CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

MATHEMATICS **9709/72**

Paper 7  Probability & Statistics 2  **(S2)** **October/November 2019**

**1 hour 15 minutes**

Candidates answer on the Question Paper.

Additional Materials:  List of Formulae (MF9)

## READ THESE INSTRUCTIONS FIRST

Write your centre number, candidate number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions in the space provided. If additional space is required, you should use the lined page at the end of this booklet. The question number(s) must be clearly shown.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.

This document consists of **13** printed pages and **3** blank pages.

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1    On average, 1 in 150 components made by a certain machine are faulty. The random variable $X$ denotes the number of faulty components in a random sample of 500 components.

(i) Describe fully the distribution of $X$. [2]

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(ii) State a suitable approximating distribution for $X$, giving a justification for your choice. [2]

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(iii) Use your approximating distribution to find the probability that the sample will include at least 3 faulty components. [3]

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**2** The heights of a certain species of animal have been found to have mean 65.2 cm and standard deviation 7.1 cm. A researcher suspects that animals of this species in a certain region are shorter on average than elsewhere. She takes a large random sample of $n$ animals of this species from this region and finds that their mean height is 63.2 cm. She then carries out an appropriate hypothesis test.

   **(i)** She finds that the value of the test statistic $z$ is $-2.182$, correct to 3 decimal places.

      **(a)** Stating a necessary assumption, calculate the value of $n$. [4]

**(b)** Carry out the hypothesis test at the 4% significance level. [3]

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**(ii)** Explain why it was necessary to use the Central Limit theorem in carrying out the test. [1]

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3   The masses, in grams, of bags of flour are normally distributed with mean $\mu$. The masses, $m$ grams, of a random sample of 50 bags are summarised by $\Sigma m = 25\,110$ and $\Sigma m^2 = 12\,610\,300$.

(i) Calculate a 96% confidence interval for $\mu$, giving the end-points correct to 1 decimal place. [6]

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Another random sample of 50 bags of flour is taken and a 99% confidence interval for $\mu$ is calculated.

**(ii)** Without calculation, state whether this confidence interval will be wider or narrower than the confidence interval found in part **(i)**. Give a reason for your answer. [1]

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**4** A random variable $X$ has probability density function given by

$$f(x) = \begin{cases} \frac{1}{2}x & 0 \leqslant x \leqslant a, \\ 0 & \text{otherwise,} \end{cases}$$

where $a$ is a constant.

**(i)** Find $a$. [2]

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**(ii)** Show that $E(X) = \frac{4}{3}$. [3]

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The median of $X$ is denoted by $m$.

**(iii)** Find $P(E(X) < X < m)$. [4]

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5  The masses, in grams, of large boxes of chocolates and small boxes of chocolates have the distributions N(325, 6.1) and N(167, 5.6) respectively.

   **(i)** Find the probability that the total mass of 10 randomly chosen large boxes of chocolates is less than 3240 g. [4]

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**(ii)** Find the probability that the mass of a randomly chosen large box of chocolates is more than twice the mass of a randomly chosen small box of chocolates. [5]

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**6** The number of accidents per month, $X$, at a factory has a Poisson distribution. In the past the mean has been 1.1 accidents per month. Some new machinery is introduced and the management wish to test whether the mean has increased. They note the number of accidents in a randomly chosen month and carry out a hypothesis test at the 1% significance level.

(i) Show that the critical region for the test is $X \geqslant 5$. Given that the number of accidents is 6, carry out the test. [6]

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Later they carry out a similar test, also at the 1% significance level.

**(ii)** Explain the meaning of a Type I error in this context and state the probability of a Type I error.

[2]

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**(iii)** Given that the mean is now 7.0, find the probability of a Type II error.

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**Additional Page**

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