Cambridge International Examinations
Cambridge International Advanced Level

FURTHER MATHEMATICS **9231/23**

Paper 2 **May/June 2015**

3 hours

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF10)

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READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
Where a numerical value is necessary, take the acceleration due to gravity to be $10\,\mathrm{m\,s^{-2}}$.
The use of a calculator is expected, where appropriate.
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **5** printed pages and **3** blank pages.

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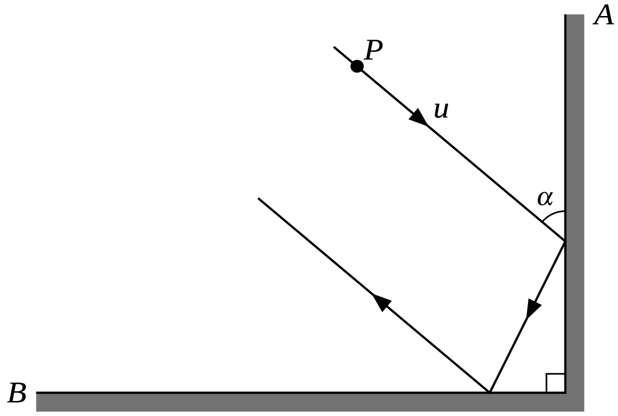
CAMBRIDGE International Examinations

1 Two uniform small smooth spheres, A and B, of equal radii and masses 2 kg and 3 kg respectively, are at rest and not in contact on a smooth horizontal plane. Sphere A receives an impulse of magnitude 8 N s in the direction AB. The coefficient of restitution between the spheres is e. Find, in terms of e, the speeds of A and B after A collides with B. [5]

Given that the spheres move in opposite directions after the collision, show that $e > \frac{2}{3}$. [1]

2



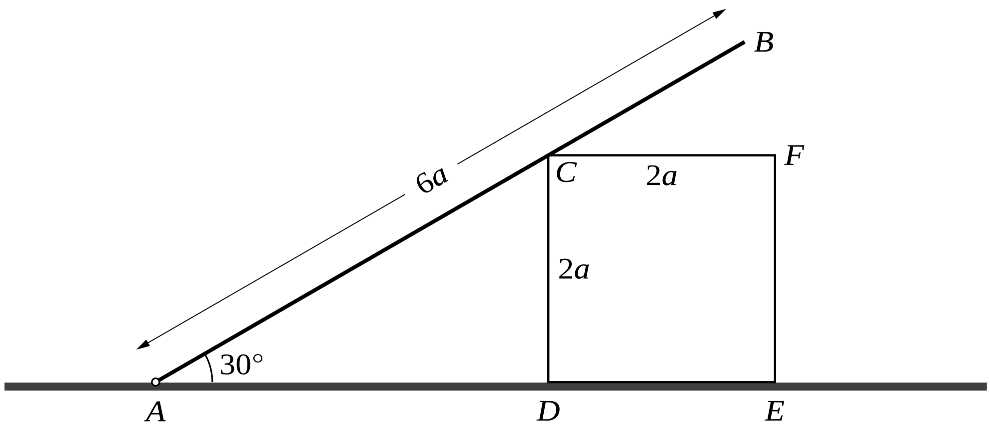
A uniform sphere P of mass m is at rest on a smooth horizontal table. The sphere is projected along the table with speed u and strikes a smooth vertical barrier A at an acute angle α. It then strikes another smooth vertical barrier B which is at right angles to A (see diagram). The coefficient of restitution between P and each of the barriers is e. Show that the final direction of motion of P makes an angle $\frac{1}{2}\pi - \alpha$ with the barrier B and find the total loss in kinetic energy as a result of the two impacts. [7]

3 A particle moves on a straight line AOB in simple harmonic motion, where $AB = 2a$ m. The centre of the motion is O and the particle is instantaneously at rest at A and B. The point M is the mid-point of OB. The particle passes through M moving towards O and next achieves its maximum speed one second later. Find the period of the motion. [4]

Find the distance of the particle from O when its speed is equal to one half of its maximum speed. [3]

At an instant 2.5 seconds after the particle passes through M moving towards O, the distance of the particle from O is $\sqrt{2}$ m. Find, in metres, the amplitude of the motion. [2]

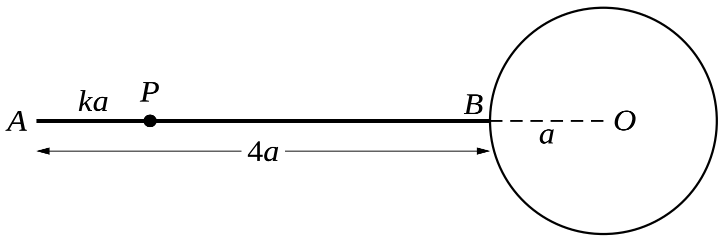
4



The diagram shows a central cross-section $CDEF$ of a uniform solid cube of weight W and with edges of length $2a$. The cube rests on a rough horizontal plane. A thin uniform rod AB, of weight W and length $6a$, is hinged to the plane at A. The rod rests in smooth contact with the cube at C, with angle CAD equal to 30°. The coefficient of friction between the plane and the cube is μ. Given that the system is in equilibrium, show that $\mu \geqslant \frac{3}{25}\sqrt{3}$. [6]

