) \leqslant 6$. Allow $0 \leqslant y \leqslant 6, [0,6]$	B1	For range / domain ignore letters
	(Domain) $1 \leqslant x \leqslant 5$. Allow $[1, 5]$	B1 [4]	unless range / domain not identified
6	$\frac{dy}{dx} = \frac{12}{\sqrt{4x+a}}$ $P(2, 14)$ Normal $3y + x = 44$		
	(i) m of normal $= -\frac{1}{3}$	B1	co
	$\frac{dy}{dx} = 3 = \frac{12}{\sqrt{4x+a}} \rightarrow a = 8$	M1 A1 [3]	Use of $m_1 m_2 = -1$. AG.
	(ii) $\int y = 12(4x + a)^{\frac{1}{2}} \div \frac{1}{2}$ $\div 4$ $(+c)$	B1 B1	Correct without "÷4". for "÷4".
	Uses (2, 14) $c = -10$	M1 A1 [4]	Uses in an integral only. Dep 'c'. co All 4 marks can be given in (i)

7	**(i)** Angle *BAC* needs sides *AB,AC* or *BA,CA* **AB.AC** = (**b** − **a**).(**c** − **a**)		Ignore *their* labels:
	$= \begin{pmatrix} 4 \\ -2 \\ 4 \end{pmatrix} . \begin{pmatrix} 0 \\ 3 \\ 4 \end{pmatrix} = 10$	B1 M1	One of **AB, BA, AC, CA** correct Use of $x_1x_2 + y_1y_2$, etc.
	$= \sqrt{36} \times \sqrt{25} \cos BAC$	M1M1	M1 prod of moduli. M1 all linked
	$\rightarrow \quad BAC = \cos^{-1} \frac{1}{3}$ AG	A1	If e.g. **BA.OC** max B1M1M1. If both vectors wrong 0/5. If e.g. **BA.AC** used → $\cos^{-1}\left(-\frac{1}{3}\right)$ final mark A0
		[5]	
	(ii) $\sin BAC = \sqrt{1-\frac{1}{9}}$	B1	Use of $s^2 + c^2 = 1$ − not decimals
	Area $= \frac{1}{2} \times 6 \times 5 \times \sqrt{\frac{8}{9}} = 5\sqrt{8}$ oe	M1 A1	Correct formula for area. Decimals seen A0
		[3]	
8	$2x^2 - 10x + 8 \rightarrow a(x + b)^2 + c$		
	(i) $a = 2,\ b = -2\frac{1}{2},\ c = -4\frac{1}{2}$	3 × B1	Or $2\left(x - 2\frac{1}{2}\right)^2 - 4\frac{1}{2}$
	→ min value is $-4\frac{1}{2}$ Allow $(2\frac{1}{2}, -4\frac{1}{2})$	B1√	Can score by sub $x = 2\frac{1}{2}$ into original but not by differentiation
		[4]	
	(ii) $2x^2 - 10x + 8 - kx = 0$ Use of "$b^2 - 4ac$" $(-10 - k)^2 - 64 < 0$ or $k^2 + 20k + 36 < 0$ $\rightarrow k = -18$ or -2 $-18 < k < -2$	 M1 M1 A1 A1	Sets equation to 0 and uses discriminant correctly Realises discriminant < 0. Allow ⩽ co Dep on 1st M1 only co
		[4]	
9	**(i)** $3x^2y = 288$ y is the height	B1	co
	$A = 2(3x^2 + xy + 3xy)$	M1	Considers at least 5 faces ($y \neq x$)
	Sub for $y \rightarrow A = 6x^2 + \frac{768}{x}$	A1	co answer given
		[3]	
	(ii) $\frac{dA}{dx} = 12x - \frac{768}{x^2}$	B1	co
	$= 0$ when $x = 4 \rightarrow A = 288$. Allow $(4, 288)$	M1 A1	Sets differential to 0 + solution. co
	$\frac{d^2A}{dx^2} = 12 + \frac{1536}{x^3}$	M1	Any valid method
	$(= 36) > 0$ Minimum	A1	co www dep on correct f″ and $x = 4$
		[5]	

10	pts of intersection $2x + 1 = -x^2 + 12x - 20$ $\rightarrow$ $x = 3, 7$	M1A1	Attempt at soln of sim eqns. co
	Area of trapezium $= \frac{1}{2}(4)(7 + 15) = 44$ (or $\int (2x+1)\,dx$ from 3 to 7 = 44)	M1A1	Either method ok. co
	Area under curve $= -\frac{1}{3}x^3 + 6x^2 - 20x$	B2,1	−1 each term incorrect
	Uses 3 to 7 $\rightarrow$ $(54\frac{2}{3})$	DM1	Correct use of limits (Dep 1st M1)
	Shaded area $= 10\frac{2}{3}$	A1 [8]	co
	OR		
	$\int_3^7 \left(-x^2+10x-21\right) = -\frac{x^3}{3}+5x^2-21x\right)$		Functions subtracted before integration
	M1 subtraction, A1A1A1 for integrated terms, DM1 correct use of limits, A1		Subtraction reversed allow A3A0. Limits reversed allow DM1A0
11	Sim eqns $\rightarrow$ A (1, 3)	M1 A1	co Allow answer only B2
	Vectors or mid-point $\rightarrow$ C (12, 14)	M1 A1√	Allow answer only B2√
	Eqn of BC $4y = x + 44$ or CD $y = 3x - 22$	M1	equation ok – unsimplified
	Sim eqns $\rightarrow$ B(4, 12) or D (9, 5)	DM1A1	Sim eqns. co
	Vectors or mid-point $\rightarrow$ B(4, 12) or D (9, 5)	DM1A1 [9]	Valid method (or sim eqns) co