FURTHER MATHEMATICS

Paper 9231/11
Further Pure Mathematics 11

There were too few candidates for a meaningful report to be produced.

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FURTHER MATHEMATICS

Paper 9231/12
Further Pure Mathematics

Key messages

- Candidates should read every question very carefully so that they answer all aspects in adequate depth.
- Candidates should show all the steps that lead to a solution, particularly when proving a given result.
- Candidates should ensure that all sketched graphs are labelled fully and drawn carefully to show significant points and behaviour at limits.

General comments

Most candidates demonstrated good knowledge across the whole syllabus. They showed their working clearly and were accurate in their handling of algebra and calculus. They also showed a good understanding of transformations. It seemed that almost all were able to complete the paper in the time allowed.

Comments on specific questions

Question 1

(a) The majority of candidates understand the structure of an induction proof.

To demonstrate the base case, a link to the statement in the case n =1 needed to be shown. The hypothesis was often written correctly, and many remembered to state that this is an assumption. Those who worked with $2A^{k+1} = 2A^k.A$ or $2A^{k+1} = A.2A^k$ managed the matrix multiplication correctly, mostly showing an intermediate step to justify the result. Some candidates tried to divide by 2 or even to add the matrices.

The conclusion was usually correct.

(b) Most candidates seem to know how to find the inverse of a 2×2 matrix. The most successful solutions began by writing down the matrix $\mathbf{A}^n$. This led to the fact that det$\mathbf{A}^n = 3^n$. Other methods often ended with an extra factor of 2.

Question 2

(a) Almost all candidates were able to answer this correctly with confident use of the relevant formulae. Those who used a substitution were also able to reach the correct answer with relative ease.

(b) The majority of candidates were able to score highly on this question. Their understanding of its requirements was usually very clear, and they proceeded confidently with the algebraic techniques needed to gain full marks. Any errors were usually in the last step when applying MF19 formulae to reach the required form of the answer.

A few candidates tried to insert the known results for the sum of powers of *n* before they had eliminated α, β, γ which made the algebra much more complicated.

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Question 3

(a) A variety of methods were used successfully to find the correct partial fractions. The presence of both k and r meant that some candidates found it awkward to make a correct substitution. When correct partial fractions were achieved, the majority could apply the method of differences to reach the correct result.

If the factor $\frac{1}{k}$ is removed for ease of writing, candidates are reminded to check that it has been replaced for the final answer.

(b) Nearly all candidates were able to find the correct limit for their answer to part **(a)**.

(c) Those candidates who used the method of differences again were usually correct. It was common for those who subtracted sums to subtract the sum to n terms instead of $n - 1$ terms.

Question 4

(a) Most attempts got no further than writing down the matrix of a general rotation about the origin. Candidates often stated that $-\sin\theta$ cannot be b^2, forgetting that θ can be a reflex angle. Particularly convincing solutions included:

(1) Stating $b^2 + c^2 = 0$ and because $b \neq 0$ then $c^2 < 0$ which is impossible.

(2) Stating $\sin\theta = c^2$ when $0 \leq \theta \leq \pi$ and $-\sin\theta = b^2$ when $\pi < \theta < 2\pi$ ($b \neq 0$). So, there are no possible values of θ which satisfy both.

(b) There were many completely correct solutions and very few errors in algebra. The question asked for lines through the origin and only a handful tried to use $y = mx + \text{constant}$.

(c) Those who knew the form of a matrix for a shear could write down the two transformations. The most common error was to get the order of the matrices wrong.

(d) Nearly all candidates who had an answer for **(c)** gained full marks with just a few not knowing the relationship between determinant and area.

Question 5

(a) Most candidates seem comfortable sketching a polar graph but many need to add more detail to obtain full marks. The pole needs to be clearly marked, and in this case the value of r is strictly decreasing. The question asked for the coordinates of the point furthest from the pole and this was often omitted, or the coordinates given in the wrong order.

(b) This question was answered correctly by the majority of candidates, who easily identified the appropriate standard integral. A few missed the $\frac{1}{2}$ at the start of the formula. Weaker candidates usually tried to involve logarithms in some way but were unsuccessful.

(c) A pleasing number of candidates recognised that they needed to consider $y = \dfrac{\sin\theta}{\sqrt{1+\theta^2}}$. The differentiation was carried out correctly and the resulting equation manipulated into the form required using convincing algebra. A few candidates found the fractional indices challenging.

The last part of the question could be answered without having proved the result, and most could obtain the correct numerical values for the function at 1.1 and 1.2 to verify the sign change and existence of a root. Candidates are advised to check that they have answered all parts of the question.

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Question 6

(a) Almost all candidates gained full marks on this.

(b) The requirement for a negative discriminant was well known and applied, and nearly all candidates were aware of its effect on stationary points. Very few spotted that their derivative was positive and so there were no turning points.

(c) Having found the equations of the asymptotes and knowing that there were no stationary points, most candidates made a reasonable attempt at the sketch. The best graphs were well labelled, and the curves were smooth and approached the asymptotes steadily.

(d) Almost all understood that the graph needed to be reflected in the x axis. There were some very neat graphs, with the intersections with the x axis shown as sharp points and the branches becoming linear as $|x|$ increased.

(e) Many candidates realised that there are two cases to consider and could find all four critical points, although a common error was to omit the solution $x = 0$. Those who solved two quadratic equations could then look at their answers and the known shape of the graph to write down the correct set of values. In contrast those who worked throughout with inequalities often wrote down overlapping regions.

Question 7

(a) This is a standard type of question, and it was generally well-answered. There were a few mistakes in finding the cross-product. Candidates are advised to check carefully as a numerical error at this early stage can affect the remainder of the question.

(b) Finding a point common to both planes proved to be quite challenging and many candidates just settled for the marks for finding the line's direction vector. The question asked for the vector **equation** of the line, and it was surprisingly rare to see an answer beginning $\boldsymbol{r}$ = as required.

(c) Quite a few candidates found the foot of the perpendicular from A to the plane and used that to write down the distance. This was often successful. Those who did try to use a known formula often omitted the constant term from the equation of the plane.

(d) The appropriate formula was more widely known in this part of the question and many candidates were able to apply it correctly and respond confidently to the resulting algebraic challenges. It was common to see the modulus of the length equated to a negative quantity, but this disappeared when the equation was squared. Most remembered to reject the negative solution.

