Cambridge
International
AS\&A Level

## Cambridge Assessment International Education

Cambridge International Advanced Subsidiary and Advanced Level

MATHEMATICS
9709/62
Paper 6
May/June 2019
MARK SCHEME
Maximum Mark: 50

## Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.
Cambridge International is publishing the mark schemes for the May/June 2019 series for most
Cambridge IGCSE ${ }^{\text {TM }}$, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

## GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.


## GENERIC MARKING PRINCIPLE 2 :

Marks awarded are always whole marks (not half marks, or other fractions).

## GENERIC MARKING PRINCIPLE 3:

Marks must be awarded positively:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.


## GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

## GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

## GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

## Mark Scheme Notes

Marks are of the following three types:
M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier $M$ or $B$ (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.
- Note: B2 or A2 means that the candidate can earn 2 or 0. $B 2 / 1 / 0$ means that the candidate can earn anything from 0 to 2 .

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.
- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking $g$ equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:
AEF/OE Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO Correct Working Only - often written by a 'fortuitous' answer
ISW Ignore Subsequent Working
SOI Seen or implied
SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

## Penalties

MR-1 A penalty of MR -1 is deducted from $A$ or $B$ marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through" marks. MR is not applied when the candidate misreads his own figures - this is regarded as an error in accuracy. An MR - 2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA -1 This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.

| Question | Answer | Marks | Guidance |
| :---: | :---: | :---: | :---: |
| 1 | $\mathrm{P}(\mathrm{~S})=\frac{1}{2}$ | B1 |  |
|  | $\mathrm{P}(T)=\frac{16}{36}\left(\frac{4}{9}\right)$ | B1 |  |
|  | $\mathrm{P}(S \cap T)=\frac{10}{36}\left(\frac{5}{18}\right)$ | M1 | $\mathrm{P}(S \cap T)$ found by multiplication scores M0 M1 awarded if their value is identifiable in their sample space diagram or Venn diagram or list of terms or probability distribution table (oe) |
|  | $\mathrm{P}(S) \mathrm{P}(T) \neq \mathrm{P}(S \cap T)$ so not independent | A1 | $8 / 36,10 / 36 \mathrm{P}(\mathrm{S}) \times \mathrm{P}(\mathrm{T})$ and $\mathrm{P}(S \cap T)$ seen in workings and correct conclusion stated, www |
|  | Alternative method for question 1 |  |  |
|  | $\mathrm{P}(\mathrm{~S})=\frac{1}{2}$ | B1 |  |
|  | $\mathrm{P}(T)=\frac{16}{36}\left(\frac{4}{9}\right)$ | B1 |  |
|  | $\mathrm{P}(S \cap T)=\frac{10}{36}\left(\frac{5}{18}\right)$ | M1 | $\mathrm{P}(S \cap T)$ found by multiplication scores M0 <br> M1 awarded if their value is identifiable in their sample space diagram or Venn diagram or list of terms or probability distribution table (oe) |
|  | $\mathrm{P}(\mathrm{~S} \mid \mathrm{T})=\frac{10}{16} \text { or } \mathrm{P}(\mathrm{~T} \mid \mathrm{S})=\frac{10}{18}$ <br> $\mathrm{P}(\mathrm{S} \mid \mathrm{T}) \neq \mathrm{P}(\mathrm{S})$ or $\mathrm{P}(\mathrm{T} \mid \mathrm{S}) \neq \mathrm{P}(\mathrm{T})$ so not independent | A1 | Either $18 / 36,10 / 16, \mathrm{P}(\mathrm{S})$ and $\mathrm{P}(S \mid T)$ seen in workings and correct conclusion stated, www Or 16/36, 10/18, $\mathrm{P}(\mathrm{T})$ and $\mathrm{P}(\mathrm{T} \mid \mathrm{S})$ seen in workings and correct conclusion stated, www |
|  |  | 4 |  |


| Question | Answer | Marks | Guidance |
| :---: | :---: | :---: | :---: |
| 2 | $\mathrm{P}(<28.9)=\mathrm{P}\left(z<\frac{28.9-30}{1.5}\right)$ | B1 | Using $\pm$ standardising formula, no continuity correction, not $\sigma^{2}$ or $\sqrt{ } \sigma$, |
|  | $\begin{aligned} & =\mathrm{P}(z<-0.733) \\ & =1-0.7682 \end{aligned}$ | M1 | Appropriate area $\Phi$ from standardisation formula $\mathrm{P}(\mathrm{z}<\ldots$.$) in final$ probability solution, <br> Must be a probability, e.g. $1-0.622$ is M0 |
|  | $=0.2318$ | A1 | Correct final probability rounding to 0.232 . (Only requires M1 not B1 to be awarded |
|  | Number of cartridges is their $0.2318 \times 8$ $=1.85$, so 2 (Also accept 1 but not both) | B1 | FT using their 4 SF (or better) value, ans. rounded or truncated to integer, no approximation indicated. |
|  |  | 4 |  |


