CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level

MARK SCHEME for the October/November 2013 series

9709 MATHEMATICS

9709/21 Paper 2 (Pure Mathematics), maximum raw mark 50

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the October/November 2013 series for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level components and some Ordinary Level components.



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Mark Scheme Notes

Marks are of the following three types:

- M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B Mark for a correct result or statement independent of method marks.
- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- The symbol
 implies that the A or B mark indicated is allowed for work correctly following
 on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
 A and B marks are not given for fortuitously "correct" answers or results obtained from
 incorrect working.
- Note: B2 or A2 means that the candidate can earn 2 or 0.
 B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.
- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

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The following abbreviations may be used in a mark scheme or used on the scripts:

AEF	Any Equivalent Form (of answer is equally acceptable)
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD	Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only – often written by a 'fortuitous' answer
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
sos	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

- MR −1 A penalty of MR −1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through \"" marks. MR is not applied when the candidate misreads his own figures this is regarded as an error in accuracy. An MR−2 penalty may be applied in particular cases if agreed at the coordination meeting.
- PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

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1 <u>Either</u> State or imply non-modular inequality $(x+1)^2 < (3x+5)^2$, or

corresponding equation or pair of linear equations

M1

Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations

M1

Obtain critical values -2 and $-\frac{3}{2}$

A1

State correct answer x < -2 or $x > -\frac{3}{2}$

A1

Obtain one critical value, e.g. x = -2, by solving a linear equation (or inequality)

or from a graphical method or by inspection

Obtain the other critical value similarly

B2

State correct answer x < -2 or $x > -\frac{3}{2}$

B1

[4]

2 (i) Consider sign of $x^4 + 2x - 9$ at x = 1.5 and x = 1.6Complete the argument correctly with appropriate calculations (f(1.5) = -0.9375, f(1.6) = 0.7536)

M1 A1 [2]

(ii) Rearrange $x^4 + 2x - 9 = 0$ to given equation or *vice versa*

B1 [1]

(iii) Use the iterative formula correctly at least once Obtain final answer 1.56

M1 A1

Show sufficient iterations to justify its accuracy to 2 d.p.

B1 [3]

$x_0 = 1.5$	$x_0 = 1.55$	$x_0 = 1.6$
1.5874	1.5614	1.5362
1.5424	1.5556	1.5685
1.5653		1.5520
1.5536		1.5604
1 5595		1.5561
1.5565		

or show there is a sign change in the interval (1.555, 1.565)

3 Obtain derivative $e^{2x} - 5e^x + 4$

B1

Equate derivative to zero and carry out recognisable solution method for a quadratic in e^x

M1

Obtain
$$e^x = 1$$
 or $e^x = 4$
Obtain $x = 0$ and $x = \ln 4$

A1 A1

Use an appropriate method for determining nature of at least one stationary point

M1

$$\left(\frac{d^2y}{dx^2} = 2e^{2x} - 5e^x, \text{ when } x = 0, \frac{d^2y}{dx^2} = -(3), x = \ln 4, \frac{d^2y}{dx^2} = +(12)\right)$$

Conclude maximum at x = 0 and minimum at $x = \ln 4$ (no errors seen)

A1 [6]

4 (i) Substitute x = 3 and equate to 14 (9a + 3b + 35 = 14)

M1 M1

Substitute x = -2 and equate to 24 (4a - 2b = 24)Obtain a correct equation in any form

A1

Obtain a correct equation in any form Solve a relevant pair of equations for *a* or for *b*

M1

Obtain a = 1 and b = -10

A1 [5]

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	(ii)	Obta Corr	mpt division by $x^2 + 2x - 8$ and reach a partial quotient of x in quotient $x - 1$ with no errors seen (can be done by observed solution method for quadratic e.g. factorisation solutions $x = 1$, $x = 2$ and $x = -4$ given and no others CWO		M1 A1 M1 A1	[4]
5	(i)		$\frac{dx}{d\theta} = -2\sin 2\theta + \sin \theta \text{ or } \frac{dy}{d\theta} = 8\sin \theta \cos \theta$		B1	
		Use	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}\theta} \div \frac{\mathrm{d}x}{\mathrm{d}\theta}$		M1	
		Use	$\sin 2\theta = 2\sin\theta \cos\theta$		M1	
		Obta	in given answer correctly		A1	[4]
	(ii)	Equa	ate derivative to -4 and solve for $\cos \theta$		M1	
	()	Obta	$\sin \cos \theta = \frac{1}{2}$		A1	
			$ \sin x = -1 $		A1	F 4 7
		Obia	$\sin y = 3$		A1	[4]
_	(-) (2)	A 44	1: :1. 1 2x - 1 - 44 2 4 2		3.7.1	
6	(a) (1)		mpt to divide by e^{2x} and attempt to integrate 2 terms grate a term of form ke^{-2x} correctly		M1 A1√	
		Fully	y correct integral $x - 3e^{-2x} (+c)$		A1	[3]
	(ii)	State	e correct expression $\frac{1}{2}\cos 2x + \frac{1}{2}$ or equivalent		B1	
			grate an expression of the form $a + b \cos 2x$, where $ab \neq 0$, correctly	M1	
		State	e correct integral $\frac{3\sin 2x}{4} + \frac{3x}{2}(+c)$		A1	[3]
	(h) Stat	te or in	mply correct ordinates 5.46143, 4.78941, 4.32808		B1	
			ect formula, or equivalent, correctly with $h = 0.5$ and three or	rdinates	M1	
	Obt	ain an	swer 4.84 with no errors seen		A1	[3]
7	(i)		$R = \sqrt{10}$		B1	
			trig formula to find α $\sin \alpha = 18.43^{\circ}$ with no errors seen		M1	[2]
		Obia	$\alpha = 18.43$ with no errors seen		A1	[3]
	(ii)	Carr	y out evaluation of $\cos^{-1}\left(\frac{2}{R}\right) \left(\approx 50.77^{\circ}\right)$		M1	
		-	y out correct method for one correct answer		M1	
			in one correct answer e.g. 34.6°		A1 M1	
			y out correct method for a further answer iin remaining 3 answers 163.8°, 214.6°, 343.8° and no other	ers in the range	A1	[5]
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