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Further Pure Mathematics

Key messages

- Candidates should read every question very carefully so that they answer all aspects in adequate depth.
- Candidates should show all the steps that lead to a solution, particularly when proving a given result.
- Candidates should ensure that all sketched graphs are labelled fully and drawn carefully to show significant points and behaviour at limits.
- Candidates need to be familiar with the formula sheet (MF 19) so that they can use the results it contains.

General comments

Most candidates demonstrated good knowledge across the whole syllabus. They showed their working clearly and were accurate in their handling of algebra and calculus. It seemed that almost all were able to complete the paper in the time allowed.

Comments on specific questions

Question 1

The majority of candidates understand the structure of an induction proof and the base case was usually proved correctly.

The wording of the hypothesis was often incomplete. Candidates need to assume that $5^{3k} + 32^k - 33$ is divisible by 31 for some positive integer *k*, rather than just assume that $5^{3k} + 32^k - 33$ is true.

The candidates who attempted to show that the (k + 1)th term is itself divisible by 31 often managed this correctly. A particularly neat solution was produced by those who stated $5^{3k} + 32^k - 33 = 31m$ where *m* is an integer and went on to show that $5^{3(k+1)} + 32^{k+1} - 33 = 31t$.

Candidates who found the difference between terms are reminded that they need to explain how this being divisible by 31, and the hypothesis for the *k*th term, together lead to the divisibility of the (k + 1)th term.

The summing up was usually correct.

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Question 2

(a) This was done well, with very few algebraic slips.

(b) Many candidates noticed that the expression in r could be written as $6-\dfrac{5}{r(r+1)}$.

From there they found correct partial fractions and the sum of n terms. There were many well-presented solutions.

A common mistake was to write $\dfrac{6r^2+6r-5}{r^2+r}=\dfrac{A}{r}+\dfrac{B}{r+1}$. The resulting partial fractions were summed and sometimes that total was multiplied by the answer for part **(a)**. Candidates might find it helpful to write down the first few terms of the sequence to check that they are summing the correct one.

(c) The majority gained a method mark for calculating $\sum_{r=1}^{2n} - \sum_{r=1}^{n}$, using their result from part **(b)**.

Some used the method of differences again, but a common error was to mistake the number of terms from n + 1 to 2n.

Question 3

(a) Most candidates were able to use the substitution $y = x^2 \Rightarrow x = y^{\frac{1}{2}}$ to arrive at the correct quartic, although there were some algebraic slips.

Some candidates did not read the question carefully and only found $\alpha^2+\beta^2+\gamma^2+\delta^2=2$, not the quartic, while others found the quartic and not the sum of squares of the roots.

(b) Most candidates recognised the relationship $\dfrac{1}{\alpha^2}+\dfrac{1}{\beta^2}+\dfrac{1}{\gamma^2}+\dfrac{1}{\delta^2}=\dfrac{\alpha^2\beta^2\delta^2+\alpha^2\beta^2\gamma^2+\beta^2\gamma^2\delta^2+\alpha^2\gamma^2\delta^2}{\alpha^2\beta^2\gamma^2\delta^2}$ and used coefficients from their answer for (**a**) to find the required sum.

A few wrote down the equation with roots $\dfrac{1}{\alpha^2}$ etc. and found the answer from that.

(c) The use of
$\alpha^4+\beta^4+\gamma^4+\delta^4=\left(\alpha^2+\beta^2+\gamma^2+\delta^2\right)^2-2\left(\alpha^2\beta^2+\alpha^2\gamma^2+\alpha^2\delta^2+\beta^2\gamma^2+\beta^2\delta^2+\gamma^2\delta^2\right)=2^2-2(11)$ and
$S_4-S_2+2S_1+5S_0=0$, were both popular and successful methods.

Question 4

Candidates are reminded that they need to read each question carefully. Many did not answer the question asked in parts **(b)**, **(c)** and **(d)**.

(a) The sequence of the transformations was understood by most of the candidates and almost all recognised the rotation. The full description needed to include not only the angle but also the origin as the centre of the rotation.

(b) The question asked for two matrices whose product was the matrix $\mathbf{M}^{-1}$. Candidates needed to write down the inverse of each matrix and then use the result $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$ to write them in the correct order. There were many good solutions.

A common error was for candidates to combine the two given matrices and then find the inverse of their answer.

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(c) Most candidates could write down the determinant **M** – **I** and set it equal to zero. A pleasing number of candidates worked through the algebra to obtain at least the first 3 marks in the question. From there, the majority of those who substituted the double angle identities obtained the required result. Errors in algebra and in use of the identities sometimes led to an equation which could not be solved.

There were also cases in which the question was wrongly interpreted as **M** singular, $\mathbf{M}^{-1}$ singular, $\mathbf{M}^{-1} - \mathbf{I}$ singular, and det **M** = 1.

(d) Most candidates found the invariant lines of the transformation rather than the invariant points that the question required.

Those who had the correct idea of invariant points wrote $\mathbf{M}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$ and successfully derived two equations in *x* and y. From there they should move on to deriving the equation using both these equations - usually only one equation was used to derive the result. Instead, both equations should be shown to give the same equation.

Question 5

(a) The result was proved well, with most candidates starting with the cartesian equation and changing it to polar form.

(b) There were a few very good sketches shown. The pole and initial line were clear, and *r* was strictly increasing over the correct domain. The curvature of the graph and the behaviour as $\theta \to \frac{\pi}{4}$ were more challenging, but many candidates produced acceptable drawings. It was common to omit giving the minimum distance, or to give the answer *a* rather than $\sqrt{a}$.

(c) The majority of candidates used the correct formula and limits for the area. The question suggested the use of the formula list (MF 19) and those candidates who took the hint could write down

$\frac{1}{2}a\int_0^{\frac{1}{12}\pi} \sec 2\theta \mathrm{d}\theta = \frac{1}{4}a\left[\ln\tan\left(\theta + \frac{1}{4}\pi\right)\right]_0^{\frac{1}{12}\pi}$ or $\frac{1}{4}a\left[\ln(\tan 2\theta + \sec 2\theta)\right]_0^{\frac{1}{12}\pi}$.

The most common error in this case was to forget the factor of $\frac{1}{2}$.

Many candidates tried other methods to integrate the function and often spent much time on unsuccessful attempts. Stronger candidates did find alternative correct ways of performing the integration.

Question 6

(a) This is a familiar question and most candidates scored full marks. The errors were slips in arithmetic.

(b) Usually correct. Most candidates used the formula, and some decided to find the foot of the perpendicular and its distance from *O*.

