Cambridge Assessment International Education
Cambridge International Advanced Level

MATHEMATICS **9709/31**

Paper 3 **May/June 2019**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2019 series for most Cambridge IGCSE™, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)
CWO	Correct Working Only – often written by a ‘fortuitous’ answer
ISW	Ignore Subsequent Working
SOI	Seen or implied
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question	Answer	Marks	Guidance
1	State or imply ordinates 3, 2, 0, 4	**B1**	These and no more Accept in unsimplified form $\left\|2^0 - 4\right\|$ etc.
	Use correct formula, or equivalent, with h = 1 and four ordinates	**M1**	
	Obtain answer 5.5	**A1**	
		3	

Question	Answer	Marks	Guidance
2	Use law for the logarithm of a product, quotient or power	**M1**	Condone $\ln\dfrac{x}{x-1}$ for M1
	Obtain a correct equation free of logarithms	**A1**	e.g. $(2x-3)(x-1) = x^2$ or $x^2 - 5x + 3 = 0$
	Solve a 3-term quadratic obtaining at least one root	**M1**	Must see working if using an incorrect quadratic $\left(\dfrac{5 \pm \sqrt{13}}{2}\right)$
	Obtain answer x = 4.30 only	**A1**	Q asks for 2 dp. Do not ISW. Overspecified answers score A0 Overspecified and no working can score M1A0
		4	

Question	Answer	Marks	Guidance
3	State or imply $3y^2 + 6xy\dfrac{dy}{dx}$ as derivative of $3xy^2$	B1	
	State or imply $3y^2\dfrac{dy}{dx}$ as derivative of y^3	B1	
	Equate derivative of LHS to zero, substitute (1, 3) and find the gradient	M1	$\left(\dfrac{dy}{dx} = \dfrac{x^2 + y^2}{y^2 - 2xy}\right)$ For incorrect derivative need to see the substitution
	Obtain final answer $\dfrac{10}{3}$ or equivalent	A1	3.33 or better. Allow $\dfrac{30}{9}$ ISW after correct answer seen
		4	

Question	Answer	Marks	Guidance
4	Use correct trig formula and obtain an equation in tan θ	**M1**	Allow with $45°$ e.g. $\frac{1}{\tan\theta}-\frac{1}{\frac{\tan\theta+\tan 45°}{1-\tan\theta\tan 45°}}=3$
	Obtain a correct horizontal equation in any form	**A1**	e.g. $1+\tan\theta-\tan\theta(1-\tan\theta)=3\tan\theta(1+\tan\theta)$
	Reduce to $2\tan^2\theta+3\tan\theta-1=0$	**A1**	or 3-term equivalent
	Solve 3-term quadratic and find a value of θ	**M1**	Must see working if using an incorrect quadratic
	Obtain answer 15.7°	**A1**	One correct solution (degrees to at least 3 sf)
	Obtain answer 119.(3)°	**A1**	Second correct solution and no others in range (degrees to at least 3 sf) Mark 0.274, 2.082 as MR: A0A1
		6	

Question	Answer	Marks	Guidance
5(i)	Use chain rule	M1	$k\cos\theta\sin^{-3}\theta\left(=-k\text{cosec}^2\theta\cot\theta\right)$ Allow M1 for $-2\cos\theta\sin^{-1}\theta$
	Obtain correct answer in any form	A1	e.g. $-2\text{cosec}^2\theta\cot\theta$, $\dfrac{-2\cos\theta}{\sin^3\theta}$ Accept $\dfrac{-2\sin\theta\cos\theta}{\sin^4\theta}$
		2	
5(ii)	Separate variables correctly and integrate at least one side	B1	$\int x\,\text{d}x=\int -\text{cosec}^2\theta\cot\theta\,\text{d}\theta$
	Obtain term $\frac{1}{2}x^2$	B1	
	Obtain term of the form $\dfrac{k}{\sin^2\theta}$	M1*	or equivalent
	Obtain term $\dfrac{1}{2\sin^2\theta}$	A1	or equivalent
	Use $x=4$, $\theta=\frac{1}{6}\pi$ to evaluate a constant, or as limits, in a solution with terms ax^2 and $\dfrac{b}{\sin^2\theta}$, where $ab\neq 0$	DM1	Dependent on the preceding M1
	Obtain solution $x=\sqrt{\left(\text{cosec}^2\theta+12\right)}$	A1	or equivalent
		6	

