Cambridge IGCSE

Cambridge International Examinations
Cambridge International General Certificate of Secondary Education

CANDIDATE NAME									
CENTRE NUMBER						CANDIDATE NUMBER				

CAMBRIDGE INTERNATIONAL MATHEMATICS **0607/63**

Paper 6 (Extended) **October/November 2016**

1 hour 30 minutes

Candidates answer on the Question Paper.

Additional Materials: Graphics Calculator

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
Do not use staples, paper clips, glue or correction fluid.
You may use an HB pencil for any diagrams or graphs.
DO **NOT** WRITE IN ANY BARCODES.

Answer both parts **A** and **B**.
You must show all relevant working to gain full marks for correct methods, including sketches.
In this paper you will also be assessed on your ability to provide full reasons and communicate your mathematics clearly and precisely.
At the end of the examination, fasten all your work securely together.
The total number of marks for this paper is 40.

This document consists of **16** printed pages.

CAMBRIDGE International Examinations

THE INVESTIGATION STARTS ON PAGE 3.

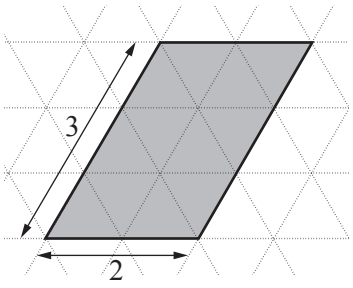
Answer **both** parts **A** and **B**.

A INVESTIGATION TRIANGULAR GRIDS (20 marks)

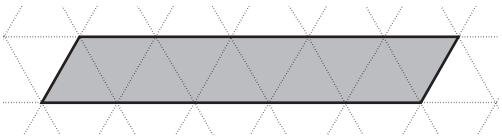
You are advised to spend no more than 45 minutes on this part.

This investigation looks at geometric results using grids of equilateral triangles.

1 The area of this 2 by 3 parallelogram is 12 triangles.

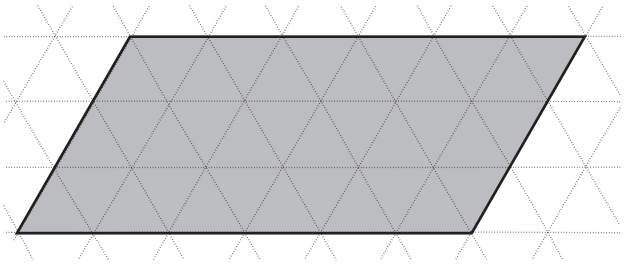


(a) Write down the area of this 5 by 1 parallelogram.



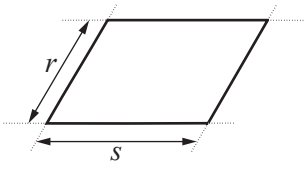
..... triangles

(b) Find the area of this 6 by 3 parallelogram.



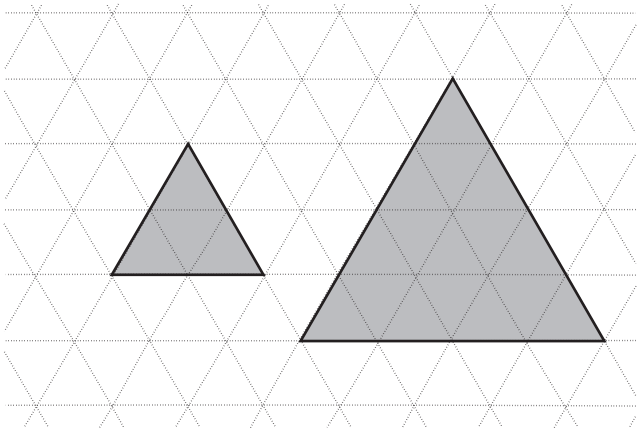
..... triangles

(c) Find a formula for the area, A, of a parallelogram measuring s by r on a triangular grid.



.....

(d) Two equilateral triangles are drawn on the grid below.



