

CANDIDATE
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MATHEMATICS

9709/31

Paper 3 Pure Mathematics 3 (P3)

October/November 2017

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name in the spaces at the top of this page.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

DO NOT WRITE IN ANY BARCODES.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 75.

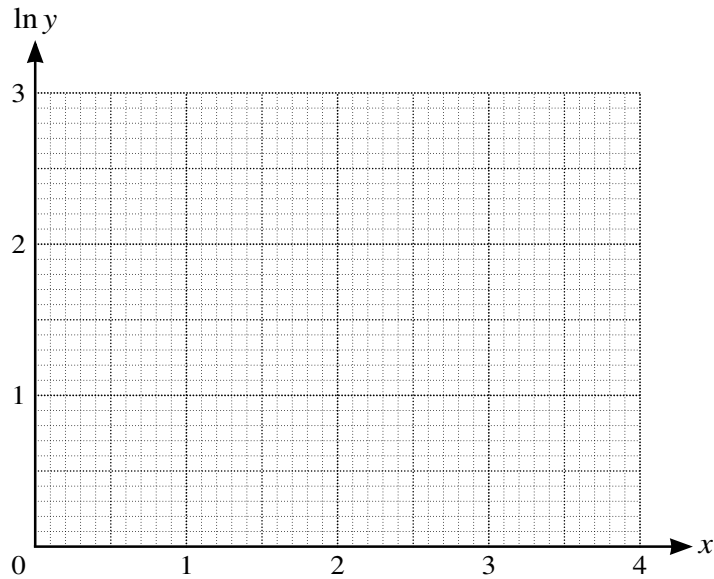
This document consists of **19** printed pages and **1** blank page.



- 2 Two variable quantities x and y are believed to satisfy an equation of the form $y = C(a^x)$, where C and a are constants. An experiment produced four pairs of values of x and y . The table below gives the corresponding values of x and $\ln y$.

x	0.9	1.6	2.4	3.2
$\ln y$	1.7	1.9	2.3	2.6

By plotting $\ln y$ against x for these four pairs of values and drawing a suitable straight line, estimate the values of C and a . Give your answers correct to 2 significant figures. [5]



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Two iterative formulae, A and B , derived from this equation are as follows:

$$x_{n+1} = (3x_n + 7)^{\frac{1}{3}}, \quad (A)$$

$$x_{n+1} = \frac{x_n^3 - 7}{3}. \quad (B)$$

Each formula is used with initial value $x_1 = 2.5$.

- (ii)** Show that one of these formulae produces a sequence which fails to converge, and use the other formula to calculate α correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [4]

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4 (i) Prove the identity $\tan(45^\circ + x) + \tan(45^\circ - x) \equiv 2 \sec 2x$. [4]

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(ii) Hence sketch the graph of $y = \tan(45^\circ + x) + \tan(45^\circ - x)$ for $0^\circ \leq x \leq 90^\circ$.

[3]

5 The equation of a curve is $2x^4 + xy^3 + y^4 = 10$.

(i) Show that $\frac{dy}{dx} = -\frac{8x^3 + y^3}{3xy^2 + 4y^3}$. [4]

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- (b) On an Argand diagram, shade the region whose points represent complex numbers satisfying both the inequalities $|z - 2 - i| \leq 2$ and $0 \leq \arg(z - i) \leq \frac{1}{4}\pi$. [4]

8 Let $f(x) = \frac{4x^2 + 9x - 8}{(x + 2)(2x - 1)}$.

(i) Express $f(x)$ in the form $A + \frac{B}{x + 2} + \frac{C}{2x - 1}$. [4]

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(ii) Hence show that $\int_1^4 f(x) dx = 6 + \frac{1}{2} \ln\left(\frac{16}{7}\right)$. [5]

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(ii) Show that the exact value of the area of R is $18 - \frac{42}{e}$.

[5]

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10 The equations of two lines l and m are $\mathbf{r} = 3\mathbf{i} - \mathbf{j} - 2\mathbf{k} + \lambda(-\mathbf{i} + \mathbf{j} + 4\mathbf{k})$ and $\mathbf{r} = 4\mathbf{i} + 4\mathbf{j} - 3\mathbf{k} + \mu(2\mathbf{i} + \mathbf{j} - 2\mathbf{k})$ respectively.

(i) Show that the lines do not intersect. [3]

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(ii) Calculate the acute angle between the directions of the lines. [3]

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