(c) There were many good solutions and a real variety of interesting approaches. Structure is needed in this type of question and a pleasing proportion of candidates defined the points they used and made their method clear.

The majority defined the points *P* and *Q* on the lines *OC* and *AB* respectively and formed two simultaneous equations in parameters λ and μ by using the zero scalar-product of *PQ* with the directions of the lines. Those who used this approach almost always successfully obtained the correct values of λ and μ and scored high marks.

Another popular approach was to use the cross-product to find the vector perpendicular to both lines, *OC* and *AB,* and then set $\overrightarrow{PQ}$ equal to a scalar multiple of this vector. This gave simultaneous equations in three parameters which were solved to complete the solution.

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One unusual approach was to find the distance between skew lines and scale the common perpendicular. Candidates using this method are reminded that they must consider both the positive and negative square root as only one led to the correct solution.

Weaker candidates often got no further than finding the direction of the common perpendicular.

The question asked for the vector **equation** of the common perpendicular and it was surprisingly rare to see an answer beginning ***r*** = as required.

Question 7

(a) This familiar question was well-answered.

(b) The most popular method was to differentiate the original equation and set $\frac{dy}{dx} = 0$.

Some candidates used part **(a)** and wrote $\frac{dy}{dx} = 1 - \frac{16}{(x-3)^2} = 0 \Rightarrow (x-3)^2 = 16$.

Whichever method was used, the majority of candidates found the correct turning points.

(c) There were many very good graphs here. Candidates had usually labelled the asymptotes, and both branches of the graph approached them correctly.

(d) This part of the question asked for both graphs to be drawn on the same set of axes and clearly identified. Sometimes the two graphs were drawn on different sets of axes and sometimes they were added to the graph in part **(c)**. It was only possible to give credit for one of the sketched curves if it could be identified. Every branch of each curve needed to be clearly labelled.

Many candidates drew only the graph of $y = \left|\frac{x^2+2x+1}{x-3}\right|$. This was usually correct.

The graph of $y^2 = \frac{x^2+2x+1}{x-3}$ was more challenging. Those candidates who realised that *y* could only exist when $\frac{x^2+2x+1}{x-3} \geq 0$ drew a curve in the first quadrant with *y* values smaller than those of the original curve and increasing less quickly.

It was rare to see the other branch of this curve as the reflection in the *x* axis. Instead, there were incorrect curves corresponding to negative values of *x*.

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FURTHER MATHEMATICS

Paper 9231/21
Further Pure Mathematics 21

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FURTHER MATHEMATICS

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Further Pure Mathematics 22

Key messages

- Candidates should show all the steps in their solutions, particularly when proving a given result.
- Candidates should read questions carefully so that they answer all aspects in adequate depth, particularly when an answer is required in a certain form or in terms of a given variable.
- Candidates should make use of results derived or given in earlier parts of a question or given in the List of formulae (MF19).

General comments

Most candidates demonstrated very good knowledge across the whole syllabus. They showed their working clearly and were accurate in their handling of algebra and calculus. They also showed understanding of linear algebra. It seemed that all were able to complete the paper in the time allowed. Sometimes candidates did not fully justify their answers and jumped to conclusions without justification, particularly where answers were given within the question. Gaps in knowledge were evident in some scripts.

Comments on specific questions

Question 1

(a) The majority of candidates formed a matrix using the coefficients of the equations and showed its determinant is 0. Those who instead performed row operations were usually also successful.

(b) Most candidates used all three equations in their manipulation to reduce to one equation with two unknowns, such as $y = -2x - 1$, justifying that the system is consistent. Alternatively, a few candidates correctly combined their working with **part (a)** and found just one solution or parameterised the general solution. Good candidates maintained accuracy throughout and correctly interpreted the situation geometrically as a sheaf.

Question 2

Most candidates were able to use the given substitution to change the differential equation to a separable one in terms of *x* and *z*. However, some did not separate the variables afterwards or did not change their expression to the required format, highlighting the need to read the question carefully.

Question 3

(a) After expanding $\left(z + z^{-1}\right)^4$ using the binomial expansion, good candidates grouped together terms clearly before applying the identity $z^n + z^{-n} = 2\cos n\theta$ to fully justify their answer. For the left-hand side, the majority of candidates clearly applied $z + z^{-1} = 2\cos\theta$.

(b) Most candidates successfully applied the given substitution to change the integral to $\int \cos^4 \theta d\theta$. Good candidates maintained accuracy when changing limits and integrating using **part (a)**.

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Question 4

(a) The majority of candidates accurately differentiated the product given in the question. Those who then replaced x^5 with $1+x^5-1$ successfully derived the given reduction formula. Others who integrated $x^5\left(1+x^5\right)^{n-1}$ by parts were unsuccessful.

(b) Most candidates correctly applied the reduction formula from **part (a)** and used I_1 as their initial value. Some used I_0 or I_2 as their initial value and were usually successful also.

Question 5

(a) This was a given answer, so evidence was needed that the characteristic equation had come from the calculation of a determinant. Most candidates maintained accuracy when expanding the determinant and showed enough working. A few did not find the roots of the given characteristic equation and so did not state the eigenvalues of $\mathbf{A}$.

(b) Candidates who used the vector product method to find the eigenvectors tended to be most successful, although sign errors were common. Candidates should be encouraged to check that their proposed eigenvector does have the required property by performing matrix multiplication. Almost all candidates showed an awareness of how to find the matrices $\mathbf{P}$ and $\mathbf{D}$.

Question 6

Almost all knew how to approach this question and completed it to a high standard. There was some inaccuracy when substituting and comparing both sides of the equation and some problems with notation. A few candidates gave expressions instead of equations as their answer.

Question 7

(a) Almost all applied the given substitution correctly. A few were inaccurate when substituting or left their answer in terms of u.

(b) Good candidates showed consideration of the sum of the areas of the rectangles to justify the left-hand side of the inequality. When dealing with the right-hand side, the most common approach was to integrate by parts and to use the result from **part (a)** to justify the given answer. A small number of candidates worked directly with the natural logarithm and were also usually successful.

(c) Good candidates adapted their solution to **part (b)**, used correct limits and derived a suitable lower bound.

Question 8

(a) Almost all, after writing sech and tanh in terms of exponentials, showed enough working to fully justify the given identity. For good mathematical communication, candidates should be reminded that, when proving an identity, they should work on just one side at a time.

(b) The majority of candidates found the first derivative correctly using parametric differentiation, although sign errors were quite common when deriving the given answer.