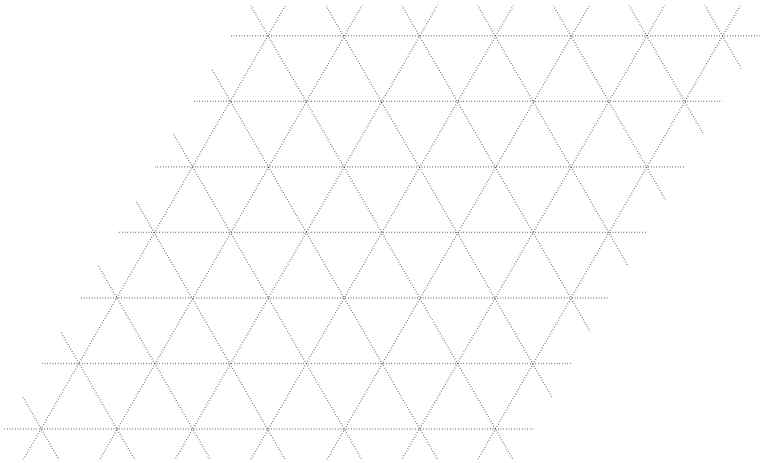
The area of the smaller triangle is 4.
Find the area of the larger triangle.

.....

(e) Find a formula for the area, A, of an equilateral triangle with side x.

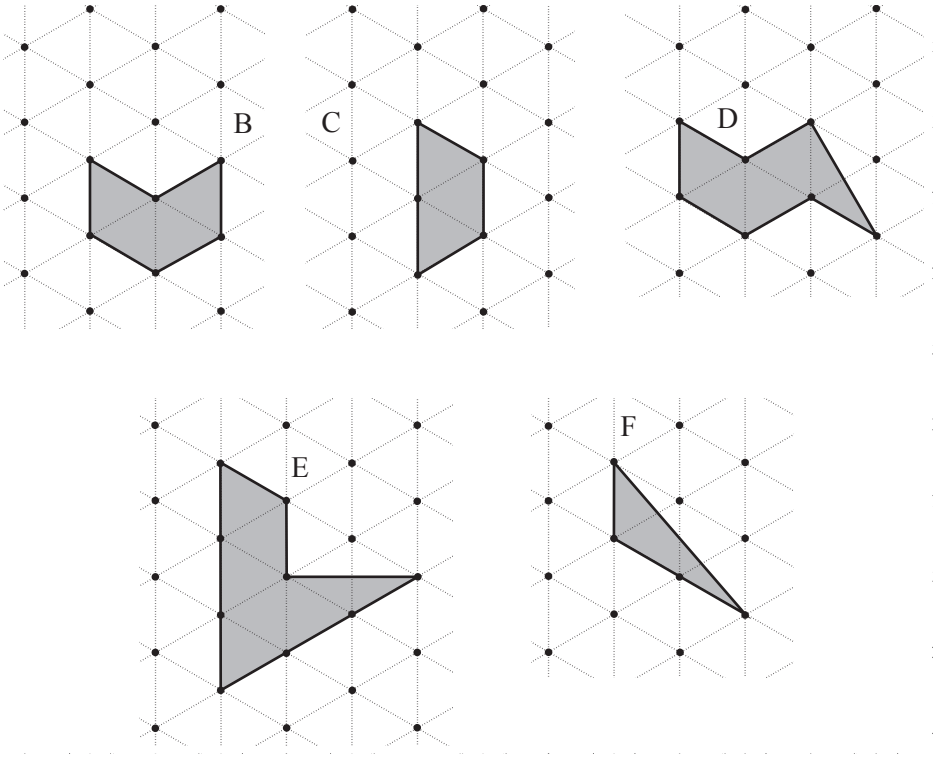
.....

(f)



Show that your formula in **part (e)** works.
Start by drawing another equilateral triangle on the grid above.

2 **(a)** Each shape is made by joining dots on a triangular grid.



Complete this table.

Shape	Number of dots inside shape (R)	Number of dots on perimeter (P)	Area in triangles (A)
B	0	6	
C	0		
D	0		5
E			
F	0	4	2

(b) For shapes on a **square** grid, Pick's rule is

$$A = R + \frac{P}{2} - 1 .$$

Does Pick's rule work for shapes on triangular grids?
Use numbers from the table in **part (a)** to support your answer.

(c) Write down a formula for A in terms of P.

.....

(d) The table below shows some values for R, P and A for shapes drawn on triangular grids.

