Cambridge IGCSE

Cambridge International Examinations
Cambridge International General Certificate of Secondary Education

CANDIDATE NAME									
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CAMBRIDGE INTERNATIONAL MATHEMATICS **0607/62**

Paper 6 (Extended) **May/June 2017**

1 hour 30 minutes

Candidates answer on the Question Paper.

Additional Materials: Graphics calculator

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
Do not use staples, paper clips, glue or correction fluid.
You may use an HB pencil for any diagrams or graphs.
DO **NOT** WRITE IN ANY BARCODES.

Answer both parts **A** and **B**.
You must show all relevant working to gain full marks for correct methods, including sketches.
In this paper you will also be assessed on your ability to provide full reasons and communicate your mathematics clearly and precisely.
At the end of the examination, fasten all your work securely together.
The total number of marks for this paper is 40.

This document consists of **12** printed pages.

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Answer **both** parts **A** and **B**.

A INVESTIGATION NUMBER STEMS (20 marks)

You are advised to spend no more than 45 minutes on this part.

This investigation is about finding numbers that have the same *Number Stem*.

The possible *Number Stems* are the nine integers from 1 to 9.

Here is how to calculate the *Number Stem* of a number.

Step 1 Add the digits of the number to get a total.
Step 2 If the total is 9 or less, STOP.
Otherwise, add the digits of the total.
Step 3 Repeat Step 2.

Examples

Number	124	Number	893
Step 1	1 + 2 + 4 = 7	Step 1	8 + 9 + 3 = 20
Step 2	STOP	Step 2	2 + 0 = 2
Number Stem is 7.		Step 3	STOP
		Number Stem is 2.	

1 **(a)** Complete the tables to show the *Number Stems* for these multiples of 3 and 12.

Multiple of 3	3	6	9	12	15	18	21	24	27	30
Number Stem	3	6	9	3		9	3	6	9	3

Multiple of 12	12	24	36	48	60	72	84	96	108	120
Number Stem	3	6	9							

(b) **(i)** Complete the sequence to show the first four numbers **greater than 3** that have a *Number Stem* of 3.

12, 21, 30, ...

(ii) Find the *n*th term of the sequence in **part (b)(i)**.

.....

(iii) Find the 87th number **greater than 3** that has a *Number Stem* of 3.

.....

(c) (i) Complete this table.

Number	*Number Stem*	Calculation	Answer
3	3	3 ÷ 9	0 remainder 3
19	1	19 ÷ 9	2 remainder 1
22	.....	22 ÷	2 remainder 4
35	.....	35 ÷ 9	..... remainder .....
.....	7	..... ÷ 9	..... remainder .....

(ii) A number, that is not a multiple of 9, is divided by 9.

What is the connection between its *Number Stem* and the remainder?

.....

(iii) Using your answer to **part (c)(ii)** write down the remainder when 104 020 100 is divided by 9.

.....

2 The sequence shows the first three numbers **greater than 2** with a *Number Stem* of 2.

11, 20, 29, …

(a) Write down the next two numbers of the sequence.

..... , ...

(b) Find the nth term of this sequence.

.....

(c) Using your answer to **part (b)**, find the largest number less than 10 000 that has a *Number Stem* of 2.

.....

3 The integer k is a *Number Stem*.

(a) Write down, in terms of k, the first four numbers **greater than k** with a *Number Stem* of k.

..... , ... , ... , ...

(b) Write down, in terms of n and k, the nth term for the sequence of numbers **greater than k** with a *Number Stem* of k.

.....

4 **(a)** Complete this table.

Calculation	Answer
7 ÷ 12	0 remainder 7
15 ÷ 12	1 remainder .....
23 ÷ 12	..... remainder .....

(b) An integer, that is not a multiple of 12, has remainder f when it is divided by 12.

Find, in terms of n and f, the nth term for the sequence of numbers **greater than** f with a remainder of f.

.....

(c) Show that $f + 10$ cannot be a term of the sequence of numbers **greater than** f with a remainder of f.

B MODELLING ELEVATORS (20 marks)

You are advised to spend no more than 45 minutes on this part.

This task is about the mass an elevator carries and the time it takes to move between floors.

EasyUp is a company that makes elevators.

For each type of elevator, the company uses two mathematical models.

Model 1
The company models the masses of the passengers using the elevator.

Model 2
The company models the time it takes the elevator to move between floors.

1 The *EasyUp*-5 elevator carries a maximum of 5 passengers.

(a) For Model 1, the company estimates that

- $\frac{2}{10}$ of the passengers have a mass of 50 kg
- $\frac{4}{10}$ of the passengers have a mass of 70 kg
- $\frac{4}{10}$ of the passengers have a mass of 85 kg.

From the numbers 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9

- 0 and 1 give a mass of 50 kg
- 2, 3, 4 and 5 give a mass of 70 kg
- 6, 7, 8 and 9 give a mass of 85 kg.

(i) Numbers are chosen at random from 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9.
Each number models the mass of a passenger.
The 5 passengers are A, B, C, D and E.