(c) The attempts to find the second derivative varied in length, with strong candidates showing the required level of algebraic fluency and remembering to divide by $\frac{\mathrm{d}x}{\mathrm{d}t}$ after differentiating with respect to t. The strongest candidates maintained accuracy when setting the second derivative equal to $-\frac{9}{2}$, then finding $t=\ln 3$ and both x and y in the required form.

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FURTHER MATHEMATICS

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Further Pure Mathematics 23

Key messages

- Candidates should show all the steps in their solutions, particularly when proving a given result.
- Candidates should read questions carefully so that they answer all aspects in adequate depth and note which algebraic form answers are required to take.
- Candidates should make use of results derived in earlier parts of a question or given in the List of formulae (MF19).

General comments

The majority of candidates demonstrated very good knowledge across the whole syllabus. They showed their working clearly and were accurate in their handling of algebra and calculus. They also showed understanding of linear algebra. It seemed that all were able to complete the paper in the time allowed. Sometimes candidates did not fully justify their answers and jumped to conclusions without justification, particularly where answers were given within the question. There were many scripts of a very high standard.

Comments on specific questions

Question 1

(a) Many candidates gained full marks in this question. Where errors were made, it was often a sign error with the second derivative or losing a factor of x when applying the product rule to find the third derivative. The few candidates who found the derivatives implicitly were less successful, often losing a term such as $\sec x$ in the process.

(b) Most candidates substituted $\frac{1}{5}$ into their expression from **part (a)** and gave their answer as a fraction, as required. A few incorrectly left their answer as $\sin^{-1}\left(\frac{1}{5}\right)$ or the sum of two fractions.

Question 2

(a) Almost all knew how to approach this question and completed it to a high standard. There were some inaccuracies when comparing coefficients to find the particular integral and some problems with notation. A few candidates gave expressions instead of equations as their answer.

(b) Most candidates correctly used their particular integral from **part (a)**. There were also some problems with notation for this part, with a few candidates using an arrow instead of an equals sign.

Question 3

After expanding $\left(z+z^{-1}\right)^4$ and $\left(z-z^{-1}\right)^4$ using the binomial expansion, good candidates grouped together terms clearly before applying the identity $z^n + z^{-n} = 2\cos n\theta$ to fully justify their answer. Similarly, for the left-hand side, evidence was needed that candidates had used both $z - z^{-1} = 2\mathrm{i}\sin\theta$ and $z + z^{-1} = 2\cos\theta$; it was sometimes not apparent that they knew these rules.

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Question 4

(a) The majority of candidates accurately differentiated both sides of the equation implicitly and showed enough working to justify the given answer. In particular, the substitution of the values needed to be demonstrated. A few candidates made a lot of work for themselves by expanding $(x+y)^6$ before differentiating and this method should be discouraged.

(b) Good candidates accurately used implicit differentiation again to find an equation involving the second derivative. Most successful attempts kept the expressions factorised. A common error when substituting values was to lose the minus sign on the final answer.

Question 5

(a) Almost all, after writing cosh in terms of exponentials, showed enough working to fully justify the given identity. For good mathematical communication when proving an identity, candidates should be reminded that they should work on just one side at a time.

(b) The majority of candidates found and simplified the integrating factor correctly by using the required formula provided in the List of formulae (MF19). After multiplying both sides of the equation by $\cosh^2 x$, most candidates were able to apply the required hyperbolic identity needed to integrate the RHS. A few candidates who divided by $cosh^2x$ before evaluating the constant lost accuracy when substituting in the initial conditions.

Question 6

(a) Most candidates formed a correct expression for the sum of the areas of the rectangles and good candidates applied the standard result for the sum of squares to accurately derive the given result.

(b) Good candidates correctly adapted their solution to **(a**) and derived a suitable lower bound.

(c) Most candidates showed enough working to justify that the difference between U_n and L_n is proportional to $\frac{1}{n}$, hence justifying the given limit.

Question 7

(a) Almost all recognised the integral as $\sinh^{-1} x$ and, after substituting in the limits, fully simplified the logarithmic form to $\ln 3$. A few times $\ln 2$ was obtained as the final answer due to carelessness in the use of logarithm rules to simplify $\ln 3 - \ln 1$.

(b) The majority of candidates accurately differentiated the product given in the question. Those who then replaced x^2 with $1+x^2-1$ successfully derived the given reduction formula. Others who integrated $x^2\left(1+x^2\right)^{\frac{1}{2}n-1}$ by parts were unsuccessful.

(c) Good candidates started from the formula for the arc length of the curve in terms of x and clearly applied the given substitution, changing the limits, to show that $s=\frac{1}{2}I_1$. Most candidates accurately applied the given reduction formula to find s.

Question 8

(a) The most efficient method here was to calculate the determinant of the given matrix and comment that it was non-zero. Candidates who attempted to solve the equations instead often made errors, particularly in finding the value of x. For the geometrical interpretation, an appreciation that the planes intersected at just one point was needed, so the statement 'three planes intersect at a point' was insufficient.

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(b) This was a given answer so complete justification was needed. Candidates writing down just $(a-\lambda)(1-a-\lambda)(-1-\lambda)=0$ followed by the eigenvalues gained no credit because they had not provided evidence that it had come from the calculation of a determinant.

(c) Candidates who used the vector product method to find the eigenvectors tended to be most successful, although sign errors were common. Candidates should be encouraged to check that their proposed eigenvector does have the required property by performing matrix multiplication. Almost all candidates showed an awareness of how to find the matrices $\mathbf{P}$ and $\mathbf{D}$.

(d) Algebraic errors in expanding the brackets were quite common here, particularly sign errors. Most candidates knew that a square matrix satisfies its own characteristic equation and showed an understanding of the required method.

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FURTHER MATHEMATICS

Paper 9231/31
Further Mechanics 31

There were too few candidates for a meaningful report to be produced.

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FURTHER MATHEMATICS

Paper 9231/32
Further Mechanics 32

Key messages

A diagram is often an invaluable tool in helping a candidate to make good progress. This is particularly the case when forces or velocities are involved. If a diagram is given on the question paper, then it may be sufficient to annotate that diagram, although a candidate is always free to draw their own diagram as well. When a result is given in a question, candidates must take care to give sufficient detail in their working so that the offered solution is clear and complete. In all questions, candidates are advised to show all their working, as credit is given for method as well as accuracy.

