ADDITIONAL MATHEMATICS

Paper 0606/12
Paper 12

Key messages

Candidates are to be reminded of the importance of working to a suitable level of accuracy throughout a question in order to be able to give their final answer correctly to the required level of accuracy. It is also essential that candidates ensure that they have met the requirements of each question. In questions that require the candidate to show a specific result, it is essential that each step of the solution is shown clearly.

General comments

There were many scripts of a high standard showing a good understanding of the syllabus and the correct applications of the techniques required. Most solutions were set out clearly and, where the need arose, the blank page in the question/answer booklet was utilised as intended, or extra pages were used.

Comments on specific questions

Question 1

(a) In both parts of this question, it was essential that the correct notation was used. A single digit answer was required as the question asked for the number of elements in the given sets. An answer of the form {6} or {1} was therefore considered to be incorrect. However, most candidates answered correctly.

(b) Most candidates were able to produce a correct Venn diagram with set *P* enclosed in set *Q* and sets *Q* and *R* separate.

(c) Any correct answers were acceptable, with most candidates obtaining at least one of the results given below.

Answers: **(a)(i)** 6 **(ii)** 1 **(c)** $S' \cup T'$ or $(S \cup T)'$ and $(X \cap Y) \cup (X \cap Z)$ or $X \cap (Y \cup Z)$

Question 2

The following criteria were needed to gain marks: a maximum point in the first quadrant, intercepts on the axes either labelled or written below the graph, cusps on the *x*-axis and a correct shape of the curve for $x < -\frac{1}{2}$ and $x > 3$. Marks were usually lost when the cusps were drawn as stationary points and the outer parts of the curve were the incorrect shape. It is suggested that the graph of the quadratic equation without the modulus is drawn faintly or with construction (dotted) lines to start with. A reflection in the *x*-axis should then ensure the correct shape throughout. The appearance of such construction lines will not be penalised but seen as an aid to obtaining a correct sketch.

Question 3

(i) Most candidates were able to obtain the first three terms of the required expansion. There was the occasional arithmetic slip or sign error in some cases. It must be noted that a few candidates chose to take out a factor of 3, giving their final answer as $243 - 54x + 5x^2$. These candidates had not answered the question correctly, not understanding that a factor of 3 could only be taken out if the final answer was written as $3\left(243-54x+5x^2\right)$.

(ii) A correct expansion of $\left(x-\frac{2}{x}\right)^2$ was obtained by most candidates and then used correctly with their answer to **Part (i)** to obtain the two terms independent of *x* which would then lead to the final answer.

Answers: **(i)** $729-162x+15x^2$ **(ii)** –2856

Question 4

(i) The majority of candidates recognised the notation $\mathrm{p}'(x)$ and used it correctly with the remainder theorem to obtain the given result.

(ii) The factor theorem was used by most candidates to obtain a second equation in *a* and *b*. This equation, together with the given result from **Part (i)**, were usually solved correctly.

(iii) It was intended that candidates use either algebraic long division or observation to write $\mathrm{p}(x)$ in the required form. Some candidates chose to use synthetic division. This method will only be correct if the resulting extra factor of 2 (the result $(2x-1)\left(2x^2+28x+49\right)$ is obtained) is taken into account. Candidates must be careful when using synthetic division by a factor of the form $ax+b$ where $a \neq \pm 1$ or 0.

(iv) This is an example of where some candidates did not take note of the demand of the question. The demand was to factorise $\mathrm{p}(x)$. Some candidates factorised $Q(x)$ only and some chose to write down the solutions to $\mathrm{p}(x)=0$.

Answers: **(ii)** $a=27$, $b=84$, **(iii)** $(2x-1)\left(x^2+14x+49\right)$ **(iv)** $(2x-1)(x+7)^2$

Question 5

(i) The product rule for logarithms was applied appropriately by the majority of candidates to obtain the correct result.

(ii) The product rule and the power rule for logarithms were applied appropriately by the majority of candidates to obtain the correct result.

(iii) Many candidates did not obtain full marks for this part of the question as they did not answer the question completely. Most obtained the correct result of $p=\frac{1}{6}$, but some candidates stopped at this point and gave a final answer of 0.17. Candidates should be guided by the mark allocation. In this case there is a mark allocation of 3 marks which is too generous for the solution of a simple linear equation. Of those candidates that did continue and solve to obtain a value for x, most were successful, giving their final answer to the required level of accuracy.