Here is a random number table, arranged in groups of 5 numbers.

1	6	8	5	2
4	7	9	1	1
6	9	8	7	4
9	1	4	6	8
4	2	1	9	0
8	2	8	8	0
8	9	4	7	6
7	4	1	6	5
1	0	4	7	6
6	0	0	4	5

Use the last four rows of the random number table to complete the table of trials below.
The first six trials have been completed for you.

	Mass of passenger (kg)					Total mass (kg)
	A	B	C	D	E	
Trial 1	50	85	85	70	70	360
Trial 2	70	85	85	50	50	340
Trial 3	85	85	85	85	70	410
Trial 4	85	50	70	85	85	375
Trial 5	70	70	50	85	50	325
Trial 6	85	70	85	85	50	375
Trial 7	85				85	
Trial 8						
Trial 9						
Trial 10						

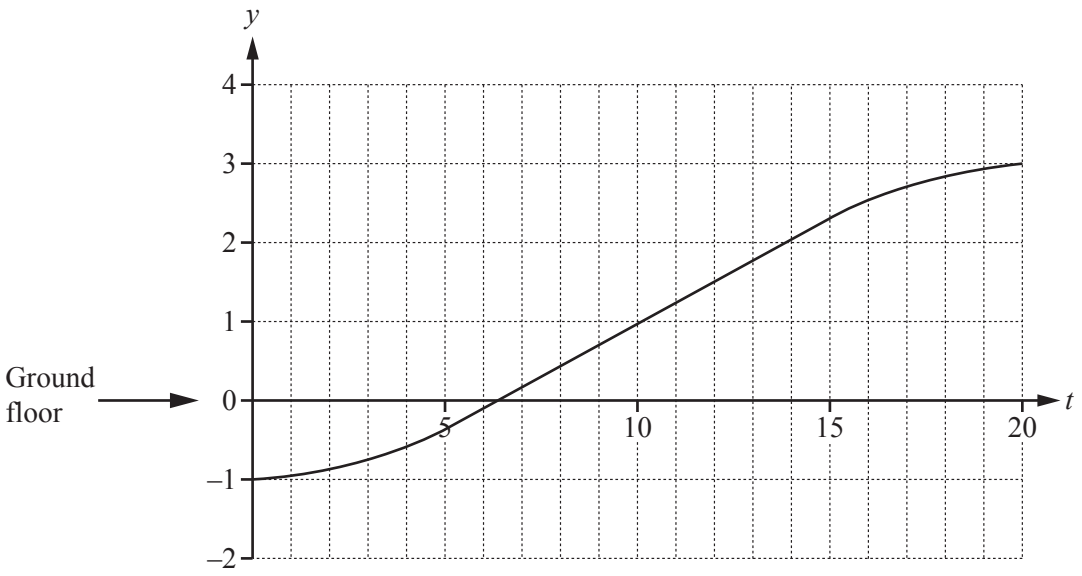
(ii) The *EasyUp*-5 elevator carries a maximum total mass of 400 kg.

Use the results of trials 1 to 10 to work out the relative frequency that the total mass will be more than 400 kg.

.....

[Turn over

(b) For Model 2, the diagram below is a distance-time graph for the *EasyUp*-5 elevator.
The graph modelling the movement is $y = f(t)$, where $f(t)$ is the number of floors above or below the ground floor (floor 0) at time t seconds.



(i) The graph shows the elevator starting one floor below the ground floor.

At which floor does it stop?

.....

(ii) Between which two floors does the elevator have the greatest average speed?

..... and .....

(iii) Find the average time it takes the elevator to move from one floor to the next.

.....

2 The *EasyUp*-3 elevator carries a maximum of 3 passengers.
The maximum total mass is 240 kg.

(a) For Model 1, the company now uses different proportions of passengers for each mass, as shown in the table.
A number is chosen from 0, 1, 2, 3, 4, 5, 6 and 7 to give the mass.
For example, the numbers 1, 2, 3, 4 and 5 each give the mass 70 kg.

(i) Complete the table below.

Proportion of passengers		$\frac{5}{8}$	$\frac{2}{8}$
Amount of numbers		5	
Numbers	0	1, 2, 3, 4, 5	
Mass of passenger (kg)	50	70	85

(ii) Random numbers model the masses of the 3 passengers, A, B and C.