General comments

Candidates are encouraged to draw a suitable diagram or, in case a diagram is provided, to annotated it, as this helps understand the problem and model it correctly. For example, in **Question 1 part (a)**, the candidates who drew a diagram realised that when the particle is at a distance of $\frac{3}{4}a$ below O, it has no gravitational potential energy, as the elastic string has become slack. Also, in **Question 7 part (b)**, the candidates who correctly drew a trajectory of the particle realised at time $t = 3$ the particle's height is $\frac{3}{4}H$ and set up a correct equation accordingly.

Candidates should be encouraged to check that the equations they write are dimensionally consistent. This is particularly important when writing moments and conservation of energy equations. When applying Newton's second law, for example to set up a differential equation or in questions involving collisions, they must ensure they explicitly mention the mass or masses involved.

Candidates should be reminded that, when the answer is given, they are expected to show their working in full, even if it involves the use of elementary algebra, as in **Question 4 part (a)**.

Comments on specific questions

Question 1

(a) Most candidates understood that, to obtain the correct answer, they had to apply the conservation of energy principle. Many good responses identified that when the particle is at a distance of $\frac{3}{4}a$ below O, it has no gravitational potential energy as the elastic string has become slack. The best responses included annotated diagrams of the system at the two points of interest.

(b) To answer this question, the expected method was to use Hooke's law to determine the tension on the string when the particle is released from rest and substitute this into Newton's second law. The best responses did this in an efficient, and often elegant, manner and included weight in Newton's second law calculations.

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Question 2

This question was answered using a variety of approaches. The most common approach was to start with the equations for the two components of the velocity before and after the collision to establish a link between α, θ and the coefficient of restitution *e*. The next step in many approaches was to then substitute into an energy equation representing the fact that after the collision the particle loses 20 per cent of its kinetic energy. The final answer was obtained using differing routes.

Some responses did not state that if the particle loses 20 per cent of its energy, it is left with 80 per cent of it, and instead worked with an equation corresponding to the situation where the particle has 20 per cent of its kinetic energy after the collision (that is, it loses 80 per cent of its energy).

The best responses used very elegant approaches, such as one whose key steps are outlined here:

$$sin(\theta) = \frac{\sqrt{5}}{5} \ , \ cos(\theta) = \frac{2\sqrt{5}}{5}$$

$$v\cos(\theta) = u\cos(\alpha)$$

$$v \ \sin(\theta) = eu\sin(\alpha)$$

$$e = \frac{1}{2\tan(\alpha)}$$

$$\frac{1}{2}mv^2 = \frac{4}{5}\times\frac{1}{2}mu^2$$

$$v = \frac{2\sqrt{5}}{5}u$$

$$u\cos(\alpha) = v\times\cos(\theta) = \frac{2\sqrt{5}}{5}u\times\frac{2\sqrt{5}}{5} = \frac{4}{5}u$$

$$\tan(\alpha) = \frac{3}{4}$$

$$e = \frac{2}{3}$$

Question 3

(a) This part question was answered correctly by many candidates. A common error (avoided by the candidates who drew a good quality diagram) was to use $\cos(\theta)$ instead of $\sin(\theta)$ for the radial component of the weight in Newton's second law. Some responses considered the equilibrium of forces at point *A* instead of at point *B*.

(b) To answer this part question the candidates had to rearrange the conservation of energy equation and substitute it into Newton's second law. The candidates who answered **part (a)** correctly usually went on to score full marks in this part question too. The most common errors were in the determination of the components of the gravitational potential energy, often made by candidates who omitted to draw a diagram.

Question 4

(a) In this part question one of the answers was given, therefore the candidates were expected to show their working with particular care to clearly communicate their understanding. The best answers involved the use of tables with volumes (as mass is proportional to volume here) and distances of the centre of mass for the hemisphere, the cylinder, and the object all explicitly stated. Many good responses were seen to this question, with most realising that the centre of mass of the hemisphere was below the *x*-axis. Some responses included equations that were not dimensionally correct. Candidates should be encouraged to check that the equations they write are dimensionally consistent.

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(b) This part question proved challenging, with few responses seen that were fully correct. Again, the most successful responses made use of a suitable diagram. By drawing such a diagram, it could be seen that application of the tangent ratio was not appropriate here. The better responses made use of the hint that $\sin(\theta) = \frac{1}{6}$ and hence applied the sine ratio instead, often with good results.

Question 5

(a) To obtain the correct answer for this part question, responses were expected to contain three key steps.

- Determine the tensions on the strings in both scenarios.
- Use Newton's second law in both scenarios.
- Solve the resulting system of simultaneous equations to obtain an expression for x in terms of a.

Most responses included all three key steps with varying degrees of success. A common error was to consider that the radius of the horizontal circle was a instead of x or, respectively, $\frac{3}{4}x$. Another error was to assume the mass of the particle was m in both scenarios. Some responses worked with the particle moving in a vertical circle instead of moving in a horizontal plane.

(b) The candidates who answered **part (a)** correctly generally had more success in answering this part. Better responses adopted a very efficient and elegant approach involving the substitution of $x = 4a$ and $v = \sqrt{12ag}$ in the equation of Newton's second law in one scenario. The best answers performed the substitution in both scenarios and checked that the value for $\boldsymbol{\lambda}$ was the same.

Question 6

(a) This part question was answered well by many candidates, who showed a good understanding of how to set up and successfully solve a differential equation. Some responses showed incorrect separation of the variables, or included the derivative of the function rather than its integral. In the weakest responses, the acceleration was expressed as $\frac{dv}{dt}$ instead of $v\frac{dv}{dx}$.

(b) This part question was also answered well, even though it proved more challenging than **part (a)**. Most candidates realised that they had to substitute $v = \frac{dx}{dt}$ in their answer to **part (a)** and solve the corresponding differential equation. A good proportion of them obtained an answer of the correct form, showing good understanding of the laws of logarithms and good algebraic manipulative skills. The most common error was in the coefficient that multiplied the natural logarithm function representing the solution of the differential equation.

Question 7

(a) Many candidates made a good attempt at this part question using a variety of approaches, mostly correct. A common error was to provide the answers in terms of u and θ, rather than just θ.