Answers: **(i)** $2+p$ **(ii)** $7p-4$ **(iii)** 1.26

Question 6

(a) An answer of products of two matrices was expected. Those candidates that gave the extra answers of **CBA** and **AA** were not able to obtain credit for these answers but are to be commended on recognising that these were valid options.

(b) (i) Most candidates obtained a correct inverse matrix.

(ii) A few candidates did not take note of the word 'Hence' in the instruction to find the matrix **Z**. Using a method involving the solution of four simultaneous equations in four unknowns was not an acceptable answer. It was intended that pre-multiplication of the given equation by the inverse matrix obtained in **Part (i)** was used. Many candidates did just this, with very few instances of post-multiplication being seen.

Answers: **(a)** **BA** and **CB** **(b)(i)** $\frac{1}{16}\begin{pmatrix} 3 & 2 \\ -5 & 2 \end{pmatrix}$ **(ii)** $\frac{1}{16}\begin{pmatrix} 16 & 3 \\ -16 & -5 \end{pmatrix}$

Question 7

It was essential that no use of calculators was made in this question. It was therefore necessary that each step in the working of the solution be shown.

(i) Most candidates used either the area of a trapezium or the area of a triangle and a rectangle with sufficient evidence of expansion without a calculator to obtain the correct result.

(ii) Most candidates were able to obtain $\cot\theta = \frac{4}{10 - 2\sqrt{5}}$ or equivalent. Some errors in the length involving *DC* were made, but this did not preclude the awarding of a method mark for a correct attempt at rationalisation. This was another part of the paper where some candidates did not read the requirements of the question and did not give their final answer in the required form. A check on this should be made at the end of each question.

Answers: **(i)** $10 + 22\sqrt{5}$ **(ii)** $\frac{1}{2} + \frac{\sqrt{5}}{10}$

Question 8

(a) (i) There were quite a few candidates who did not appreciate the fact that when a particle is travelling at constant velocity, the acceleration is zero. It was not intended that the gradient at the instant $t = 5$ be calculated, although if it was done correctly the mark was awarded.

(ii) Most candidates realised that they needed to find the total area under the graph. There was only the occasional arithmetic slip made by some candidates.

(b) (i) This part of the question was intended to test whether candidates were aware of the difference between velocity and speed. Unfortunately, there were many incorrect answers of –2.5.

(ii) This part of the question was intended to test whether candidates realised that they needed to be working in radians as well as the need for differentiation. Fortunately, there were many completely correct solutions. It should be noted that some candidates, when choosing not to give an exact answer, did not give their final answer to the correct level of accuracy.

Answers: **(a)(i)** 0 **(ii)** 110 m **(b)(i)** 2.5ms^{-1} **(ii)** $\frac{\pi}{4}$ or 0.785

Question 9

(i) Very few incorrect solutions were seen, with most candidates making correct use of the area of the sector to obtain an expression for the angle of the sector first and then making use of the arc length and the radius to obtain the given result for the perimeter. It should be noted that candidates should be working in radians for efficiency, but as this was not a requirement of the question, those that chose to work in degrees were not penalised.

(ii) Most candidates realised the need to differentiate the expression for the perimeter and equate it to zero to find the value of *r* for which the perimeter has a stationary value. This was another example of candidates not reading the requirements of the question as many lost a mark by not finding this value of P. It was also essential that working be shown to determine the nature of the stationary point. Most chose to use the second derivative method and arrive at the correct conclusion appropriately. If the second derivative method is not being used and the method of inspection of the gradient either side of the stationary point is being considered, it is essential that candidates make this clear, usually by using a table which has clear headings.

Answer: **(ii)** 24, minimum

Question 10

(i) There were many completely correct solutions to this part, with candidates integrating correctly and making correct use of arbitrary constants. Candidates must ensure, however, that they give their final answer as an equation as required, not as an expression.

(ii) Apart from those candidates that mistakenly thought that the gradient of the tangent was 10, most applied a correct method. The final accuracy mark was not awarded if the correct level of accuracy was not used. Answers in exact form were acceptable as were unsimplified answers as the form of the final answer was not specified..

