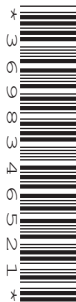




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0607/52

October/November 2015

1 hour

Additional Materials: Graphics Calculator

READ THESE INSTRUCTIONS FIRST

DO **NOT** WRITE IN ANY BARCODES.

The total number of marks for this paper is 24.

This document consists of **7** printed pages and **1** blank page.

THE INVESTIGATION STARTS ON PAGE 3.

Answer **all** the questions.

INVESTIGATION

SUMS OF TWO SQUARES

This investigation looks at the results when two square numbers are added together.

- 1** Here is a list of the first 11 prime numbers.

2 3 5 7 11 13 17 19 23 29 31

- (a) In the list there are 4 numbers that are one more than a multiple of 4.
These are called *Pythagorean Primes*.
The smallest one is 5 and the largest one is 29.

Write down the other two.

5, , , 29

- (b) The 17th century French mathematician Albert Girard proved that every Pythagorean Prime equals the sum of two square numbers.

Write your answers to **part (a)** as the sum of two square numbers.
Two have been written down for you.

$$5 = 1^2 + 2^2$$

$$\dots\dots\dots = \dots\dots\dots + \dots\dots\dots$$

$$\dots\dots\dots = \dots\dots\dots + \dots\dots\dots$$

$$29 = 2^2 + 5^2$$

- (c) Another Pythagorean Prime is 101.
Write 101 as the sum of two square numbers.

$$101 = \dots\dots\dots + \dots\dots\dots$$

- 2 The sum of two square numbers can equal another square number.
For example,

$$\begin{aligned} 3^2 + 4^2 &= 9 + 16 \\ &= 25 \\ &= 5^2 \end{aligned}$$

We say that **3, 4, 5** is a *Pythagorean Triple*.

- (a) Show, by calculation, that 7, 24, 25 is a Pythagorean Triple.

- (b) Each row in this table is a Pythagorean Triple.

Complete the table.

Use patterns of numbers in the table to help you.

3	4	5
5	12	13
7	24	25
9	40	
11	60	
13		
		113

- (c) What is the connection between the **square** of the smallest number and the other two numbers in each Pythagorean Triple in the table?

.....

.....

- (d) Use your answer to **part (c)** and the patterns of numbers in the table to complete the following Pythagorean Triple.

..... , , 421

3 $2\sqrt{x}$, $x - 1$, $x + 1$ is a Pythagorean Triple when x is a square number.

(a) (i) Find the Pythagorean Triple when $x = 16$.

.....,,

(ii) Check that your answer to **part (a)(i)** is a Pythagorean Triple.
Use the method of the example in **question 2**.

(b) In the table, x is the square of an even number.
Each row is a Pythagorean Triple.

	$2\sqrt{x}$	$x - 1$	$x + 1$
$(x = 16)$			
$(x = 36)$	12		37
	16	63	65
		99	
	24		145

Write your answer to **part (a)(i)** in the first row of this table.

Complete the three columns of the table.

You may use patterns or the fact that $2\sqrt{x}$, $x - 1$, $x + 1$ is a Pythagorean Triple to help you.

- (c) What is the connection between the **square** of the smallest number and the sum of the other two numbers in each of the Pythagorean Triples in the table?

.....

.....

- (d) Show algebraically that $2\sqrt{x}$, $x - 1$, $x + 1$ satisfies your connection in **part (c)**.

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