(b) Candidates answered this part question using a variety of approaches. The most common one used the formula $s = ut + \frac{1}{2}at$ with $s = \frac{1}{4}H$, $u = 0$, $a = g$, $t = (3 - T)$ and obtained the equation $\frac{1}{4}H = 5(3 - T)^2$. Other popular approaches considered $\frac{1}{4}H = y(T) - y(3)$ or $\frac{3}{4}H = y(3)$. The candidates then substituted their expression from **part (a)** for H and, when applicable, for T, and obtained a quadratic trigonometric equation in $\sin(\theta)$ which was solved flawlessly in most cases. Stronger responses included good quality diagrams of the trajectory, annotated in terms of H and

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T. Weaker responses did not include a step identifying that $\frac{1}{4}H$ was the difference between the height of the particle at the two time points, writing instead equations such as $\frac{1}{4}H = y(3)$.

(c) The best responses calculated both components of the velocity and used them to determine the speed, expressing these either in an exact surd form, or as an approximated decimal. Weaker responses included only the vertical component of the velocity.

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FURTHER MATHEMATICS

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Key messages

In all questions, candidates are advised to show all their working, as credit is given for method as well as accuracy. This is particularly the case when a result is given in a question.

A diagram is often an invaluable tool in helping a candidate to make good progress. This is particularly the case when forces or velocities are involved. If a diagram is given on the question paper, then it may be sufficient to annotate that diagram, although a candidate is always free to draw their own diagram as well.

General comments

In most questions the majority of candidates understood what method to use, however some omitted to draw a suitable diagram, or to annotate the given diagram, and this resulted in writing incorrect equations. This was particularly the case in **Question 1** and **Question 4**.

Candidates are reminded that, when the answer is given, they are expected to show their working in full, even if it involves the use of simple algebra.

Comments on specific questions

Question 1

Many good responses were seen to this question on motion in a vertical circle. As is usual in questions on this topic, candidates need to write down an energy equation and a Newton's second law equation. Those candidates who drew a diagram labelled with all the given information almost always followed with accurate equations. A common error was to assume that the particle was at rest when the string went slack. Other candidates confused initial and final velocities, usually when no diagram was drawn.

Question 2

This problem involved the motion of a particle attached to the end of an elastic string with the other end of the string attached to a fixed point. The system is on a horizontal surface so only elastic potential energy and kinetic energy need to be considered in writing down an energy equation. There will be four terms in total: two terms of each type. Many candidates worked accurately and solved their equation to find the value of λ as 4.5. One common error was to use λ as the modulus of elasticity instead of λmg given in the question. This leads to an incorrect answer for λ of $4.5mg$. A second common error was to use the extended length of the string rather than the extension in the expression for the elastic potential energy. So $\frac{5}{3}a$ and $\frac{4}{3}a$ were seen instead of $\frac{2}{3}a$ and $\frac{1}{3}a$.

Question 3

(a) Almost all candidates knew how to answer this question, and most did so accurately. The common method was to write down the area and distance of the centre of mass from *AC* for each of the triangles *ABC* and *DEC*. A moments equation was then formed and simplified to give the distance of the centre of mass of the shape *ADEB* from *AC*. The errors that occurred were usually in the

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distances of the centre of mass of each of the triangles from *AC* or in the algebraic manipulation of the moments equation.

Some candidates found the distance of the centre of mass of the shape *ADEB* from points other than *AC.* An appropriate subtraction could have then been used, but this was not seen.

(b) Most candidates realised that the object would be on the point of toppling about *E* when the distance found in **part (a)** was equal to *x*. This was followed by some rearrangement to a quadratic equation and a single valid solution $x = \frac{12}{5}a$. Some candidates used an inequality and gave a range of values $x \leqslant \frac{12}{5}a$, however there is only one expression for *x* when the object is on the point of toppling.

Question 4

(a) Most candidates made a good attempt at this question and a pleasing number scored full marks. As always on this topic, equations resulting from the conservation of linear momentum and Newton's law of restitution, both along the line of centres, were required. The next step was to interpret the fact that the spheres are moving in the same direction after the collision. This was the step that some candidates found challenging, and which led to their solution being abandoned. Those who did negotiate this step usually went on to combine their three equations to find the value of the coefficient of restitution.

(b) Many candidates obtained the correct total loss in kinetic energy. The common error was to omit one of the components of the speed in one or more of the kinetic energy terms. A few candidates noted that because the components of velocity perpendicular to the line of centres were unchanged by the collision, then it was sufficient to consider only the components of velocity along the line of centres. This was acceptable as long as this justification was stated.

Question 5

(a) Many different approaches were seen in the solutions to this question. Most candidates resolved in two directions and substituted for *r* using trigonometry. This gained the first three marks. At this point, solutions diverged. The most efficient approach was to eliminate the tension *T* from the two resolution equations to obtain $\tan\theta = \frac{3}{4}$. From this it can be deduced that $\cos\theta = \frac{4}{5}$ and substituting this back into the vertical resolution equation gives *T* immediately. Some responses missed this deduction and instead included lengthy work involving Pythagoras' theorem and trigonometry without success.

Another approach that was seen quite often was to write *r* in terms of the extension *x*. Combining this with the resolution equations leads to a homogeneous quadratic equation in *x* and *a*. This can be solved to give $x = 3a$. Many of those who followed this approach made errors in the algebra and the substitution for the trigonometric functions and did not reach a solution.

Many candidates wrote down Hooke's law although it was not needed in this part. There is a way ahead using Hooke's law, but the algebra involved proved very challenging for those who attempted this method.

(b) The best responses used the expression, in terms of *a*, for the extension of the string as found in **part (a)**. Application of Hooke's law together with the vertical resolution equation leads to the value of *k*.

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Question 6

(a) Many candidates were able to set up the correct differential equation, to solve it, include the application of the boundary condition and to find the required expression for v. Sometimes there were sign errors or algebraic errors, but most candidates obtained a logarithmic term on integration.

A few candidates attempted to solve the problem by using *suvat* equations, but these equations are not appropriate as the acceleration was not constant.

(b) There were two common methods for finding x in terms of t. In the first method, which was the most common, the candidate integrated their expression for v obtained in **part (a)**. This was usually done accurately although the constant of integration was sometimes omitted. To complete the solution, it was necessary to find the value of t when $v = 12$, from the expression obtained in **part (a)** and then use this to find the value of x.

Candidates who had been unsuccessful in answering **part (a)** correctly were still able to gain method marks in this part as long as their expression for v contained a term of the form be^{ct}.

In the second method, the candidate started again from the original differential equation, but with the acceleration expressed as $v\dfrac{dv}{dx}$. Some candidates ended their response after separating the variables because they were not able to integrate the function $\dfrac{v}{1-kv}$.