Answers: **(i)** $y = e^{2x} + \frac{3x^2}{2} + 8x - 6$ **(ii)** $y + 2.26 = -\frac{1}{12}\left(x - \frac{1}{4}\right)$

Question 11

(a) Most candidates were able to obtain at least one mark in this part of the question, by obtaining an equation in terms of $\sin x$ and $\cos x$ together with one correct solution. Too many candidates divided their equation through by $\sin x$ and thus did not consider the solutions of the equation $\sin x = 0$. Some candidates were also unable to obtain both solutions to $\cos x = \pm\frac{1}{\sqrt{2}}$, not having considered the solution obtained from $\cos x = -\frac{1}{\sqrt{2}}$.

(b)(i) Many correct solutions were seen with most candidates showing enough working to obtain the given result.

(ii) Most candidates realised that they needed to use the result from **Part (i)** and hence attempted to solve the equation $\cos 3\theta = \frac{1}{2}$. It was pleasing to see that many candidates are now a lot more confidant in dealing with negative angles. Most chose to leave their answers in terms of π.

Answers: **(a)** $0^\circ, 45^\circ, 135^\circ, 180^\circ$ **(b)(ii)** $\pm\frac{\pi}{9}$, $\pm\frac{5\pi}{9}$

ADDITIONAL MATHEMATICS

Paper 0606/22
Paper 22

Key messages

To succeed in this examination, candidates need to be able to interpret and use all the information given in a problem. Candidates should read each question carefully and identify key statements. Candidates also need to show sufficient method so that marks can be awarded. Candidates need to be aware of instructions in questions such as '...showing all your working'. Such instructions mean that when a solution is incomplete, often through calculator use, a significant loss of marks will result. Candidates should ensure that their answers are given to at least the accuracy demanded in a question. When no particular accuracy is required, candidates should ensure that they follow the instructions printed on the front page of the examination paper. Candidates need to take care to ensure that their calculator is in the appropriate mode when working with trigonometric expressions.

General comments

Most candidates were well prepared for this examination and many excellent solutions were offered. Candidates were able to recall and use manipulative technique when needed. Most candidates were also able to formulate problems into mathematical terms and select and apply appropriate techniques of solution.

The presentation of work was generally clear and logical. Some candidates made good use of the blank pages at the end of the paper or used additional paper. This ensured that their work was legible and could be marked. Candidates who did this usually added a note in their script to indicate that their answer was written, or continued, elsewhere. This was very helpful.

Many candidates offered complete solutions, with all working shown. Candidates who relied on their calculator to solve equations or evaluate definite integrals, for example, often lost marks. This was because key steps in the method, which were required, were omitted. Showing clear and full method is essential if a question asks candidates to 'Show that…' a result is in a particular form. This instruction indicates that the answer has been given and that the marks will be awarded for the method. Working back from the given answer is rarely successful in this case. The need for this was highlighted in **Questions 2(i)** and **7(i)** in this examination.

When candidates are required to 'Explain why' something is valid or correct, it is important that any explanation is not contradictory or does not contain incorrect statements. This was required in **Questions 7(ii)**, **9(a)(i)** and **9(b)(iii)** in this paper.

In order for final answers to be accurate to three significant figures, working values must be given to a greater accuracy. This avoids a premature approximation error. This was evident in **Question 2(ii)**, **10(ii)** and **11(b)(ii)** in this paper.

Most candidates attempted to answer all questions. Candidates seemed to have sufficient time to attempt all questions within their capability.

Comments on specific questions

Question 1

Generally, this question was well answered. A few candidates reversed the answers to **Parts (i)** and **(ii)**. The most common error was to find ${}^{15}C_3$ in **Part (iii)**.

Answers: **(i)** 1 081 575 **(ii)** 40 320 **(iii)** 2730

Question 2

(i) Almost all candidates understood the need to apply the quotient rule, or rearranged correctly and used the product rule. Most earned 3 or 4 marks. The few candidates who were attempting to work back from the given answer often stated the derivative of e^x as xe^x. Some candidates omitted brackets or did not show convincing working to find the given answer. A few candidates omitted to state the given answer as part of their solution. This was penalised.