Number of dots inside shape (R)	Number of dots on perimeter (P)	Area in triangles (A)
0	4	2
0	6	4
0	7	5
1	4	4
1	6	6
1	8	8
1	10	10
2	4	6
2	5	7
2	8	10
2	10	12
3	8	12
3	6	10
3	10	14
3	12	16
4	11	17
4	14	20

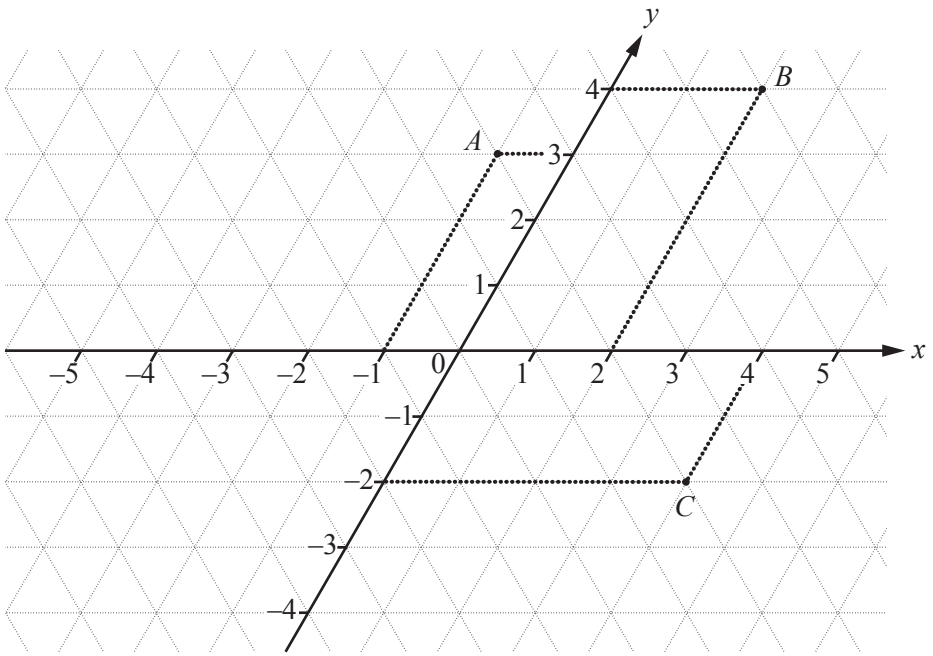
Find a formula for A in terms of R and P.

.....

(e) When $A = 100$, find a possible pair of values of R and P.

.....

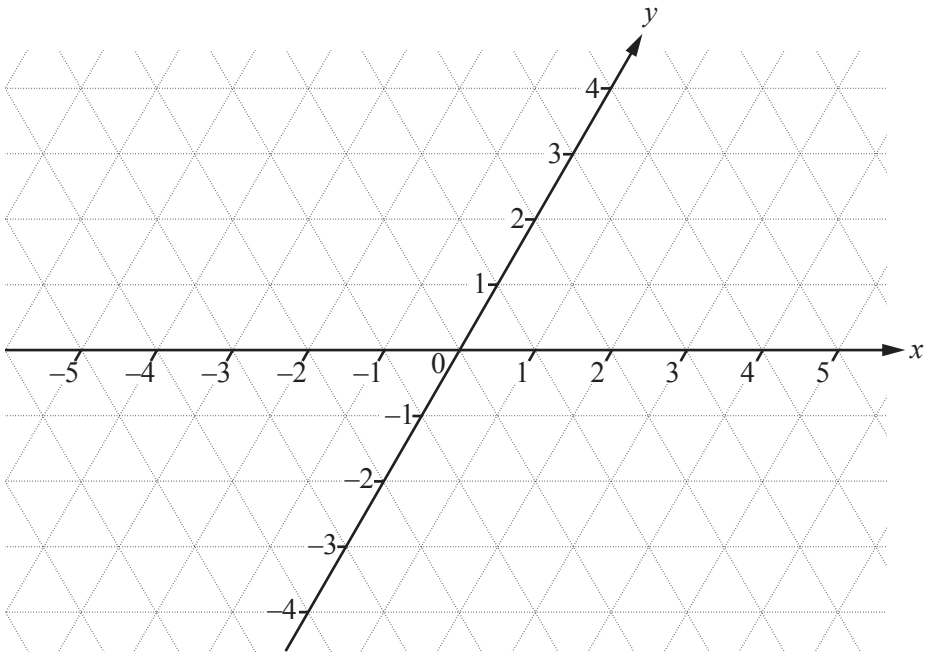
3 Co-ordinates can be used on triangular grids.



