

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

## MATHEMATICS

9709/73
Paper 7 Probability \& Statistics 2 (S2)
May/June 2011
1 hour 15 minutes
Additional Materials: Answer Booklet/Paper Graph Paper List of Formulae (MF9)

## READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.
Answer all the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.
At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [] at the end of each question or part question.
The total number of marks for this paper is 50 .
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

1 A hotel kitchen has two dish-washing machines. The numbers of breakdowns per year by the two machines have independent Poisson distributions with means 0.7 and 1.0. Find the probability that the total number of breakdowns by the two machines during the next two years will be less than 3 .

2 In a random sample of 70 bars of Luxcleanse soap, 18 were found to be undersized.
(i) Calculate an approximate $90 \%$ confidence interval for the proportion of all bars of Luxcleanse soap that are undersized.
(ii) Give a reason why your interval is only approximate.

3 At an election in 2010, 15\% of voters in Bratfield voted for the Renewal Party. One year later, a researcher asked 30 randomly selected voters in Bratfield whether they would vote for the Renewal Party if there were an election next week. 2 of these 30 voters said that they would.
(i) Use a binomial distribution to test, at the $4 \%$ significance level, the null hypothesis that there has been no change in the support for the Renewal Party in Bratfield against the alternative hypothesis that there has been a decrease in support since the 2010 election.
(ii) (a) Explain why the conclusion in part (i) cannot involve a Type I error.
(b) State the circumstances in which the conclusion in part (i) would involve a Type II error.

4 On average, 1 in 2500 people have a particular gene.
(i) Use a suitable approximation to find the probability that, in a random sample of 10000 people, more than 3 people have this gene.
(ii) The probability that, in a random sample of $n$ people, none of them has the gene is less than 0.01 . Find the smallest possible value of $n$.

5 Each drink from a coffee machine contains $X \mathrm{~cm}^{3}$ of coffee and $Y \mathrm{~cm}^{3}$ of milk, where $X$ and $Y$ are independent variables with $X \sim \mathrm{~N}\left(184,15^{2}\right)$ and $Y \sim \mathrm{~N}\left(50,8^{2}\right)$. If the total volume of the drink is less than $200 \mathrm{~cm}^{3}$ the customer receives the drink without charge.
(i) Find the percentage of drinks which customers receive without charge.
(ii) Find the probability that, in a randomly chosen drink, the volume of coffee is more than 4 times the volume of milk.

6 The distance travelled, in kilometres, by a Grippo brake pad before it needs to be replaced is modelled by $10000 X$, where $X$ is a random variable having the probability density function

$$
\mathrm{f}(x)= \begin{cases}-k\left(x^{2}-5 x+6\right) & 2 \leqslant x \leqslant 3 \\ 0 & \text { otherwise }\end{cases}
$$

The graph of $y=\mathrm{f}(x)$ is shown in the diagram.

(i) Show that $k=6$.
(ii) State the value of $\mathrm{E}(X)$ and find $\operatorname{Var}(X)$.
(iii) Sami fits four new Grippo brake pads on his car. Find the probability that at least one of these brake pads will need to be replaced after travelling less than 22000 km .

7 Previous records have shown that the number of cars entering Bampor on any day has mean 352 and variance 121.
(i) Find the probability that the mean number of cars entering Bampor during a random sample of 200 days is more than 354 .
(ii) State, with a reason, whether it was necessary to assume that the number of cars entering Bampor on any day has a normal distribution in order to find the probability in part (i).
(iii) It is thought that the population mean may recently have changed. The number of cars entering Bampor during the day was recorded for each of a random sample of 50 days and the sample mean was found to be 356 . Assuming that the variance is unchanged, test at the $5 \%$ significance level whether the population mean is still 352 .

BLANK PAGE

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.
University of Cambridge International Examinations is part of the Cambridge Assessment Group. Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.

