Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

FURTHER MATHEMATICS **9231/13**

Paper 1 **May/June 2016**

3 hours

Additional Materials: List of Formulae (MF10)

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READ THESE INSTRUCTIONS FIRST

An answer booklet is provided inside this question paper. You should follow the instructions on the front cover of the answer booklet. If you need additional answer paper ask the invigilator for a continuation booklet.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a calculator is expected, where appropriate.
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **4** printed pages and **1** insert.

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1 Verify that $\dfrac{1}{(3r+1)(3r+4)} = \dfrac{1}{3}\left(\dfrac{1}{3r+1} - \dfrac{1}{3r+4}\right)$. [1]

Let S_N denote $\displaystyle\sum_{r=1}^{N} \frac{1}{(3r+1)(3r+4)}$ and let S denote $\displaystyle\sum_{r=1}^{\infty} \frac{1}{(3r+1)(3r+4)}$. Find the least value of N such that $S - S_N < \frac{1}{10\,000}$. [5]

2 It is given that a diagonal of a polygon is a line joining two non-adjacent vertices. Prove, by mathematical induction, that an n-sided polygon has $\frac{1}{2}n(n-3)$ diagonals, where $n \geqslant 3$. [6]

3 Find the two values of the constant k for which the equations

$$\begin{aligned} kx + y + z &= 2, \\ x + ky + z &= -1, \\ x + y + kz &= -1, \end{aligned}$$

have no unique solution. [4]

Show that, for one of these values of k, the equations have no solution, and solve the equations for the other value of k. [3]

4 The curve C has equation $y = -\ln(1 - x^2)$ for $-\frac{1}{2} \leqslant x \leqslant \frac{1}{2}$. Show that

$$1 + \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2 = \left(\frac{1 + x^2}{1 - x^2}\right)^2. \quad [2]$$

Show further that $\dfrac{1 + x^2}{1 - x^2}$ may be expressed in the form $\dfrac{P}{1 + x} + \dfrac{Q}{1 - x} + R$, where P, Q and R are constants to be determined. [2]

Find the exact arc length of C. [4]

5 The curve C has equation $y = \dfrac{x + 2}{x^2 - 9}$. Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} < 0$ at all points on C. [3]

State the equations of the asymptotes of C. [2]

Sketch C, showing the coordinates of any points of intersection with the coordinate axes. [3]

6 Let $I_n = \displaystyle\int_0^2 x^n (4 - x^2)^{\frac{1}{2}}\,\mathrm{d}x$, for $n \geqslant 1$. By considering $\dfrac{\mathrm{d}}{\mathrm{d}x}\left\{x^n (4 - x^2)^{\frac{3}{2}}\right\}$, show that

$$(n + 3)I_{n+1} = 4nI_{n-1}, \text{ where } n \geqslant 2. \quad [4]$$

Find the value of I_1 and deduce the exact value of I_3. [4]

7 A curve has polar equation $r = \dfrac{1}{1 - \cos\theta}$, for $0 < \theta < 2\pi$. Find, in the form $y^2 = \mathrm{f}(x)$, the cartesian equation of the curve. [3]

Hence sketch the curve, and shade the region whose area is given by $\dfrac{1}{2}\displaystyle\int_{\frac{1}{2}\pi}^{\frac{3}{2}\pi} \dfrac{1}{(1 - \cos\theta)^2}\,\mathrm{d}\theta$. [3]

Using the cartesian equation of the curve, find the area of this region. [3]

8 The cubic equation

$$z^3 - z^2 - z - 5 = 0$$

has roots α, β and γ. Show that the value of $\alpha^3 + \beta^3 + \gamma^3$ is 19. [4]

Find the value of $\alpha^4 + \beta^4 + \gamma^4$. [2]

Show that the cubic equation with roots $\dfrac{\alpha - 1}{\alpha}$, $\dfrac{\beta - 1}{\beta}$ and $\dfrac{\gamma - 1}{\gamma}$ may be found using the substitution $z = \dfrac{1}{1 - x}$, and find this equation, giving your answer in the form $px^3 + qx^2 + rx + s = 0$, where p, q, r and s are constants to be determined. [4]

9 Use de Moivre's theorem to show that $\cos^4\theta = \frac{1}{8}(\cos 4\theta + 4\cos 2\theta + 3)$. [4]

Find the corresponding expression for $\sin^4\theta$ in terms of $\cos 4\theta$ and $\cos 2\theta$. [4]

Hence find the exact value of $\displaystyle\int_0^{\frac{1}{8}\pi} (\cos^4\theta + \sin^4\theta)\,\mathrm{d}\theta$. [3]

10 Given that y is a function of x and that $x = \mathrm{e}^u$, show that

$$x\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}u} \qquad \text{and} \qquad x^2\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \frac{\mathrm{d}^2y}{\mathrm{d}u^2} - \frac{\mathrm{d}y}{\mathrm{d}u}.$$ [3]

Given also that

$$x^2\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 3x\frac{\mathrm{d}y}{\mathrm{d}x} + 17y = 34\ln x + 21,$$

deduce that

$$\frac{\mathrm{d}^2y}{\mathrm{d}u^2} + 2\frac{\mathrm{d}y}{\mathrm{d}u} + 17y = 34u + 21.$$ [1]

Find y in terms of x given that $y = 0$ and $\dfrac{\mathrm{d}y}{\mathrm{d}x} = -1$ when $x = 1$. [9]

[Question 11 is printed on the next page.]

11 Answer only **one** of the following two alternatives.

EITHER

It is given that 1 and 4 are eigenvalues of the matrix $\mathbf{A}$, where

$$\mathbf{A} = \begin{pmatrix} 1 & -3 & -3 \\ -8 & 6 & -3 \\ 8 & -2 & 7 \end{pmatrix}.$$

Find eigenvectors corresponding to each of these eigenvalues. [3]

Given further that $\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$ is an eigenvector of $\mathbf{A}$, find the corresponding eigenvalue. [2]

Write down matrices $\mathbf{P}$ and $\mathbf{D}$ such that $\mathbf{P}^{-1}\mathbf{A}\mathbf{P} = \mathbf{D}$, where $\mathbf{D}$ is a diagonal matrix, and find $\mathbf{P}^{-1}$. [5]

Write down a matrix $\mathbf{C}$ such that $\mathbf{C}^2 = \mathbf{D}$, and deduce a matrix $\mathbf{B}$ such that $\mathbf{B}^2 = \mathbf{A}$. [4]

OR

The position vectors of the points A, B, C, D are

$$\mathbf{a} = 2\mathbf{i} + \lambda\mathbf{j} - 3\mathbf{k}, \quad \mathbf{b} = 6\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}, \quad \mathbf{c} = \mathbf{i} + 2\mathbf{j} - \mathbf{k}, \quad \mathbf{d} = \mathbf{i} + 7\mathbf{j} + 4\mathbf{k}$$

respectively. It is given that the shortest distance between the lines AB and CD is 3.

(i) Show that $\lambda^2 + \lambda - 20 = 0$. [7]

(ii) The planes p_1 and p_2 are the planes through A, B and D corresponding to the two values of λ satisfying the equation in part **(i)**. Find the acute angle between p_1 and p_2. [7]