The points $A(-1, 3)$, $B(2, 4)$ and $C(4, -2)$ are shown on the grid.

(a) A regular hexagon can be drawn with integer co-ordinates for vertices.

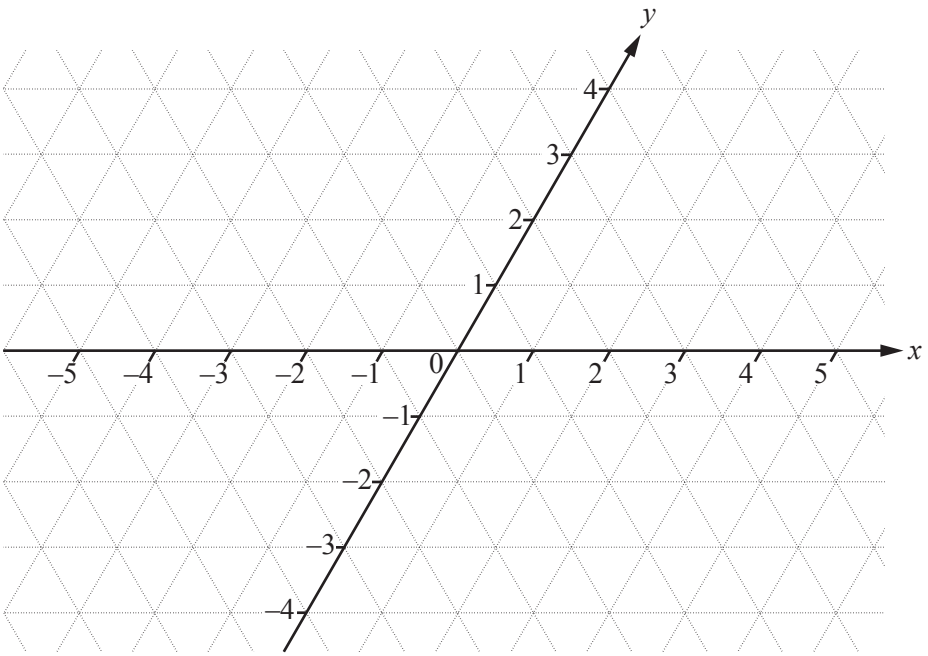
This statement is not true for square grids.
Show, using the grid, whether the statement is true for triangular grids.



The statement is .....

(b) The point (x, y) is rotated about $(0, 0)$ through 180° to the point $(-x, -y)$.

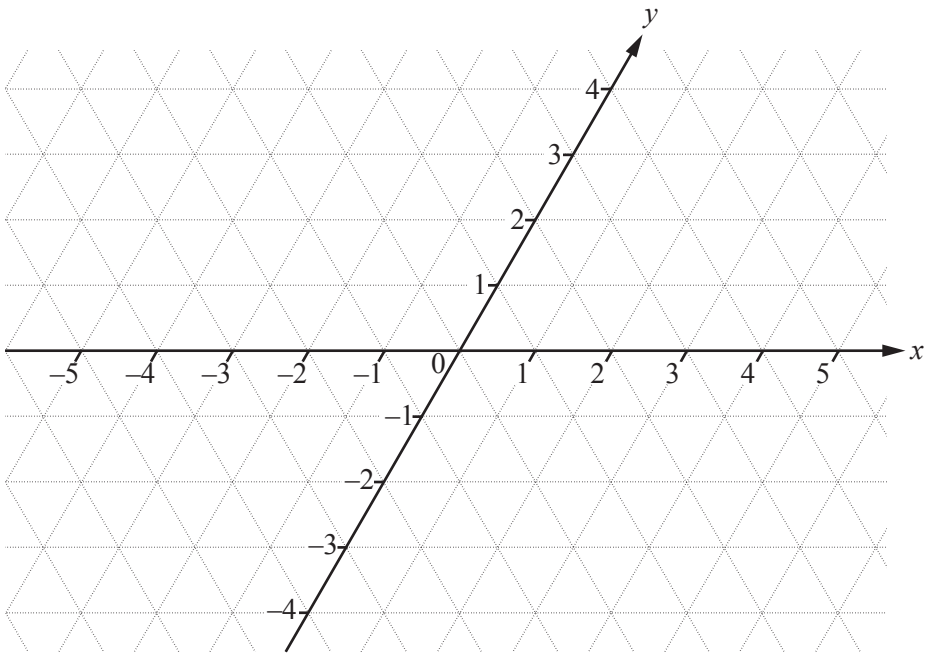
This statement is true for square grids.
Using the grid, investigate whether the statement is true for triangular grids.



