Cambridge International Examinations
Cambridge International Advanced Level

FURTHER MATHEMATICS **9231/13**

Paper 1 **May/June 2014**

3 hours

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF10)

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READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a calculator is expected, where appropriate.
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **4** printed pages.

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1 The vectors $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$ and $\mathbf{d}$ in $\mathbb{R}^3$ are given by

$$\mathbf{a} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad \mathbf{c} = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} \quad \text{and} \quad \mathbf{d} = \begin{pmatrix} 3 \\ -2 \\ 0 \end{pmatrix}.$$

Show that $\{\mathbf{a}, \mathbf{b}, \mathbf{c}\}$ is a basis for $\mathbb{R}^3$. [3]

Express $\mathbf{d}$ in terms of $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$. [2]

2 Show that the difference between the squares of consecutive integers is an odd integer. [1]

Find the sum to n terms of the series

$$\frac{3}{1^2 \times 2^2} + \frac{5}{2^2 \times 3^2} + \frac{7}{3^2 \times 4^2} + \ldots + \frac{2r+1}{r^2(r+1)^2} + \ldots$$

and deduce the sum to infinity of the series. [5]

3 It is given that $\phi(n) = 5^n(4n+1) - 1$, for $n = 1, 2, 3, \ldots$. Prove, by mathematical induction, that $\phi(n)$ is divisible by 8, for every positive integer n. [7]

4 The curve C has cartesian equation $\left(x^2 + y^2\right)^2 = 2a^2xy$, where a is a positive constant. Show that the polar equation of C is $r^2 = a^2 \sin 2\theta$. [3]

Sketch C for $-\pi < \theta \leqslant \pi$. [2]

Find the area enclosed by one loop of C. [2]

5 State the sum of the series $z + z^2 + z^3 + \ldots + z^n$, for $z \neq 1$. [1]

By letting $z = \cos\theta + \mathrm{i}\sin\theta$, show that

$$\cos\theta + \cos 2\theta + \cos 3\theta + \ldots + \cos n\theta = \frac{\sin\frac{1}{2}n\theta}{\sin\frac{1}{2}\theta}\cos\tfrac{1}{2}(n+1)\theta,$$

where $\sin\frac{1}{2}\theta \neq 0$. [7]

6 The curve C has parametric equations

$$x = \mathrm{e}^t - 4t + 3, \quad y = 8\mathrm{e}^{\frac{1}{2}t}, \quad \text{for } 0 \leqslant t \leqslant 2.$$

(i) Find, in terms of e, the length of C. [5]

(ii) Find, in terms of π and e, the area of the surface generated when C is rotated through 2π radians about the x-axis. [5]

7 The curve C has parametric equations

$$x = \sin t, \quad y = \sin 2t, \quad \text{for } 0 \leqslant t \leqslant \pi.$$

Find $\dfrac{d^2y}{dx^2}$ in terms of t. [5]

Hence, or otherwise, find the coordinates of the stationary points on C and determine their nature. [5]

8 It is given that λ is an eigenvalue of the non-singular square matrix $\mathbf{A}$, with corresponding eigenvector $\mathbf{e}$. Show that λ^{-1} is an eigenvalue of $\mathbf{A}^{-1}$ for which $\mathbf{e}$ is a corresponding eigenvector. [2]

Deduce that $\lambda + \lambda^{-1}$ is an eigenvalue of $\mathbf{A} + \mathbf{A}^{-1}$. [1]

It is given that 1 is an eigenvalue of the matrix $\mathbf{A}$, where

$$\mathbf{A} = \begin{pmatrix} 2 & 0 & 1 \\ -1 & 2 & 3 \\ 1 & 0 & 2 \end{pmatrix}.$$

Find a corresponding eigenvector. [2]

It is also given that $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$ are eigenvectors of the matrix $\mathbf{A}$. Find the corresponding eigenvalues. [2]

Hence find a matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that

$$(\mathbf{A} + \mathbf{A}^{-1})^3 = \mathbf{PDP}^{-1}.$$ [4]

9 Using the substitution $u = \cos\theta$, or any other method, find $\int \sin\theta \cos^2\theta \, d\theta$. [1]

It is given that $I_n = \int_0^{\frac{1}{2}\pi} \sin^n\theta \cos^2\theta \, d\theta$, for $n \geqslant 0$. Show that, for $n \geqslant 2$,

$$I_n = \frac{n-1}{n+2} I_{n-2}.$$ [5]

Hence find the exact value of $\int_0^{\frac{1}{2}\pi} \sin^4\theta \cos^2\theta \, d\theta$. [4]

10 Find the particular solution of the differential equation

$$\frac{d^2x}{dt^2} + 0.16\frac{dx}{dt} + 0.0064x = 8.64 + 0.32t,$$

given that when $t = 0$, $x = 0$ and $\dfrac{dx}{dt} = 0$. [10]

Show that, for large positive t, $\dfrac{dx}{dt} \approx 50$. [2]

11 Answer only **one** of the following two alternatives.

EITHER

Express $\dfrac{2x^2 - x + 5}{x^2 - 1}$ in the form $2 + \dfrac{A}{x-1} + \dfrac{B}{x+1}$, where A and B are integers to be found. [3]

The curve C has equation $y = \dfrac{2x^2 - x + 5}{x^2 - 1}$. Show that there are two distinct values of x for which $\dfrac{dy}{dx} = 0$. [4]

Sketch C, stating the equations of the asymptotes and giving the coordinates of any points of intersection with the coordinate axes and with the asymptotes. You do not need to find the coordinates of the turning points. [7]

OR

With respect to an origin O, the point A has position vector $4\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$ and the plane Π_1 has equation

$$\mathbf{r} = (4 + \lambda + 3\mu)\mathbf{i} + (-2 + 7\lambda + \mu)\mathbf{j} + (2 + \lambda - \mu)\mathbf{k},$$

where λ and μ are real. The point L is such that $\overrightarrow{OL} = 3\overrightarrow{OA}$ and Π_2 is the plane through L which is parallel to Π_1. The point M is such that $\overrightarrow{AM} = 3\overrightarrow{ML}$.

(i) Show that A is in Π_1. [1]

(ii) Find a vector perpendicular to Π_2. [2]

(iii) Find the position vector of the point N in Π_2 such that ON is perpendicular to Π_2. [5]

(iv) Show that the position vector of M is $10\mathbf{i} - 5\mathbf{j} + 5\mathbf{k}$ and find the perpendicular distance of M from the line through O and N, giving your answer correct to 3 significant figures. [6]