Find the magnitude of the force acting on the rod at A. [4]

5



The end B of a uniform rod AB, of mass $3M$ and length $4a$, is rigidly attached to a point on the circumference of a uniform disc. The disc has centre O, mass $2M$ and radius a, and ABO is a straight line. The disc and the rod are in the same vertical plane. A particle P, of mass M, is attached to the rod at a distance ka from A, where k is a positive constant (see diagram). Show that the moment of inertia of this system, about a fixed horizontal axis l through A perpendicular to the plane of the disc, is $(67 + k^2)Ma^2$. [5]

The system is free to rotate about l and performs small oscillations of period $4\pi\sqrt{\left(\frac{a}{g}\right)}$. Find the possible values of k. [6]

6 The reliability of the broadband connection received from two suppliers, A and B, is classified as good, fair or poor by a random sample of householders. The information collected is summarised in the following table.

		Reliability		
		Good	Fair	Poor
Supplier	A	65	63	33
	B	51	44	44

Test, at the 5% significance level, whether reliability is independent of supplier. [8]

7 For a random sample of 10 observations of pairs of values (x, y), the equation of the regression line of y on x is $y = 3.25x - 4.27$. The sum of the ten x values is 15.6 and the product moment correlation coefficient for the sample is 0.56. Find the equation of the regression line of x on y. [7]

Test, at the 5% significance level, whether there is evidence of non-zero correlation between the variables. [4]

8 A large number of long jumpers are competing in a national long jump competition. The distances, in metres, jumped by a random sample of 7 competitors are as follows.

6.25 7.01 5.74 6.89 7.24 5.64 6.52

Assuming that distances are normally distributed, test, at the 5% significance level, whether the mean distance jumped by long jumpers in this competition is greater than 6.2 metres. [7]

The distances jumped by another random sample of 8 long jumpers in this competition are recorded. Using the data from this sample of 8 long jumpers, a 95% confidence interval for the population mean, μ metres, is calculated as $5.89 < \mu < 6.75$. Find the unbiased estimates for the population mean and population variance used in this calculation. [5]

9 The continuous random variable X has probability density function given by

$$f(x) = \begin{cases} 0 & x < 2, \\ ae^{-(x-2)} & x \geqslant 2, \end{cases}$$

where a is a constant. Show that $a = 1$. [1]

Find the distribution function of X and hence find the median value of X. [5]

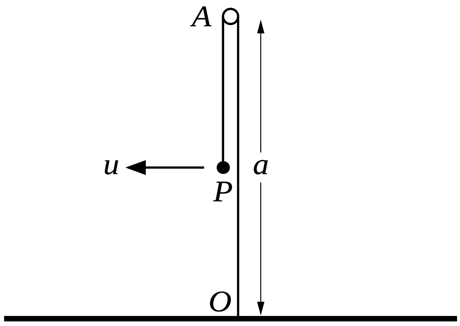
The random variable Y is defined by $Y = e^X$. Find

(i) the probability density function of Y, [4]

(ii) $P(Y > 10)$. [2]

10 Answer only **one** of the following two alternatives.

EITHER



One end of a light inextensible string of length $\frac{3}{2}a$ is attached to a fixed point O on a horizontal surface. The other end of the string is attached to a particle P of mass m. The string passes over a small fixed smooth peg A which is at a distance a vertically above O. The system is in equilibrium with P hanging vertically below A and the string taut. The particle is projected horizontally with speed u (see diagram). When P is at the same horizontal level as A, the tension in the string is T. Show that $T = \frac{2m}{a}(u^2 - ag)$. [3]

The ratio of the tensions in the string immediately before, and immediately after, the string loses contact with the peg is 5 : 1.

(i) Show that $u^2 = 5ag$. [6]

(ii) Find, in terms of m and g, the tension in the string when P is next at the same horizontal level as A. [5]

OR

The times taken, in hours, by cyclists from two different clubs, A and B, to complete a 50 km time trial are being compared. The times taken by a cyclist from club A and by a cyclist from club B are denoted by t_A and t_B respectively. A random sample of 50 cyclists from A and a random sample of 60 cyclists from B give the following summarised data.

$$\Sigma t_A = 102.0 \qquad \Sigma t_A^2 = 215.18 \qquad \Sigma t_B = 129.0 \qquad \Sigma t_B^2 = 282.3$$

Using a 5% significance level, test whether, on average, cyclists from club A take less time to complete the time trial than cyclists from club B. [10]

A test at the α% significance level shows that there is evidence that the population mean time for cyclists from club B exceeds the population mean time for cyclists from club A by more than 0.05 hours. Find the set of possible values of α. [4]

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