The statement is .....

(c) The point (x, y) is reflected in the y-axis to the point $(x, -y)$.

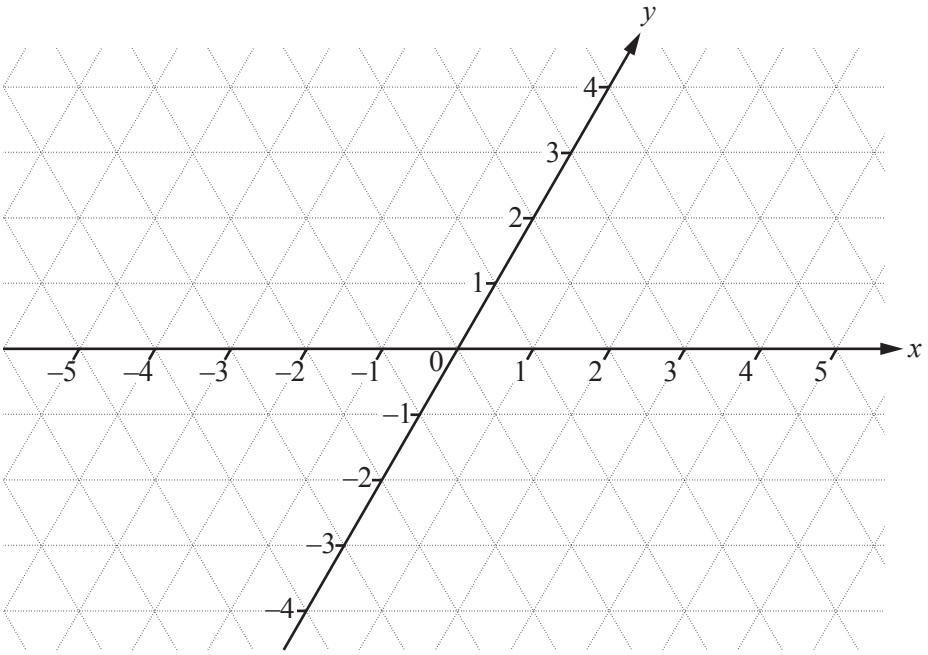
This statement is true for square grids.
Using the grid, investigate whether the statement is true for triangular grids.



The statement is .....

(d) The midpoint of the line joining the points (x_1, y_1) and (x_2, y_2) is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

This statement is true for square grids.
Using the grid, investigate whether the statement is true for triangular grids.



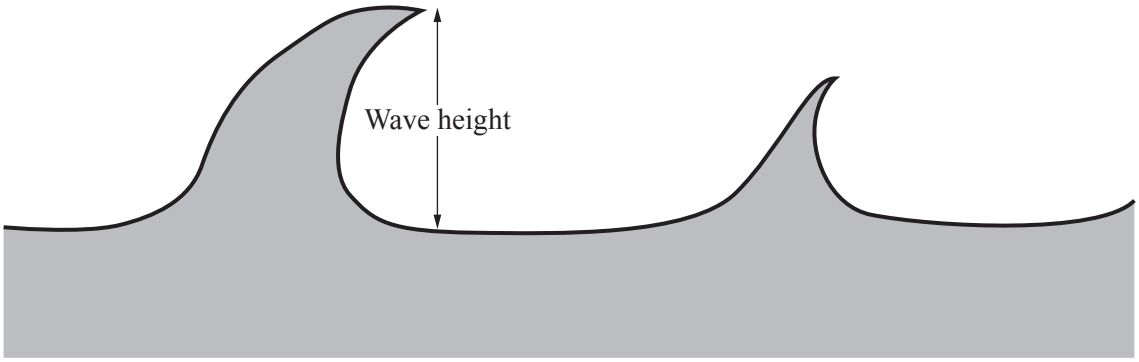
The statement is .....