(ii) A good number of candidates found the value of the derivative when $x = 2$ and evaluated this to at least 3 significant figures, as needed. The solution was usually completed by multiplying this by h. A few candidates rounded the value of the derivative when $x = 2$ to fewer than 3 significant figures and seemed to be applying the idea of approximation to the answer when, in fact, the method being used was the approximation.

Answer: **(ii)** $-0.0261h$

Question 3

(i) Some excellent sketches were seen, with candidates taking care over the axis of symmetry of the curve as well as the y-intercept and amplitude. A few candidates initiated their sketch at the correct point but then clearly used the x-axis as the axis of symmetry. Most candidates attempted a graph of correct period and only a few sketches had incorrect amplitude.

(ii) Almost all candidates answered this part correctly. Those few who were incorrect usually arranged the three values in a different order.

Answer: **(ii)** $a = -1$, $b = 5$, $c = 3$

Question 4

(a) Candidates all understood that the brackets on the left needed to be expanded and the terms collected. Mostly this was managed correctly. Occasional sign slips were made and a few arithmetic errors were seen. The majority of candidates were able to find the critical values for their quadratic expression and most of these were able to give an inequality of the correct form for their answer. When stating critical values, it may be less confusing for some candidates to write, for example,

CV: $x = -1$, $x = \frac{3}{4}$, as many better candidates did. This may have reduced the errors made by a few candidates who wrote, for example, CV: $x \leqslant -1, \frac{3}{4}$, which they then stated as their answer.

(b) The majority of candidates used the given equation, correctly wrote down the values for a, b and c and applied $b^2 - 4ac$. Most of these candidates were able to find the discriminant as -1. The best candidates understood that, as the value was independent of k, there were no real roots *whatever the value of k*. Indication of this was required for full marks to be given. A few candidates multiplied through by 4 and worked with $x^2 + 4kx + 4k^2 + 4 = 0$. This was allowed. Some of these candidates did not multiply through correctly and this was not permitted as it was unnecessary. A few candidates were unable to state the correct a, b and c. These candidates usually included xs in their expressions or incorrectly grouped the x^2 and k^2 terms, for example.

Answer: **(a)** $-1 \leqslant x \leqslant \frac{3}{4}$

Question 5

In this question, candidates needed to apply problem solving skills and work their way through the correct, multi-step solution. Care needed to be taken at all stages to ensure that the information given was used in the correct way. Most candidates were able to correctly find the gradient of *AB*. A few candidates inverted the calculation and usually did this consistently, appearing to have a correct solution as they had made a repeated error. Almost all candidates understood that the gradient of *CD* could be found using the product of the gradients being equal to –1. Many candidates then either formed an equation using the gradient in terms of k or formed the equation of the line *CD* and substituted $x = 3$. A good number of candidates found the x-coordinate of *D* in this way. At this point in the solution, a few candidates would have benefitted from rereading the question to check which equation they were trying to find. While many candidates went on to state an acceptable form of the correct equation, a few found the equation of *CD* or *AB* rather than the perpendicular bisector. These candidates usually omitted to find the mid-point of *CD*. Some candidates used an incorrect gradient, after finding the mid-point correctly.

Answer: $y = -\frac{3}{2}\left(x - \frac{13}{2}\right)$

Question 6

(i) This part was almost universally correct. A few candidates omitted brackets and stated $\ln y = \ln A + \ln bx$. This was penalised unless there was clear evidence that $\ln y = \ln A + (\ln b)x$ was intended.

(ii) Many candidates would have benefitted from rereading the question as the equation of the line of best fit was often omitted. Some candidates needed to take more care with the form of the line of best fit as, after stating $Y = 1.4X + 2.2$, they often stopped and did not replace Y with $\ln y$. Many candidates did not round their correct values of A and b to 1 significant figure, as required. A few candidates used points which were not on the line, commonly (0.8, 3.4), to find the value of m and/or c. This was not permitted. Some candidates needed to take more care with reading the scale when reading the value of the y-intercept as it was often stated as 2 or 2.1. Occasionally the value of the intercept was calculated using their gradient instead of reading it from the graph. This introduced an unnecessary opportunity to make an error. Weaker candidates tended to confuse A with $\ln A$ and b with $\ln b$. A few weaker candidates anti-logged by incorrectly using the base 10 or worked with lg throughout, instead of ln.