| Question | Answer | Marks | Guidance |
| :---: | :---: | :---: | :---: |
| 3(i) | $\begin{aligned} & \mathrm{P}(\text { at most } 7)=1-\mathrm{P}(8,9,10) \\ & =1-{ }^{10} \mathrm{C} 8(0.35)^{8}(0.65)^{2}-{ }^{10} \mathrm{C}_{9}(0.35)^{9}(0.65)^{1}-(0.35)^{10} \end{aligned}$ | M1 | Use of normal approximation M0 <br> Binomial term of form ${ }^{10} \mathrm{C}_{x} p^{x}(1-p)^{10-x} \quad 0<p<1$ any $p, x \neq 10,0$ |
|  | [ $=1-0.004281-0.0005123-0.00002759]$ | A1 | Correct unsimplified (or individual terms evaluated) answer seen Condone $1-\mathrm{A}+\mathrm{B}+\mathrm{C}$ leading to correct solution |
|  | $=0.995$ | B1 | B1 not dependent on previous marks. |
|  | Alternative method for question 3(i) |  |  |
|  | $\mathrm{P}($ at most 7$)=\mathrm{P}(0,1,2,3,4,5,6,7)$ | M1 | Binomial term of form ${ }^{10} \mathrm{C}_{x} p^{x}(1-p)^{10-x} \quad 0<p<1$ any $p, x \neq 10,0$ |
|  | $=(0.65)^{10}+{ }^{10} \mathrm{C} 1(0.35)^{1}(0.65)^{9}+\ldots+{ }^{10} \mathrm{C}_{7}(0.35)^{7}(0.65)^{3}$ | A1 | Correct unsimplified answer or individual terms evaluated seen |
|  | $=0.995$ | B1 |  |
|  |  | 3 |  |
| 3(ii) | $\begin{aligned} & 1-(0.65)^{n}>0.99 \\ & 0.01>(0.65)^{n} \end{aligned}$ | M1 | Equation or inequality with $(0.65)^{n}$ and 0.01 or $(0.35)^{n}$ and 0.99 only (Note $1-0.99$ is equivalent to 0.01 etc.) |
|  | $n>10.69$ | M1 | Solving their $a^{n}=c, 0<a, c<1$ using logs or Trial and Error If answer inappropriate, at least 2 trials are required for Trial and Error M mark |
|  | smallest $n=11$ | A1 | CAO |
|  |  | 3 |  |


| Question | Answer | Marks | Guidance |
| :---: | :---: | :---: | :---: |
| 4 | $z=0.842=\left(\frac{121-\mu}{\sigma}\right) \text { so } 0.842 \sigma=121-\mu$ | B1 | $\pm 0.842$ seen but B0 if $1 \pm 0.842$ oe seen |
|  |  | M1 | One appropriate standardisation equation with a $z$-value, $\mu, \sigma$ and 121 or 102 , condone continuity correction. Not $0.158,0.42, \ldots$ |
|  | $z=-0.58=\left(\frac{102-\mu}{\sigma}\right)$ so $-0.58 \sigma=102-\mu$ | B1 | $\pm 0.58(0)$ seen but B0 if $1 \pm 0.58$ oe seen |
|  | Solving | M1 | Correct algebraic elimination of $\mu$ or $\sigma$ from their two simultaneous equations to form an equation in one variable, condone 1 numerical slip |
|  | $\sigma=13.4 \mu=110$ | A1 | If M0A0 scored (i.e. no algebraic elimination seen), SC B1 can be awarded for both answers correct <br> Consistent use of $\sigma^{2}$ or $\sqrt{ } \sigma$ throughout apply MR penalty to A mark or SC B mark. |
|  |  | 5 |  |



| Question | Answer | Marks | Guidance |
| :---: | :---: | :---: | :---: |
| 5(iv) | $\mathrm{P}\left(1^{\text {st }} \mathrm{C} \mid 2^{\text {nd }} \mathrm{T}\right)=\frac{P(C \cap T)}{P(T)}=\frac{\frac{1}{7} \times \frac{6}{9}}{\frac{1}{7} \times \frac{6}{9}+\frac{6}{7} \times \frac{5}{9}}=\frac{\frac{6}{63}}{\frac{36}{63}}$ | B1 | $\mathrm{P}(\mathrm{C} \cap \mathrm{T})$ attempt seen as numerator of a fraction, consistent with their tree diagram or correct |
|  |  | M1 | Summing 2 appropriate two-factor probabilities, consistent with their tree diagram or correct seen anywhere |
|  |  | A1 | $\frac{36}{63}$ oe or correct unsimplifed expression seen as numerator or denominator of a fraction |
|  | $\frac{1}{6}$ oe | A1 | Final answer |
|  |  | 4 |  |