B MODELLING WAVES (20 marks)

You are advised to spend no more than 45 minutes on this part.

This part is about modelling sea waves.

The sketch shows part of a sea wave.



Here are the wave heights, in metres, of a sample of 60 waves in order of size.

0.27	0.30	0.50	0.56	0.57	0.73	0.77	0.78	0.87	0.96
0.99	1.00	1.09	1.16	1.20	1.21	1.34	1.49	1.50	1.51
1.51	1.52	1.55	1.57	1.60	1.61	1.63	1.65	1.69	1.71
1.73	1.76	1.77	1.78	1.83	1.84	1.86	1.92	1.97	1.98
2.06	2.15	2.18	2.20	2.30	2.47	2.49	2.49	2.51	2.63
2.80	2.83	2.98	3.15	3.21	3.23	3.26	3.47	4.76	5.20

1 The mean height of the highest one-third of the waves in a sample is H.

(a) For the sample of 60 waves, calculate H and show that it rounds to 2.92.

(b) Scientists use H to make estimates.

Comment on the accuracy of the following estimates.

(i) The highest wave is approximately $2H$.

(ii) The highest 10% of waves have a mean height of approximately $1.27H$.

[Turn over

2 This frequency table shows 60 wave heights.

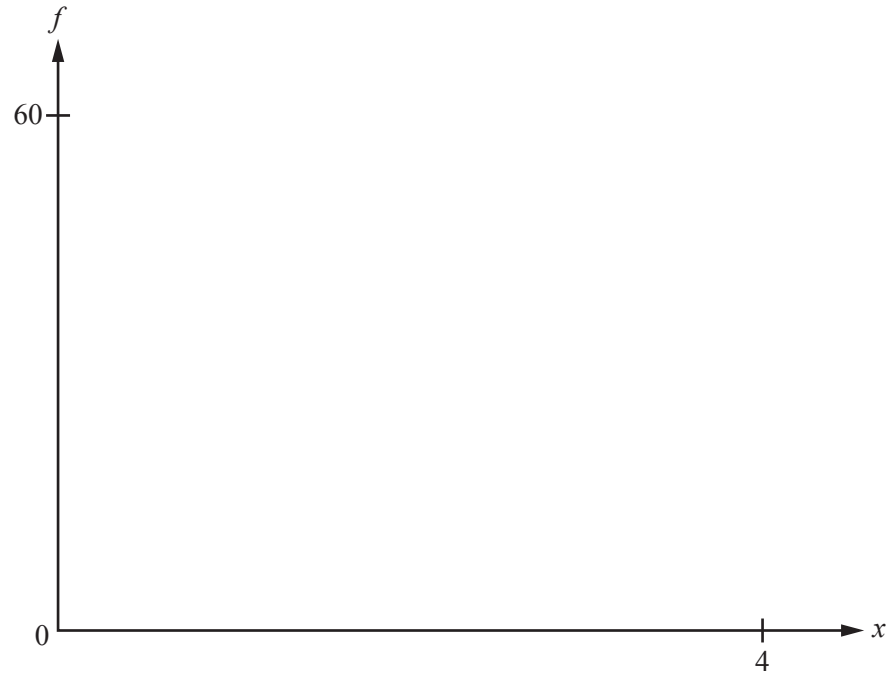
Wave height (x metres)	Frequency (f)
$0 < x \leqslant 0.5$	2
$0.5 < x \leqslant 1.0$	7
$1.0 < x \leqslant 1.5$	9
$1.5 < x \leqslant 2.0$	22
$2.0 < x \leqslant 2.5$	8
$2.5 < x \leqslant 3.0$	5
$3.0 < x \leqslant 3.5$	5
$3.5 < x \leqslant 4.0$	0
$4.0 < x \leqslant 4.5$	0
$4.5 < x \leqslant 5.0$	1
$5.0 < x \leqslant 5.5$	1

Two models for the frequency, f, are

A $\quad f = 52 \times 10^{-(x-1.8)^2}$

B $\quad f = 14x^3 \times 2^{-(x-0.7)^2}$

(a) On the axes, sketch and label the graph of Model A and the graph of Model B.