Question	Answer	Marks	Guidance
6(i)	State correct expansion of $\sin(2x+x)$	B1	
	Use trig formulae and Pythagoras to express $\sin 3x$ in terms of $\sin x$	M1	
	Obtain a correct expression in any form	A1	e.g. $2\sin x(1-\sin^2 x)+\sin x(1-2\sin^2 x)$
	Obtain $\sin 3x \equiv 3\sin x - 4\sin^3 x$ correctly **AG**	A1	Accept $=$ for $\equiv$
		4	
6(ii)	Use identity, integrate and obtain $-\frac{3}{4}\cos x+\frac{1}{12}\cos 3x$	B1 B1	One mark for each term correct
	Use limits correctly in an integral of the form $a\cos x + b\cos 3x$, where $ab \neq 0$	M1	$\left(-\frac{3}{8}-\frac{1}{12}+\frac{3}{4}-\frac{1}{12}=-\frac{11}{24}+\frac{2}{3}\right)$
	Obtain answer $\frac{5}{24}$	A1	Must be exact. Accept simplified equivalent e.g. $\frac{15}{72}$ Answer only with no working is 0/4
		4	

Question	Answer	Marks	Guidance
7(i)	State at least one correct derivative	B1	$-2\sin\frac{1}{2}x$, $\frac{1}{(4-x)^2}$
	Equate product of derivatives to – 1	M1	or equivalent
	Obtain a correct equation, e.g. $2\sin\frac{1}{2}x=(4-x)^2$	A1	
	Rearrange correctly to obtain $a=4-\sqrt{2\sin\frac{a}{2}}$ **AG**	A1	
		4	
7(ii)	Calculate values of a relevant expression or pair of expressions at $a = 2$ and $a = 3$	M1	e.g. $\begin{matrix} a=2 & 2<2.7027.. \\ a=3 & 3>2.587.. \end{matrix}$ $\begin{pmatrix} 0.703 \\ -0.412 \end{pmatrix}$ $\begin{pmatrix} 2.317 \\ -0.995 \end{pmatrix}$ Values correct to at least 2 dp
	Complete the argument correctly with correct calculated values	A1	
		2	
7(iii)	Use the iterative formula $a_{n+1}=4-\sqrt{(2\sin\frac{1}{2}a_n)}$ correctly at least once	M1	
	Obtain final answer 2.611	A1	
	Show sufficient iterations to 5 d.p. to justify 2.611 to 3 d.p., or show there is a sign change in the interval (2.6105, 2.6115)	A1	2, 2.70272, 2.60285, 2.61152, 2.61070, 2.61077 2.5, 2.62233, 2.60969, 2.61087, 2.61076 3, 2.58756, 2.61301, 2.61056, 2.61079 Condone truncation. Accept more than 5 dp
		3	

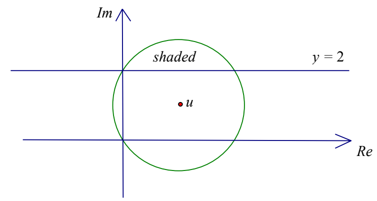
Question	Answer	Marks	Guidance
8(i)	State or imply the form $\frac{A}{2+x}+\frac{B}{3-x}+\frac{C}{(3-x)^2}$	**B1**	
	Use a correct method to obtain a constant	**M1**	
	Obtain one of $A=2$, $B=2$, $C=-7$	**A1**	
	Obtain a second value	**A1**	
	Obtain the third value	**A1**	[Mark the form $\frac{A}{2+x}+\frac{Dx+E}{(3-x)^2}$, where $A=2$, $D=-2$ and $E=-1$, B1M1A1A1A1.]
		5	
8(ii)	Use a correct method to find the first two terms of the expansion of $(2+x)^{-1}$, $(3-x)^{-1}$ or $(3-x)^{-2}$, or equivalent, e.g. $\left(1+\frac{1}{2}x\right)^{-1}$	**M1**	
	Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction	**A1** **A1** **A1**	**FT** on A, B and C $1-\frac{x}{2}+\frac{x^2}{4}$ $\frac{2}{3}\left(1+\frac{x}{3}+\frac{x^2}{9}\right)$ $-\frac{7}{9}\left(1+\frac{2x}{3}+\frac{3x^2}{9}\right)$
	Obtain final answer $\frac{8}{9}-\frac{43}{54}x+\frac{7}{108}x^2$	**A1**	
			For the A, D, E form of fractions give M1A1**ft**A1**ft** for the expanded partial fractions, then, if $D \neq 0$, M1 for multiplying out fully, and A1 for the final answer.
		5	