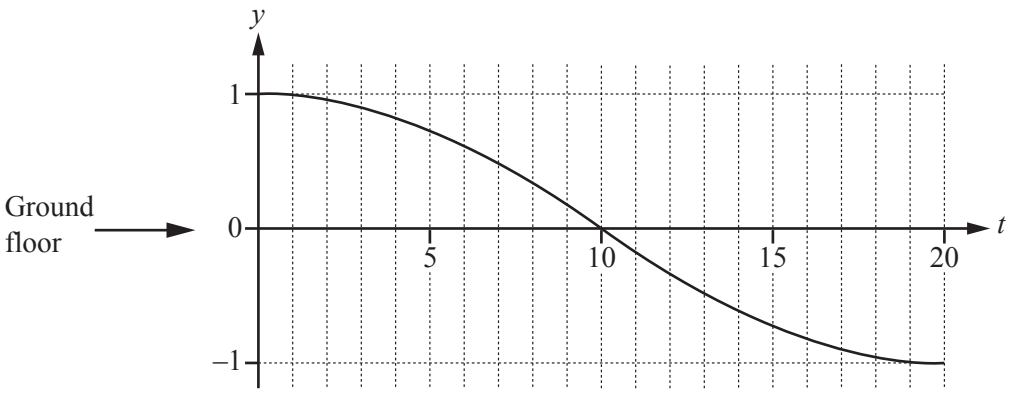
Here is a random number table, arranged in groups of four numbers.
Three random numbers are needed for each trial.
The numbers 8 and 9 are not used. Cross out the numbers 8 and 9 in the table.
When four numbers remain in a row, cross out the last number.

~~8~~	2	1	5
1	6	3	~~3~~
6	7	0	~~5~~
0	~~9~~	1	5
2	0	8	6
1	0	1	1
3	4	8	2
9	0	4	3

Use the last four rows of the random number table to complete the table of trials below.
The first four trials have been completed for you.

	Mass of passenger (kg)			Total mass (kg)
	A	B	C	
Trial 1	70	70	70	210
Trial 2	70	85	70	225
Trial 3	85	85	50	220
Trial 4	50	70	70	190
Trial 5				
Trial 6				
Trial 7				
Trial 8				

(b) For Model 2, the diagram below is a distance-time graph for the *EasyUp*-3 elevator.
The graph modelling the movement is $y = \mathrm{h}(t)$, where $\mathrm{h}(t)$ is the number of floors above or below the ground floor (floor 0) at time t seconds.



(i) The graph shows that the *EasyUp*-3 elevator starts to move from floor 1.

Find the average time it takes the elevator to move from one floor to the next.

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(ii) $\mathrm{h}(t) = \cos kt$.

Find the value of the integer k.

.....

3 The mass carried by an elevator is x kilograms.

EasyUp say that an elevator is well designed when

- the probability that x is less than the maximum total mass is greater than 0.95

and

- it takes at most 5 seconds on average to move between floors.

Explain whether

(a) the *EasyUp*-5 elevator is well designed,

.....

.....

(b) the *EasyUp*-3 elevator is well designed.

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4 Write down **two** ways to improve Model 1 in **question 1(a)**.

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.....

.....

5 The *EasyUp-n* elevator carries a maximum of n passengers.
The maximum total mass is $80n$ kilograms.

(a) For Model 1, the company uses different proportions of passengers for each mass, as shown in the table.
A number is chosen at random from m integers to give the mass.

(i) Complete the table.

Proportion of passengers		$\frac{2}{m}$	$\frac{m-3}{m}$
Amount of numbers			
Mass of passenger (kg)	50	70	85

(ii) Explain why $m \geqslant 4$.

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Question 5(b) and question 5(c) are printed on the next page.

(b) For Model 2, the distance-time graph for the *EasyUp-7* elevator is modelled by $y = -\mathrm{h}(2t)$.

(i) Use your answer to **question 2 (b)(ii)** to write down the equation of this distance-time graph.

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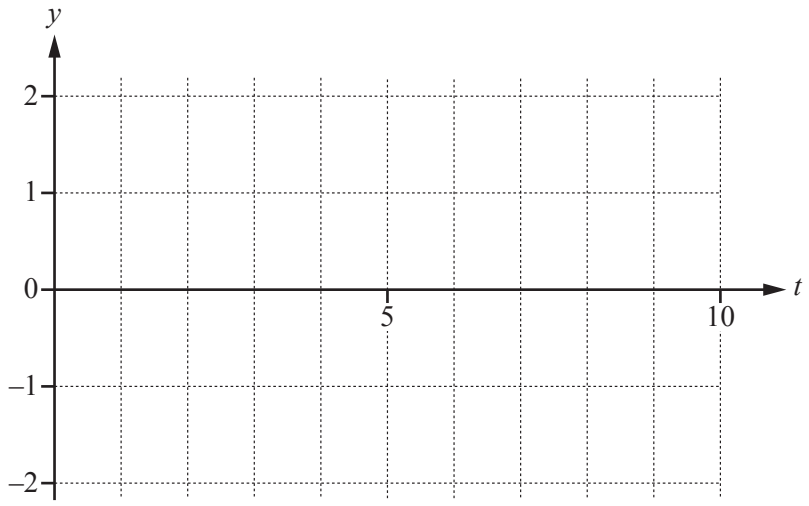
(ii) On the grid below, sketch the graph of $y = -\mathrm{h}(2t)$, for $0 \leqslant t \leqslant 10$.



(c) The mass carried by an elevator is x kilograms.
The probability that x is less than the maximum mass is 0.99.

Use this information and your graph in **part (b)(ii)** to explain why the *EasyUp-7* elevator is well designed.

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