(b) For Model A, find the wave height that has the maximum frequency.

.....

(c) For Model B, find the wave height that has the maximum frequency.

.....

(d) Which model best fits the data in the table?
Give two reasons for your choice.

Model .....

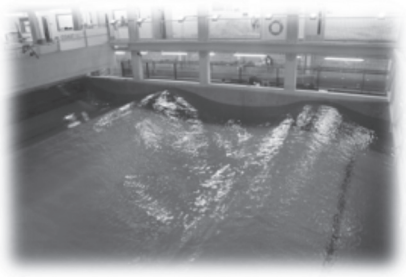
Reason 1 .....

.....

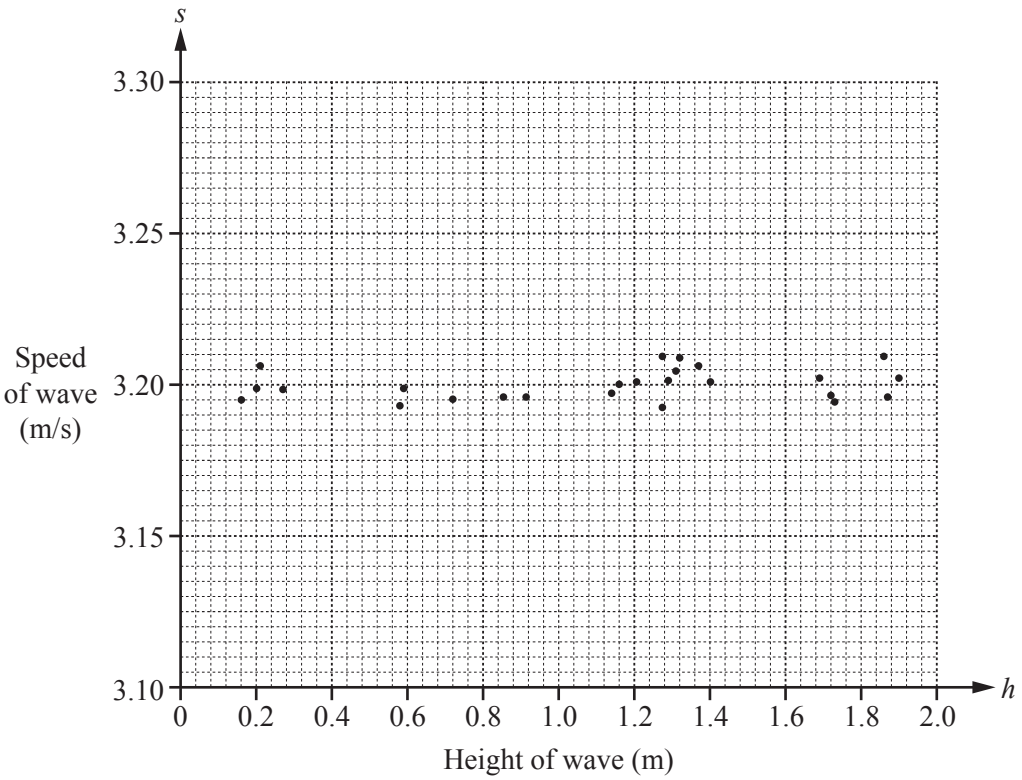
Reason 2 .....

.....

3 Wave machines make waves of different heights and speeds.



(a) This diagram shows the speed, s metres per second, for waves of different heights, h metres.



The graph of a model connecting s and h is a horizontal line.

(i) Without doing any calculations write down a possible model.

(ii) What does your model tell you about the connection between s and h?

