



Cambridge International AS & A Level

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FURTHER MATHEMATICS

9231/23

Paper 2 Further Pure Mathematics 2

October/November 2023

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.

1 Show that the system of equations

$$\begin{aligned}14x - 4y + 6z &= 5, \\ x + y + kz &= 3, \\ -21x + 6y - 9z &= 14,\end{aligned}$$

where k is a constant, does not have a unique solution and interpret this situation geometrically. [4]

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- 3 Find the first three terms in the Maclaurin's series for $\tanh^{-1}\left(\frac{1}{2}e^x\right)$ in the form $\frac{1}{2}\ln a + bx + cx^2$, giving the exact values of the constants a , b and c . [6]

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- 4 Find the particular solution of the differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 3y = 27x^2,$$

given that, when $x = 0$, $y = 2$ and $\frac{dy}{dx} = -8$.

[10]

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5 The curve C has parametric equations

$$x = \frac{2}{3}t^{\frac{3}{2}} - 2t^{\frac{1}{2}}, \quad y = 2t + 5, \quad \text{for } 0 < t \leq 3.$$

(a) Find the exact length of C .

[5]

- 6 (a) Starting from the definitions of cosh and sinh in terms of exponentials, prove that

$$\sinh 2x = 2 \sinh x \cosh x. \quad [3]$$

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- (b) Using the substitution $u = \sinh x$, find $\int \sinh^2 2x \cosh x \, dx$. [4]

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- (c) Find the particular solution of the differential equation

$$\frac{dy}{dx} + y \tanh x = \sinh^2 2x,$$

given that $y = 4$ when $x = 0$. Give your answer in the form $y = f(x)$. [7]

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7 The matrix **A** is given by

$$\mathbf{A} = \begin{pmatrix} -6 & 2 & 13 \\ 0 & -2 & 5 \\ 0 & 0 & 8 \end{pmatrix}.$$

(a) Find a matrix **P** and a diagonal matrix **D** such that $\mathbf{A}^{-1} = \mathbf{PDP}^{-1}$. [7]

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(b) Use the characteristic equation of \mathbf{A} to find \mathbf{A}^{-1} . [4]

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8 (a) State the sum of the series $1 + z + z^2 + \dots + z^{n-1}$, for $z \neq 1$. [1]

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(b) By letting $z = \cos \theta + i \sin \theta$, where $\cos \theta \neq 1$, show that

$$1 + \cos \theta + \cos 2\theta + \dots + \cos (n-1)\theta = \frac{1}{2} \left(1 - \cos n\theta + \frac{\sin n\theta \sin \theta}{1 - \cos \theta} \right). \quad [7]$$

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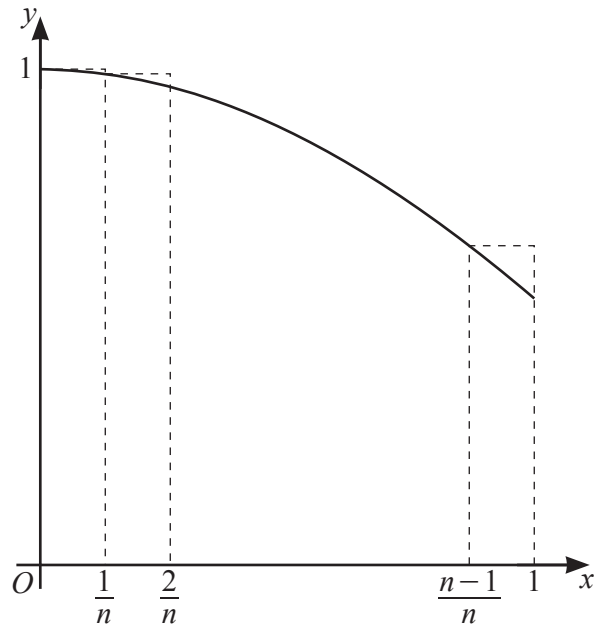
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The diagram shows the curve with equation $y = \cos x$ for $0 \leq x \leq 1$, together with a set of n rectangles of width $\frac{1}{n}$.

(c) By considering the sum of the areas of these rectangles, show that

$$\int_0^1 \cos x dx < \frac{1}{2n} \left(1 - \cos 1 + \frac{\sin 1 \sin \frac{1}{n}}{1 - \cos \frac{1}{n}} \right). \quad [4]$$

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(d) Use a similar method to find, in terms of n , a lower bound for $\int_0^1 \cos x dx$. [3]

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Additional page

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