Cambridge IGCSE

Cambridge International Examinations
Cambridge International General Certificate of Secondary Education

CANDIDATE NAME	
CENTRE NUMBER	
CANDIDATE NUMBER	

3861217397

CAMBRIDGE INTERNATIONAL MATHEMATICS **0607/61**

Paper 6 (Extended) **October/November 2014**

1 hour 30 minutes

Candidates answer on the Question Paper

Additional Materials: Graphics Calculator

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
Do not use staples, paper clips, glue or correction fluid.
You may use an HB pencil for any diagrams or graphs.
DO **NOT** WRITE IN ANY BARCODES.

Answer both parts **A** and **B**.
You must show all relevant working to gain full marks for correct methods, including sketches.
In this paper you will also be assessed on your ability to provide full reasons and communicate your mathematics clearly and precisely.
At the end of the examination, fasten all your work securely together.
The total number of marks for this paper is 40.

This document consists of **11** printed pages and **1** blank page.

CAMBRIDGE International Examinations

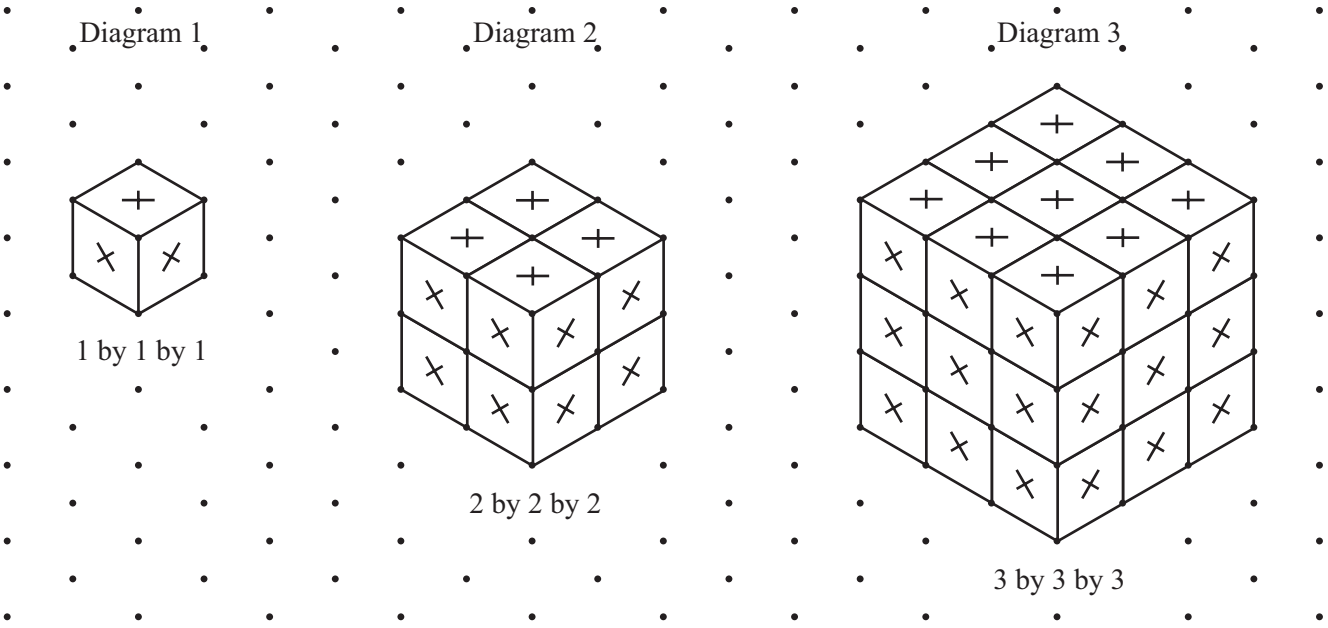
Answer **both** parts **A** and **B**.

A INVESTIGATION CUBES (20 marks)

You are advised to spend no more than 45 minutes on this part.

Identical small cubes fit together to make larger cubes.
There are no gaps between these small cubes.
For each cube that is made, a cross is marked on each **outside** face of each small cube.

The diagram shows the first three cubes that can be made.



This investigation is about the number of crosses that can be marked on cubes.

Look at the 1 by 1 by 1 cube.
It is made from 1 small cube.
It has 6 crosses on it (3 crosses are on the faces not seen on the diagram).

1 Look at the 2 by 2 by 2 cube.

(a) How many small cubes is this cube made from?

.....

(b) Explain why there are only 3 crosses on each small cube.

.....

(c) Find the total number of crosses on the 2 by 2 by 2 cube.

.....

2 Look at the 3 by 3 by 3 cube.

(a) How many small cubes is this cube made from?

.....

(b) How many of these small cubes have 3 crosses on them?

.....

(c) There are 12 small cubes with 2 crosses on them.
There is 1 small cube with no crosses on it.

