Cambridge International Examinations
Cambridge International Advanced Level

FURTHER MATHEMATICS **9231/11**

Paper 1 **May/June 2016**

3 hours

Additional Materials: List of Formulae (MF10)

4158100014

READ THESE INSTRUCTIONS FIRST

An answer booklet is provided inside this question paper. You should follow the instructions on the front cover of the answer booklet. If you need additional answer paper ask the invigilator for a continuation booklet.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a calculator is expected, where appropriate.
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **4** printed pages and **1** insert.

1 The roots of the cubic equation $2x^3 + x^2 - 7 = 0$ are α, β and γ. Using the substitution $y = 1 + \dfrac{1}{x}$, or otherwise, find the cubic equation whose roots are $1 + \dfrac{1}{\alpha}$, $1 + \dfrac{1}{\beta}$ and $1 + \dfrac{1}{\gamma}$, giving your answer in the form $ay^3 + by^2 + cy + d = 0$, where a, b, c and d are constants to be found. [4]

2 Express $\dfrac{4}{r(r+1)(r+2)}$ in partial fractions and hence find $\displaystyle\sum_{r=1}^{n} \frac{4}{r(r+1)(r+2)}$. [5]

Deduce the value of $\displaystyle\sum_{r=1}^{\infty} \frac{4}{r(r+1)(r+2)}$. [1]

3 Prove by mathematical induction that, for all positive integers n, $10^n + 3 \times 4^{n+2} + 5$ is divisible by 9. [6]

4 A curve C has polar equation $r^2 = 8 \operatorname{cosec} 2\theta$ for $0 < \theta < \frac{1}{2}\pi$. Find a cartesian equation of C. [3]

Sketch C. [2]

Determine the exact area of the sector bounded by the arc of C between $\theta = \frac{1}{6}\pi$ and $\theta = \frac{1}{3}\pi$, the half-line $\theta = \frac{1}{6}\pi$ and the half-line $\theta = \frac{1}{3}\pi$. [3]

[It is given that $\displaystyle\int \operatorname{cosec} x \, dx = \ln\left|\tan \tfrac{1}{2}x\right| + c$.]

5 Let $I_n = \displaystyle\int_0^{\frac{1}{2}\pi} \cos^n x \sin^2 x \, dx$, for $n \geqslant 0$. By differentiating $\cos^{n-1} x \sin^3 x$ with respect to x, prove that

$$(n+2)I_n = (n-1)I_{n-2} \quad \text{for } n \geqslant 2.$$ [5]

Hence find the exact value of I_4. [4]

6 Use de Moivre's theorem to express $\cot 7\theta$ in terms of $\cot \theta$. [4]

Use the equation $\cot 7\theta = 0$ to show that the roots of the equation

$$x^6 - 21x^4 + 35x^2 - 7 = 0$$

are $\cot\left(\frac{1}{14}k\pi\right)$ for $k = 1, 3, 5, 9, 11, 13$, and deduce that

$$\cot^2\left(\tfrac{1}{14}\pi\right)\cot^2\left(\tfrac{3}{14}\pi\right)\cot^2\left(\tfrac{5}{14}\pi\right) = 7.$$ [5]

7 A curve C has equation $y = \dfrac{x^2}{x-2}$. Find the equations of the asymptotes of C. [3]

Show that there are no points on C for which $0 < y < 8$. [4]

Sketch C, giving the coordinates of the turning points. [3]

8 Find a cartesian equation of the plane Π_1 passing through the points with coordinates $(2, -1, 3)$, $(4, 2, -5)$ and $(-1, 3, -2)$. [4]

The plane Π_2 has cartesian equation $3x - y + 2z = 5$. Find the acute angle between Π_1 and Π_2. [3]

Find a vector equation of the line of intersection of the planes Π_1 and Π_2. [4]

9 Find the value of the constant k such that $y = kx^2e^{2x}$ is a particular integral of the differential equation

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 4e^{2x}. \qquad (*)$$ [4]

Hence find the general solution of $(*)$. [3]

Find the particular solution of $(*)$ such that $y = 3$ and $\dfrac{dy}{dx} = -2$ when $x = 0$. [4]

10 Write down the eigenvalues of the matrix $\mathbf{A}$, where

$$\mathbf{A} = \begin{pmatrix} -2 & 1 & -1 \\ 0 & -1 & 2 \\ 0 & 0 & 1 \end{pmatrix},$$

and find corresponding eigenvectors. [4]

Find a matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $\mathbf{P}^{-1}\mathbf{AP} = \mathbf{D}$, and hence find the matrix $\mathbf{A}^n$, where n is a positive integer. [8]

[Question 11 is printed on the next page.]

11 Answer only **one** of the following two alternatives.

EITHER

A curve C has parametric equations

$$x = e^{2t}\cos 2t, \quad y = e^{2t}\sin 2t, \quad \text{for } -\tfrac{1}{2}\pi \leqslant t \leqslant \tfrac{1}{2}\pi.$$

Find the arc length of C. [6]

Find the area of the surface generated when C is rotated through 2π radians about the x-axis. [8]

OR

The linear transformation $\mathrm{T} : \mathbb{R}^4 \to \mathbb{R}^4$ is represented by the matrix $\mathbf{M}$, where

$$\mathbf{M} = \begin{pmatrix} 1 & -2 & 3 & -4 \\ 2 & -4 & 7 & -9 \\ 4 & -8 & 14 & -18 \\ 5 & -10 & 17 & -22 \end{pmatrix}.$$

Find the rank of $\mathbf{M}$. [3]

Obtain a basis for the null space K of T. [3]

Evaluate

$$\mathbf{M}\begin{pmatrix} 1 \\ -2 \\ 2 \\ -1 \end{pmatrix},$$

and hence show that any solution of

$$\mathbf{Mx} = \begin{pmatrix} 15 \\ 33 \\ 66 \\ 81 \end{pmatrix} \qquad (*)$$

has the form $\begin{pmatrix} 1 \\ -2 \\ 2 \\ -1 \end{pmatrix} + \lambda\mathbf{e}_1 + \mu\mathbf{e}_2$, where λ and μ are scalars and $\{\mathbf{e}_1, \mathbf{e}_2\}$ is a basis for K. [2]

Hence obtain a solution $\mathbf{x}'$ of $(*)$ such that the sum of the components of $\mathbf{x}'$ is 6 and the sum of the squares of the components of $\mathbf{x}'$ is 26. [6]