Question 7

(a) Most candidates attempted to write down three equations connecting distance and time. The first equation is the equation for the motion of the model aircraft: $d = 5T$. The second and third equations represent the vertical and the horizontal motion of the ball. A common error was to write that the ball had travelled a horizontal distance $d-8$ instead of $d+8$. The latter is the case because the ball sets off before the aircraft, so must have travelled further. Another common error was to include the $5T$ in the vertical motion equation, thereby mixing up horizontal and vertical distances.

From the three correct equations, the value of T is found by algebraic elimination resulting in a quadratic equation and a single positive solution.

(b) The best responses found the direction of motion of the ball using *suvat* equations to find the vertical and horizontal components of its velocity. Division of the vertical component by the horizontal component gives the tangent of the angle which the direction of motion makes with the horizontal. In this case the tangent of the angle is negative so the ball's direction of travel is below the horizontal.

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FURTHER MATHEMATICS

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Further Probability and Statistics 41

There were too few candidates for a meaningful report to be produced.

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FURTHER MATHEMATICS

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Further Probability and Statistics 42

Key messages

In all questions candidates are advised to show all their working, as credit is given for method as well as accuracy. When a result is given in a question, candidates must take care to give sufficient detail in their working so that the offered solution is clear and complete.

When conducting a hypothesis test, candidates are strongly encouraged to determine whether there is sufficient evidence to accept or reject H_0, stating this decision explicitly, before then concluding in context. Care must be taken with the language used when interpreting the result of any test. In general, a hypothesis test is not a proof and it is not appropriate to use definitive statements. Concluding statements should always include some degree of uncertainty, for example, 'there is insufficient evidence to support the claim that….' rather than 'the test proves that….'.

General comments

Almost all candidates attempted all the questions. The standard was generally high, with many candidates presenting clear and accurate solutions throughout.

The rubric for this paper specifies that non-exact numerical answers should be given to 3 significant figures. Candidates would therefore be well-advised to work to a greater degree of accuracy while working towards the final answer. Premature rounding or working to only 3 significant figures may result in an error in the third figure in the final answer. This is particularly the case in statistics problems where several different values are calculated, each depending on the previous one. Such rounding errors were seen in **Question 2** and **Question 3a**.

Comments on specific questions

Question 1

This question was answered well by most candidates. The most common error was an incorrect t-value, usually 1.796, in the confidence interval formula. A small minority of candidates opted for a z-value, 1.645 or 1.96, which is not appropriate for a small sample with unknown population variance.

Question 2

Most candidates offered very good solutions to this question. They realised that a two-sample t-test was required and used a pooled variance when calculating the appropriate test statistic. A minority of candidates did not take note of the statement in the question that the times are assumed to be normally distributed with the same population variance and the implication that a pooled variance should be used.

Candidates are advised to evaluate quantities as they progress through the method, retaining a good degree of accuracy as they do so. This accuracy needs to be at least 4 significant figures if working in decimals so that their final answer is accurate to the required 3 significant figures.

A significant minority of responses offered a final statement which lacked any uncertainty, including some who said that they had **proved** that the mean time taken to complete the puzzle by candidates at school P is equal to the mean time taken at school Q. Candidates should be aware that all that the hypothesis test can show is that there is insufficient evidence to suggest that the mean time at P is greater than the mean time at Q.

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Question 3

(a) Most candidates knew how to find the values of *a* and *b*. The best responses used sufficient detail throughout their method to calculate a value for *a* accurately. This was a 'show that' request with an answer correct to 4 decimal places, so candidates were expected to work either exactly, in fractions, or to at least 5 decimal places.

(b) Almost all candidates demonstrated knowledge of how to apply a goodness of fit test, but there were several causes of error in many cases. The best responses combined the first two columns in the table, which was necessary because the first expected frequency was less than 5. There were some responses in which the null hypothesis was stated with insufficient detail, for example 'it is a good fit model' or 'the distribution is a good fit'. It is expected that both the distribution and the data are mentioned in the hypotheses. Examples of better responses here were 'the given distribution is a good model for the data' or, as a minimum, 'the distribution fits the data'. Some responses used an incorrect tabular value in the comparison with the calculated test statistic. Some responses did not include any level of uncertainty in the conclusion, for example by stating that 'f is a good fit' as the conclusion of the test.

Question 4

(a) Most candidates knew the methodology of a Wilcoxon matched-pairs signed-rank test. The best responses referred to the population median in the hypotheses and did so indicating an understanding of the purpose of the test and the context. In some responses, the hypotheses included the words 'population mean', but the relevance was not clear. Some responses omitted word 'population'. The correct critical value of 21 was often seen; 17 was the most common incorrect value. The best responses included a lack of uncertainty in the conclusions. A few responses used a paired sample *t*-test which was not appropriate here.

(b) Most candidates understood what was required here. They noted that the test statistic would now be 20 instead of 23 and that since 20 is less than the critical value of 21, the conclusion of the test would be different. Some responses were formed of an explanation in words only, without including 'numerical justification' as required.

Question 5

This question was answered well by almost all candidates.

(a) This part was almost always correct. A small number of candidates made errors in calculating the probability of obtaining 1 head and the probability of obtaining 2 heads. A few candidates had the correct probabilities but associated them with incorrect powers of t in the probability generating function.

(b) Candidates knew what they had to do in this part and the vast majority scored full marks. Errors were almost always numerical slips.

(c) Candidates understood that they had to differentiate their probability generating function and substitute $t = 1$ to find $E(Z)$. Many correct answers were seen.

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Question 6

(a) Almost all the candidates integrated the probability density function to find the cumulative distribution function. A common error was omitting to find the constant of integration. It was necessary to include $\mathrm{F}(x) = 0$ for $x < 0$ and $\mathrm{F}(x) = 1$ for $x > 2 \ln 2$, but often these were incorrectly combined as '$\mathrm{F}(x) = 0$ otherwise'.

(b) Most candidates carried out the change of variable successfully and then differentiated to find the probability density function $\mathrm{g}(Y)$.

(c) Most candidates equated their cumulative distribution function $\mathrm{G}(y)$ to 0.3 and then solved the resulting quadratic equation to find a single value of y. A common error was to offer also the second value of y obtained from the quadratic equation; this second value was out of the range of possible values. A few candidates equated their probability density function to 0.3.

(d) Many candidates used a correct method in this part. Common errors were confusion between $\mathrm{G}(y)$ and $\mathrm{g}(y)$ and/or incorrect limits of integration.