Question	Answer	Marks	Guidance
9(i)	Obtain a vector parallel to the plane, e.g. $\overrightarrow{CB} = 2\mathbf{i} + \mathbf{j}$	**B1**	
	Use scalar product to obtain an equation in a, b, c,	**M1**	e.g. $2a + b = 0$, $a + 5c = 0$, $a + b - 5c = 0$
	Obtain two correct equations in a, b, c	**A1**	
	Solve to obtain $a : b : c$,	**M1**	or equivalent
	Obtain $a : b : c = 5 : -10 : -1$,	**A1**	or equivalent
	Obtain equation $5x - 10y - z = -25$,	**A1**	or equivalent
	Alternative method 1		
	Obtain a vector parallel to the plane, e.g. $\overrightarrow{CD} = \mathbf{i} + 5\mathbf{k}$	**B1**	$\overrightarrow{BD} = -\mathbf{i} - \mathbf{j} + 5\mathbf{k}$
	Obtain a second such vector and calculate their vector product, e.g. $(2\mathbf{i} + \mathbf{j}) \times (\mathbf{i} + 5\mathbf{k})$	**M1**	
	Obtain two correct components	**A1**	
	Obtain correct answer, e.g. $5\mathbf{i} - 10\mathbf{j} - \mathbf{k}$	**A1**	
	Substitute to find d	**M1**	
	Obtain equation $5x - 10y - z = -25$,	**A1**	or equivalent

Question	Answer	Marks	Guidance
9(i)	**Alternative method 2**		
	Obtain a vector parallel to the plane, e.g. $\overrightarrow{DB} = \mathbf{i} + \mathbf{j} - 5\mathbf{k}$	**B1**	
	Obtain a second such vector and form correctly a 2-parameter equation for the plane	**M1**	
	State a correct equation, e.g. $\mathbf{r} = 3\mathbf{i} + 4\mathbf{j} + \lambda(\mathbf{i} + 5\mathbf{k}) + \mu(\mathbf{i} + \mathbf{j} - 5\mathbf{k})$	**A1**	
	State three equations in x, y, z, λ and μ	**A1**	
	Eliminate λ and μ	**M1**	
	Obtain equation $5x - 10y - z = -25$	**A1**	or equivalent
	Alternative method 3		
	Substitute for B and C and obtain $3a + 4b = d$ and $a + 3b = d$	**B1**	
	Substitute for D to obtain a third equation and eliminate one unknown (a, b, or d) entirely	**M1**	
	Obtain two correct equations in two unknowns, e.g. a, b, c	**A1**	
	Solve to obtain their ratio, e.g. $a : b : c$	**M1**	
	Obtain $a : b : c = 5 : -10 : -1$, $a : c : d = 5 : -1 : -25$, or $b : c : d = 10 : 1 : 25$	**A1**	or equivalent
	Obtain equation $5x - 10y - z = -25$	**A1**	or equivalent

Question	Answer	Marks	Guidance
9(i)	**Alternative method 4**		
	Substitute for B and C and obtain $3a + 4b = d$ and $a + 3b = d$	**B1**	
	Solve to obtain $a : b : d$	**M2**	or equivalent
	Obtain $a : b : d = 1 : -2 : -5$	**A1**	or equivalent
	Substitute for C to obtain c	**M1**	
	Obtain equation $5x - 10y - z = -25$	**A1**	or equivalent
		6	
9(ii)	State or imply a normal vector for the plane $OABC$ is **k**	**B1**	
	Carry out correct process for evaluating a scalar product of two relevant vectors, e.g. (5**i** – 10**j** – **k**).(**k**)	**M1**	i.e. correct process using **k** and their normal
	Using the correct process for calculating the moduli, divide the scalar product by the product of the moduli and evaluate the inverse cosine of the result	**M1**	Allow M1M1 for clear use of an incorrect vector that has been stated to be the normal to $OABC$
	Obtain answer 84.9° or 1.48 radians	**A1**	
		4	

Question	Answer	Marks	Guidance
10(i)	State or imply $r = 2$	**B1**	Accept $\sqrt{4}$
	State or imply $\theta = \frac{1}{6}\pi$	**B1**	
	Use a correct method for finding the modulus or the argument of u^4	**M1**	Allow correct answers from correct u with minimal working shown
	Obtain modulus 16	**A1**	
	Obtain argument $\frac{2}{3}\pi$	**A1**	Accept $16e^{i\frac{2\pi}{3}}$
		5	
10(ii)	Substitute u and carry out a correct method for finding u^3	**M1**	$\left(u^3 = 8i\right)$ Follow *their* u^3 if found in part **(i)**
	Verify u is a root of the given equation	**A1**	
	State that the other root is $\sqrt{3} - \mathrm{i}$	**B1**	
	Alternative method		
	State that the other root is $\sqrt{3} - \mathrm{i}$	**B1**	
	Form quadratic factor and divide cubic by quadratic	**M1**	$\left(z - \sqrt{3} - i\right)\left(z - \sqrt{3} + i\right)\left(= z^2 - 2\sqrt{3}z + 4\right)$
	Verify that remainder is zero and hence that u is a root of the given equation	**A1**	
		3	

Question	Answer	Marks	Guidance
10(iii)	Show the point representing u in a relatively correct position	**B1**	
	Show a circle with centre u and radius 2	**B1**	**FT** on the point representing u. Condone near miss of origin
	Show the line $y = 2$	**B1**	
	Shade the correct region	**B1**	
	Show that the line and circle intersect on $x = 0$	**B1**	Condone near miss
		5	