(iii) The simplest method of solution for this part was to use the graph to find $\ln y = 6$ when $x = 2.7$ and then anti-log. This method was not dependent on having the correct values for A and b. A few candidates did this, although most used the exponential or logarithmic equation they had found.

Answers: **(i)** $\ln y = \ln A + x\ln b$ **(ii)** $\ln y = 1.4x + 2.2$; $A = 9$, $b = 4$ **(iii)** $y = 400$

Question 7

(i) A good number of candidates earned full marks for this part. A few made slips with the 2 or the $\frac{1}{2}$ when applying the chain rule to $\sqrt{x^2+1}$, but this was not common. A few candidates were unable to manipulate their unsimplified answer to the form required. Again, this was not common.

(ii) This was very well answered with a high proportion of candidates earning both marks. A few candidates gave at least a partially correct explanation. Some candidates gave more comment than was required and made an error. Those candidates who kept their solutions simple and stated that $\frac{dy}{dx} = 0$ at a stationary point and that it was not possible for $2x^2$ to be –1, were the most successful.

Answer: **(i)** $\frac{dy}{dx} = \frac{2x^2+1}{(x^2+1)^{\frac{1}{2}}}$

Question 8

(i) This part was almost universally correct.

(ii) A good number of correct solutions were seen. Some candidates misunderstood the ratio and attempted to work with $\overrightarrow{AC} = 3\overrightarrow{CB}$. Candidates who formed a proportion $\frac{AC}{CB} = \frac{1}{3}$ usually avoided this error. A good proportion of candidates were able to form a correct vector route to find $\overrightarrow{OC}$. Some candidates, again, should have reread the question as, having found $\overrightarrow{OC}$ they did not complete the solution. Most candidates who attempted to find the unit vector were successful. A few candidates multiplied by the magnitude of the vector, instead of dividing by it. A few other candidates seemed to think the magnitude was the unit vector.

(iii) Again, a good number of correct answers were seen. Some formed $\overrightarrow{DA}$ and then negated it. Others used the route $\overrightarrow{OD} - \overrightarrow{OA}$ successfully. Weaker candidates often misread their own writing, using $\overrightarrow{OD}$ as $\overrightarrow{AD}$ or omitted to understand that $\overrightarrow{AD}$ was a fractional part of $\overrightarrow{OA}$ as they multiplied by λ only. Candidates who were unable to successfully interpret the ratio in **Part (ii)** generally repeated the error in this part.

Answers: **(i)** $4\mathbf{i} - 16\mathbf{j}$ **(ii)** $\frac{3\mathbf{i} + 8\mathbf{j}}{\sqrt{73}}$ **(iii)** $\frac{-\lambda}{\lambda + 1}(2\mathbf{i} + 12\mathbf{j})$

Question 9

(a) (i) The simplest explanations offered were based upon each *x* is mapped to a unique *y*, therefore a function, and the function being many-one, therefore no inverse. Most candidates were able to state a satisfactory reason to explain why the function had no inverse but very few candidates justified the mapping being a function. Weaker candidates commented that as it had input and output it was a function or tried to find the inverse function and comment on issues with the domain. This was not accepted as the inverse did not exist in this case. A few candidates were unclear in their comments as to whether they were considering g or its inverse. It was not uncommon to suggest 'it is one-many', for example.

(ii) An excellent number of correct expressions were stated for the composite function and its domain. A few candidates needed to take more care as $6(6x^4 + 5) + 5$ was not an uncommon incorrect answer amongst those seen. Weaker candidates occasionally stated the answer $(6x^4 + 5)^2$.

(iii) This part proved challenging for many. A reasonable number of correct answers were seen, but the most common incorrect answer offered was 5.

(iv) Again, this proved to be challenging. Some candidates misinterpreted the phrase *For this value of k*. Rather than understanding that this required the negative fourth root to be taken, many found the value of their expression for the inverse of h when *x* was either 5 or 0. Some candidates remembered to take $\pm$ the fourth root. Most of these omitted to discard the positive root, however.

Most candidates earned 2 marks and this was usually for giving an answer of $\mathrm{h}^{-1}(x) = \sqrt[4]{\frac{x-5}{6}}$.

(b) (i) A good number of correct answers were seen. A few candidates made a slip with the inequality sign and $p \leqslant 2$ was not uncommon from these candidates. Weaker candidates offered $p > 5$ or similar or simply stated p was real.