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FURTHER MATHEMATICS

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Key messages

In all questions candidates are advised to show all their working, as credit is given for method as well as accuracy. When a result is given in a question, candidates must take care to give sufficient detail in their working so that the offered solution is clear and complete.

When conducting a hypothesis test, candidates are strongly encouraged to determine whether there is sufficient evidence to accept or reject H_0, stating this decision explicitly, before then concluding in context. Care must be taken with the language used when interpreting the result of any test. In general, a hypothesis test is not a proof and it is not appropriate to use definitive statements. Concluding statements should always include some degree of uncertainty, for example, 'there is insufficient evidence to support the claim that….' rather than 'the test proves that….'.

General comments

Almost all candidates attempted all the questions. The standard was generally high, with many candidates presenting clear and accurate solutions throughout.

The rubric for this paper specifies that non-exact numerical answers should be given to 3 significant figures. Candidates would therefore be well-advised to work to a greater degree of accuracy while working towards the final answer. Premature rounding or working to only 3 significant figures may result in an error in the third figure in the final answer. This is particularly the case in statistics problems where several different values are calculated, each depending on the previous one. Such rounding errors were seen in **Question 4a**.

Comments on specific questions

Question 1

(a) Almost all candidates integrated the probability density function, and did so correctly, but a common error was to omit the constant term. It was necessary to include $F(x) = 0$ for $x < 0$ and $F(x) = 1$ for $x > 27$, but often these were incorrectly combined as '$F(x) = 0$ otherwise'.

(b) This part was usually done well. A minority of candidates correctly found the cumulative distribution function for Y but did not go on to differentiate to find the corresponding probability density function.

(c) The majority of candidates knew how to proceed in this part, by equating their cumulative distribution function for Y to 0.5 and solving the resulting quadratic equation. Most candidates then knew to reject the root that was outside the domain of Y. Some candidates gave an answer correct to 3 significant figures, although an exact answer was required. A few candidates equated the probability density function, rather than the cumulative distribution function, to 0.5.

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Question 2

There were many excellent clearly presented solutions to this problem. There were also solutions where it was very difficult to follow the steps in working. Candidates are strongly advised to show all the steps in their working, in a logical order, so that partial credit may be awarded.

Most candidates knew how to set up the correct equations from the endpoints of the confidence interval and most used the correct t-value of 1.771. Some used adjacent table values of 1.350 or 1.761 and others used a z-value, usually 1.645.

The value of Σx was usually correct, but errors occurred in finding Σx^2, with factors of 13, 14 or 14^2 appearing in the variance formula.

Question 3

(a) The mechanics of the Wilcoxon matched-pairs signed-rank test were well-known and accurately executed by most candidates. Most identified the correct 1-tail critical value of 8, although the 2-tail value of 5 was seen sometimes. Most candidates knew how to interpret their calculated test statistic in relation to this value.

The first of two common errors was that the hypotheses were not stated in terms of the population parameter. They often referred to the median, with the word 'population' omitted. Sometimes, the hypotheses referred to the (population) mean or simply used the symbol μ.

The second common error was to give a conclusion that was too assertive. The outcome of a test provides evidence which is either sufficient or insufficient to reject H_0; the outcome never proves anything so care should be taken to avoid definite statements in conclusions. It is also important to be aware that it is the null hypothesis H_0 that is being tested, so it is H_0 that is rejected or accepted.

(b) This part was found to be very difficult. Only a small number of candidates were able to offer an appropriate answer. Some candidates realised that they needed to comment on differences and/or symmetry, but very few realised that the assumption was about *population* differences. Many responses included unrelated comments about normality or randomness or independence.

Question 4

(a) This question was answered well. Solutions were usually presented clearly, with quantities evaluated at each step of the process. The best responses found the two individual sample variances explicitly, then a variance appropriate for a two-sample test. Then they stated the test statistic and finally compared this with 1.282, the appropriate tabular value of t, followed by a conclusion containing a degree of uncertainty. Some responses used a pooled variance in the calculation, although there was no indication in the question that the two population variances could be assumed to be equal. As in other questions, conclusions were sometimes too assertive.

Candidates are advised to evaluate quantities as they progress through the method, retaining a good degree of accuracy as they do so. This accuracy needs to be at least 4 significant figures so that their final answer is accurate to the required 3 significant figures.

(b) Many candidates correctly stated that the samples were large or that the Central Theorem could be applied. Some responses stated that the distribution was normal. Others said that the sample size was more than 30. This latter statement in itself is not a sufficient statement: there needs to be some comment to suggest that this is considered to be a large sample.

Question 5

(a) This is a 'show that' question, so candidates needed to give convincing reasoning. The best responses gave three clear steps. Firstly, they showed that $k = \frac{1}{8}$, done most simply by using $G_X(1) = 1$. Secondly, a general result for $G'_X(t)$ was found and thirdly, the elements were brought together by using the fact that $E(X) = G'_X(1)$. The most common omission was in the first step where many candidates simply stated the value of k.

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(b) The responses to this part were very good. A small minority of responses added the two probability generating functions instead of multiplying them.

(c) The responses to this part were very good.

(d) Many candidates identified the modal value of Z as the power of t which has the highest coefficient. The most common error was to give the highest valued coefficient rather than the power of t. Other candidates thought that the 'most probable value of Z' was the expected value $E(Z)$. This is 4.04 and these candidates usually rounded it down to 4.

Question 6

(a) The mechanics of carrying out the chi-squared test with a contingency table were generally well-known and executed accurately, though not always with sufficient working shown. Candidates are reminded of the rubric of this paper which states that all necessary working should be shown. The best responses gave the expected values and/or the individual contributions to the chi-squared value. The correct critical value of 7.378 was almost always used in the comparison. The hypotheses were usually stated correctly. As in other questions, the conclusions were sometimes too assertive. Candidates should also be aware, as previously, that it is the null hypothesis that is being tested. The conclusion in this part should be that there is sufficient evidence to suggest that grade is not independent of age (H_0 is rejected). The statement 'there is in*sufficient* evidence to suggest that grade is independent of age' is not correct.

(b) Most candidates made a good attempt at answering this part although again some concluding statements were too assertive.

(c) There were many different answers to this part. Most candidates opted for the result in **part (b)** based on there being more groups, more degrees of freedom or more detail. A common error was to state that there was more data which is not correct. There is the same amount of data, but it is split into more groups. Other comments that were not accepted were usually too vague, such as 'more accurate'.

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