Cambridge International Examinations
Cambridge International Advanced Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

FURTHER MATHEMATICS **9231/11**

Paper 1 **October/November 2017**

3 hours

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF10)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a calculator is expected, where appropriate.
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **23** printed pages and **1** blank page.

JC17 11_9231_11/FP

[Turn over

1 Find $\sum_{r=1}^{n}(4r-3)(4r+1)$, giving your answer in its simplest form. [4]

2 Find the general solution of the differential equation

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 5x = 4 - 5t^2.$$ [6]

3 **(i)** Show that $\dfrac{\mathrm{d}^{n+1}}{\mathrm{d}x^{n+1}}(x^{n+1}\ln x) = \dfrac{\mathrm{d}^n}{\mathrm{d}x^n}\big(x^n + (n+1)x^n \ln x\big)$. [2]

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(ii) Prove by mathematical induction that, for all positive integers n,

$$\frac{\mathrm{d}^n}{\mathrm{d}x^n}(x^n \ln x) = n!\left(\ln x + 1 + \frac{1}{2} + \ldots + \frac{1}{n}\right).$$ [5]

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4 The cubic equation $2x^3 - 3x^2 + 4x - 10 = 0$ has roots α, β and γ.

(i) Find the value of $(\alpha + 1)(\beta + 1)(\gamma + 1)$. [4]

(ii) Find the value of $(\beta + \gamma)(\gamma + \alpha)(\alpha + \beta)$. [4]

5 The curve C has equation $2x^3 + 3x^2y - 3y^3 - 16 = 0$.

(i) Find the coordinates of the point A on C at which $\frac{dy}{dx} = 0$ and $x \neq 0$. [5]

(ii) Find the value of $\frac{d^2y}{dx^2}$ at A. [3]

6 The points A, B and C have position vectors $2\mathbf{i} - \mathbf{j} + \mathbf{k}$, $3\mathbf{i} + 4\mathbf{j} - \mathbf{k}$ and $-\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}$ respectively.

(i) Find the area of the triangle ABC. [4]

(ii) Find the perpendicular distance of the point A from the line BC. [3]

(iii) Find the cartesian equation of the plane through A, B and C. [2]

7 The linear transformation $\mathrm{T} : \mathbb{R}^4 \to \mathbb{R}^4$ is represented by the matrix $\mathbf{A}$, where

$$\mathbf{A} = \begin{pmatrix} 1 & -1 & -2 & 3 \\ 5 & -3 & -4 & 25 \\ 6 & -4 & -6 & 28 \\ 7 & -5 & -8 & 31 \end{pmatrix}.$$

(i) Find the rank of $\mathbf{A}$ and a basis for the null space of T. [7]

(ii) Find the matrix product $\mathbf{A}\begin{pmatrix} -1 \\ 1 \\ -1 \\ 1 \end{pmatrix}$ and hence find the general solution of the equation $\mathbf{Ax} = \begin{pmatrix} 3 \\ 21 \\ 24 \\ 27 \end{pmatrix}$. [3]

8 Let $I_n = \int_0^{\frac{1}{4}\pi} \sec^n x \, dx$ for $n > 0$.

(i) Find the value of I_2. [2]

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(ii) Show that, for $n > 2$,

$$(n-1)I_n = 2^{\frac{1}{2}n-1} + (n-2)I_{n-2}.$$ [5]

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(iii) The curve C has equation $y = \sec^3 x$ for $0 \leqslant x \leqslant \frac{1}{4}\pi$. The region R is bounded by C, the x-axis, the y-axis and the line $x = \frac{1}{4}\pi$. Find the volume of revolution generated when R is rotated through 2π radians about the x-axis. [4]

9 The curve C has equation

$$y = \frac{3x - 9}{(x - 2)(x + 1)}.$$

(i) Find the equations of the asymptotes of C. [2]

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(ii) Show that there is no point on C for which $\frac{1}{3} < y < 3$. [4]

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(iii) Find the coordinates of the turning points of C. [3]

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(iv) Sketch C. [3]

10 **(i)** Use de Moivre's theorem to show that

$$\sin 5\theta = 5\sin\theta - 20\sin^3\theta + 16\sin^5\theta. \qquad [5]$$

(ii) Hence explain why the roots of the equation $16x^4 - 20x^2 + 5 = 0$ are $x = \pm\sin\frac{1}{5}\pi$ and $x = \pm\sin\frac{2}{5}\pi$. [3]

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(iii) Without using a calculator, find the exact values of

$$\sin\tfrac{1}{5}\pi\sin\tfrac{2}{5}\pi\sin\tfrac{3}{5}\pi\sin\tfrac{4}{5}\pi \quad \text{and} \quad \sin^2\left(\tfrac{1}{5}\pi\right)+\sin^2\left(\tfrac{2}{5}\pi\right).$$ [4]

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11 Answer only **one** of the following two alternatives.

EITHER

(i) The vector $\mathbf{e}$ is an eigenvector of the matrix $\mathbf{A}$, with corresponding eigenvalue λ, and is also an eigenvector of the matrix $\mathbf{B}$, with corresponding eigenvalue μ. Show that $\mathbf{e}$ is an eigenvector of the matrix $\mathbf{AB}$ with corresponding eigenvalue $\lambda\mu$. [3]

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(ii) Find the eigenvalues and corresponding eigenvectors of the matrix $\mathbf{A}$, where

$$\mathbf{A} = \begin{pmatrix} 0 & 1 & 3 \\ 3 & 2 & -3 \\ 1 & 1 & 2 \end{pmatrix}.$$ [6]

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(iii) The matrix **B**, where

$$\mathbf{B} = \begin{pmatrix} 3 & 6 & 1 \\ 1 & -2 & -1 \\ 6 & 6 & -2 \end{pmatrix},$$

has eigenvectors $\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$. Find the eigenvalues of the matrix **AB**, and state corresponding eigenvectors. [4]

OR

The polar equation of a curve C is $r = a(1 + \cos\theta)$ for $0 \leqslant \theta < 2\pi$, where a is a positive constant.

(i) Sketch C. [2]

(ii) Show that the cartesian equation of C is

$$x^2 + y^2 = a\big(x + \sqrt{(x^2 + y^2)}\big).$$ [2]

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(iii) Find the area of the sector of C between $\theta = 0$ and $\theta = \frac{1}{3}\pi$. [4]

(iv) Find the arc length of C between the point where $\theta = 0$ and the point where $\theta = \frac{1}{3}\pi$. [5]

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