(ii) Candidates who took care with the graph of $y = \mathrm{p}(x)$, making sure that the *y*-intercept and the asymptote were correct before reflecting in the line $y = x$ usually earned all 3 marks. Those who drew the asymptotes on the diagram were more successful than those who did not. This was often the feature that was missing, with many curves tending to the *x*-axis and the *y*-axis. A good number of candidates sketched a graph of the correct exponential shape and usually indicated that they understood that all that was required to sketch the inverse function was to reflect it in the given line. Some candidates unnecessarily found the rule for the inverse function and this was, on occasion, unhelpful, as the graphs they sketched were not symmetrical. A few candidates ignored the given line and reflected their p(x) in the x-axis.

(iii) This part of the question was very well answered, with almost all candidates understanding the connection between the graphs and the equation given.

Answers: **(a)(ii)** $6(6x^4 + 5)^2 + 5$; all real x **(iii)** 0 **(iv)** $\mathrm{h}^{-1}(x) = -\sqrt[4]{\frac{x-5}{6}}$ **(b)(i)** $p > 2$

Question 10

(i) Many fully correct answers were seen to this part of the question. Most candidates used the suggested substitution and found a pair of values for u. This usually resulted in a fully correct solution. A few candidates square-rooted their values for u, instead of squaring them or simply restated them as the values of x. Occasional slips were made in simplifying the initial equation. Many candidates who did this stated no method of solution, solving using their calculator. These candidates were penalised. It is important to show how the solutions to any quadratic equation have been found.

(ii) Candidates used various approaches to answer this part. Most commonly the difference between the area under the curve and the area of the trapezium was attempted. A good number of candidates offered fully correct and complete solutions, showing all key method steps, as required. Some candidates should take care with the accuracy of working values. These candidates made premature approximation errors, rounding their areas to 2 or 3 significant figures, before calculating their final difference of areas. The final answer was often given to 2 significant figures, when at least 3 were required. However, in this case, it was possible to work with exact values and state the exact answer. Those who insist upon rounding should write down a more accurate answer, before attempting to round, to avoid a possible loss of accuracy mark. Only a few candidates stated the integral they were attempting to find without any integration of terms being seen or without a difference of values, such as F(4) – F(0.25), being found. Candidates who worked out the difference of the expressions and then integrated were unlikely to make rounding errors. However, some candidates did not choose the correct values from **Part (i)** for their upper and lower limits. This error was also compounded by candidates who integrated to find the area under the line instead of using $\frac{1}{2}(a+b)\times h$.

Answers: **(i)** $A(0.25, 3.75)$, $B(4, 15)$ **(ii)** 2.8125

Question 11

(a) A good number of candidates understood the need to simplify the expression before integrating and were able to do this successfully. Some of these candidates thought that $x^2 \times x^6 = x^{12}$. It was a requirement that candidates stated the constant of integration in their answer. Many candidates omitted it and were penalised in this part of the question. The very weakest candidates attempted to integrate each term in the numerator and the denominator offering answers such as

$$\frac{\frac{x^3}{3}\left(\frac{x^7}{7}+x\right)}{\frac{x^7}{7}}.$$

(b) (i) This was well answered with almost all candidates stating the answer as a multiple of $\sin(4\theta - 5)$. A few candidates had clearly differentiated but most candidates were fully correct in their answer. A few candidates earned M1 only for having the negative of the correct answer or for multiplying by 4.

(ii) A reasonable number of fully correct solutions were seen. As candidates were directed to use the previous part of the question to answer this part, they needed to show full method to indicate that they had done so. Many candidates earned the method mark for a correct substitution of limits which was shown. Some candidates needed to take more care with the accuracy of their final answer. These candidates should, perhaps, write down a more accurate answer to the question before attempting to round. This may avoid the loss of an accuracy mark. Other candidates needed to take care over their presentation as they omitted brackets. A few candidates were working in degrees, which was not valid here. These candidates may do better if they checked the mode of their calculator at the start of each question. A few candidates gave a choice of answers in degrees or radians. This was not condoned.

Answers: **(a)** $\frac{x^3}{3} - \frac{1}{3x^3} + c$ **(b)(i)** $\frac{\sin(4\theta - 5)}{4}$ $(+c)$ **(ii)** 0.0353