How many small cubes have only 1 cross on them?

.....

3 Complete this table.
You may use the dotty grid on page 7 to help you.

Size of cube	Total number of small cubes	Number of small cubes with			
		0 crosses	1 cross	2 crosses	3 crosses
2 by 2 by 2		0	0		
3 by 3 by 3		1		12	
4 by 4 by 4	64	8	24		
5 by 5 by 5		27	54		8

4 **(a)** To work out the number of crosses on the 3 by 3 by 3 cube, complete the following.

1	small cube with	0	crosses gives	0	crosses
.....	small cubes with	1	cross gives	6	crosses
12	small cubes with	2	crosses gives	.....	crosses
.....	small cubes with	3	crosses gives	.....	crosses
			Total =	.....	crosses

(b) The total number of crosses can also be worked out by the following method.
Complete the following.

The number of crosses on one face of the 3 by 3 by 3 cube is .....

So the total number of crosses on all the 6 faces is .....

(c) Find the total number of crosses on a 4 by 4 by 4 cube.

.....

(d) Find an expression, in terms of n, for the total number of crosses on an n by n by n cube.

.....

5 The number of small cubes with 0 crosses forms a sequence.

Size of cube	2 by 2 by 2	3 by 3 by 3	4 by 4 by 4	5 by 5 by 5		*n* by *n* by *n*
Number of small cubes with 0 crosses	0	1	8	27		

For an n by n by n cube, find an expression, in terms of n, for the number of small cubes with 0 crosses. Write your answer in the table above.

6 For an n by n by n cube, an expression for the number of small cubes with one cross is $6(n-2)^2$.

In an n by n by n cube, can the number of small cubes with 0 crosses equal the number of small cubes with one cross? Show your working.

7 For an n by n by n cube, find an expression, in terms of n, for the number of small cubes with 2 crosses.

.....

8 **(a)** In an n by n by n cube there are 64 small cubes that have 0 crosses on them.
How many small cubes does it have altogether?

.....

(b) In another n by n by n cube there are 60 small cubes that have 2 crosses on them.
In this cube, how many small cubes have only 1 cross on them?

.....

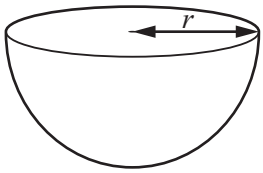
B MODELLING FISH PONDS (20 marks)

You are advised to spend no more than 45 minutes on this part.

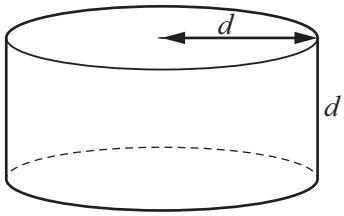
Volume of a cylinder of radius r, height h, is $\pi r^2 h$
Volume of a sphere, radius r, is $\frac{4}{3}\pi r^3$

Fish ponds can be either hemispherical or cylindrical.

hemispherical pond



cylindrical pond



1 **(a)** A hemispherical pond has radius r metres.

When $r = 3$, show that the volume of the pond is $18\pi\,\text{m}^3$.

(b) A cylindrical pond has a radius of d metres and a depth of d metres.
Show that the volume of this pond is πd^3 cubic metres.

(c) The radius and the depth of the cylindrical pond in **part (b)** is the same as the radius of the hemispherical pond in **part (a)**.
Show that the cylindrical pond holds more water than the hemispherical pond.

(d) A hemispherical pond of radius r metres has the same volume as a cylindrical pond of radius and depth d metres.

Show that $d = \sqrt[3]{\frac{2}{3}}\,r$.

2 Theo wants to work out how many fish he can put in a pond.

A fish needs 5 litres of water for every 0.1 cm of its length.

(a) The average length of 15 fish is 18 cm.
Find the number of cubic metres of water needed for these fish.

.....

(b) There are F fish in the pond, with an average length of L cm.
Find a model for the number of cubic metres of water, W, needed.

.....

(c) Theo's pond has a volume of 20 m^3.

(i) Use your model to find the maximum number of fish, of average length 24 cm, that can be put in this pond.

.....

(ii) If Theo's pond is hemispherical find its radius.

.....

(iii) If Theo's pond is cylindrical, and the radius and depth are the same, find its radius.

.....

3 Theo decides to have a cylindrical pond.
The radius, r metres, does not have to be the same as the depth, d metres.

(a) For a pond with volume 20 m^3 find a model for the depth, d, in terms of r.

.....

(b) Sketch the graph of d against r for $1 \leqslant r \leqslant 5$.

(c) What practical problem is there when the radius is less than one metre?

.....

(d) Find the radius of a pond when the depth is 1 m.

.....

4 The water level must be 30 cm below the top of the sides of the pond.

(a) Modify your model in **question 3 (a)**.

.....

(b) Explain how this affects your graph in **question 3 (b)**.

BANK PAGE