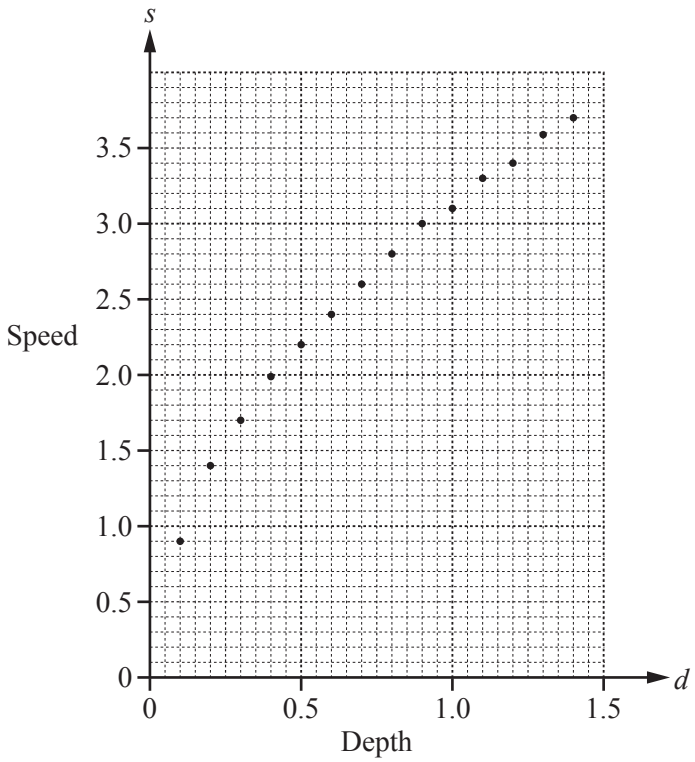
.....

.....

(b) Another model uses the connection between water depth, d metres, and wave speed, s metres per second.
Here are some results from some wave machine experiments.

d (metres)	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
s (metres per second)	0.9	1.4	1.7	2.0	2.2	2.4	2.6	2.8	3.0	3.1	3.3	3.4	3.6	3.7

The results are plotted on this grid.



Here are three possible models.

$$s = a\sqrt{d} + c \qquad s = c + a\cos d \qquad s = ad^2 + c.$$

(i) Which model best fits the data?

.....

(ii) Find suitable values for a and c in your model.

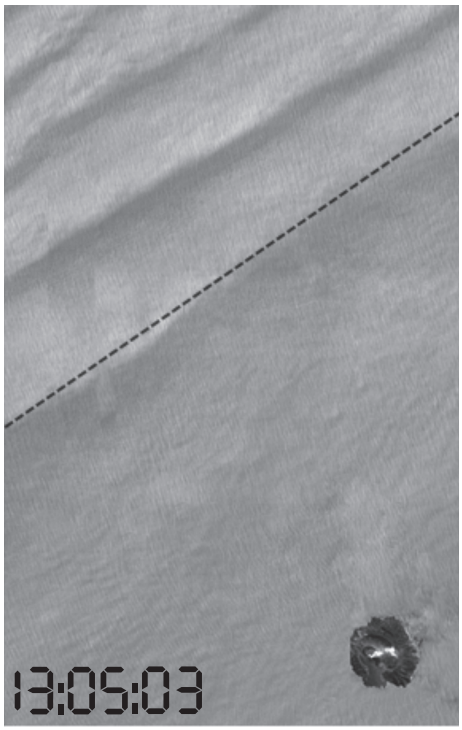
a = ...

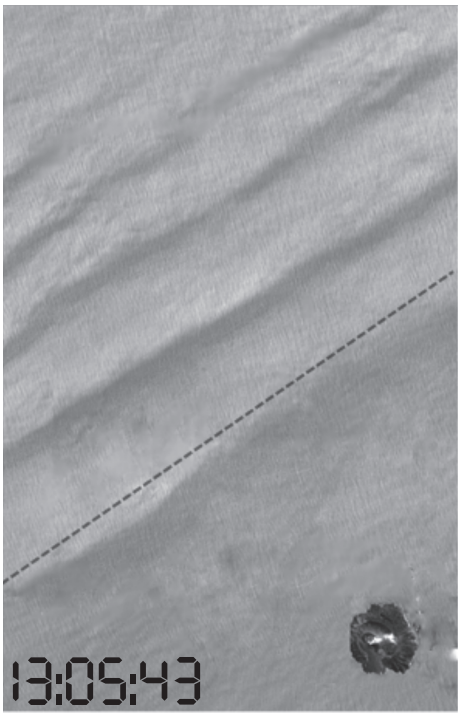
c = ...

Question 3(c) is printed on the next page.

[Turn over

(c) These photographs, taken at different times, show a small island in the bottom right-hand corner.
A wave, marked by a dotted line, travels towards the island.
On both photographs 1 cm represents 100 m.
Each photograph shows the time in the form hour : minute : second.





Use your answer to **part (b)(ii)** to calculate the depth of the sea.

..... m