| Question | Answer | Marks | Guidance |
| :---: | :---: | :---: | :---: |
| 6(i) | Advantage: comment referring to spread or range or shape | B1 | Comments referring to quartiles, IQR, Range, median, shape, skewness, data distribution, spread score B1 <br> Any comments with reference to mean or standard deviation or any other 'disadvantage' will score B0 <br> Comments referring to ' 5 -value plot', comparison with another data set, overview or ease of drawing/plotting/reading require an appropriate advantage statement. |
|  | Disadvantage: comment referring to limited data information provided | B1 | Comments referring to no individual data, no information about the number of values, unable to calculate mean, standard deviation, variance and mode score B1 <br> Any comments with reference to median, shape or any other 'advantage' will score B0 <br> Comments referring to 'size of data set' or 'average' require an appropriate disadvantage statement. <br> Comments referring to outliers are ignored in all cases (as outliers are not in the syllabus content) unless supported by an appropriate advantage / disadvantage statement. <br> If comments not clearly identified, assume first comment is the advantage. |
|  |  | 2 |  |



| Question | Answer | Marks | Guidance |
| :---: | :---: | :---: | :---: |
| $6(\mathrm{iii})(\mathrm{b})$ | $\mathrm{IQR}=$ their $329-$ their $256=73$ or 72.5 | B1 | FT Must follow through only from their stated values (condone if correct <br> quartiles stated here), not reading from graph. |
|  |  | $\mathbf{1}$ |  |


| Question | Answer | Marks | Guidance |
| :---: | :---: | :---: | :---: |
| 7(a) | ${ }^{6} \mathrm{C}_{3} \times{ }^{3} \mathrm{C}_{2} \times{ }^{1} \mathrm{C}_{1}$ | M1 | ${ }^{6} \mathrm{C}_{\mathrm{a}} \times{ }^{6-\mathrm{a}} \mathrm{C}_{\mathrm{b}} \times{ }^{6-\mathrm{ab}} \mathrm{C}_{6-\mathrm{ab}}$ seen oe ${ }^{6-\mathrm{ab}} \mathrm{C}_{6-\mathrm{ab}}$ can be implied by 1 or omission, condone use of permutations, |
|  | $=20 \times 3$ | A1 | Any correct method seen no addition/additional scenarios |
|  | $=60$ | A1 | Correct answer |
|  | Alternative method for question 7(a) |  |  |
|  | $\frac{{ }^{6} \mathrm{P}_{6}}{{ }^{\text {a }} \text { ( }}$ | M1 | ${ }^{6} \mathrm{P}_{6} /\left({ }^{n} \mathrm{P}_{n} \mathrm{x} k\right)$ with $3 \geqslant n>1$ and $6 \geqslant \mathrm{k}$ an integer $\geqslant 1, \operatorname{not} 6!/ 1$ |
|  | ${ }^{3} \mathrm{P}_{3} \times{ }^{2} \mathrm{P}_{2} \times{ }^{1} \mathrm{P}_{1} \quad=\frac{3!\times 2!}{}$ | A1 | Correct method with no additional terms |
|  | $=60$ | A1 | Correct answer |
|  |  | 3 |  |
| 7(b)(i) | $\frac{4!}{3!} \times \frac{3!}{2!} \times 2$ | M1 | A single expression with either $4!/ 3!\times k$ or $3!/ 2!\times \mathrm{k}, \mathrm{k}$ a positive integer seen oe (condone 2 identical expressions being added) |
|  |  | M1 | Correctly multiplying their single expression by 2 or 2 identical expressions being added. |
|  | $=24$ | A1 | Correct answer |
|  |  | 3 |  |


| Question | Answer | Marks | Guidance |
| :---: | :---: | :---: | :---: |
| 7(b)(ii) | Total no of arrangements $=\frac{7!}{2!3!}=420(\mathrm{~A})$ | B1 | Accept unsimplified |
|  | No with 2 s together $=\frac{6!}{3!}=120(B)$ | B1 | Accept unsimplified |
|  | With 2s not together: their (A) - their (B) | M1 | Subtraction indicated, possibly by their answer, no additional terms present |
|  | $=300$ ways | A1 | Exact value www |
|  | Alternative method for question 7(b)(ii) |  |  |
|  | $3{ }^{7}{ }^{7}{ }^{7}-{ }^{7}{ }^{8}$ - |  |  |
|  | $5!\times 6 \times 5$ | B1 | $k \times 5$ ! in numerator, $k$ a positive integer |
|  |  | B1 | $m \times 3$ ! In denominator, $m$ a positive integer |
|  |  | M1 | Their 5!/3! multiplied by ${ }^{6} \mathrm{C}_{2}$ only (no additional terms) |
|  | $=300$ ways | A1 | Exact value www |
|  |  | 4 |  |

