MATHEMATICS

Key messages

When answers are given candidates should ensure that their working fully justifies the given result. If they do not reach the given result they would be well advised to check back through their working rather than adjusting their answer to fit the given solution

General comments

The most successful candidates had a good understanding of algebraic processes and avoided becoming involved with incorrect or circuitous methods which use up valuable time.

Centres should make their students aware of the wealth of revision materials on the Cambridge International website in the form of the three time zone past-papers for March, June and November series.

Comments on specific questions

Question 1

For those candidates able to integrate the fractional indices this was a straightforward start to the paper. Most who reached this point appreciated the need to substitute $x = 4$ to find the gradient and then used an appropriate straight line equation with this gradient and the given point.

Answer: $y = 2(x - 4)$

Question 2

The application of differentiation in this question was not as obvious to candidates as that in question 1.
Of those who appreciated the values for f $'(x) = 0$ were required and deduced the range for f $'(x) > 0$ only a minority appreciated the link with the given domain and deduced that the negative limit for their range should be selected.

Answer: $a = -4/3$

Question 3

In both parts the appropriate formula were usually selected and quoted correctly.

In part **(a)** those who selected the sum to infinity formula correctly were often able to express their two sums in terms of a and r and equate these to eliminate a to find the value of r.

For part **(b)** the first terms and common differences of both series were seen to be used effectively to form an equation involving the two sums of n terms. Those who cancelled $n/2$ on both sides of their equation quickly reached the solution. Those who expanded both sides were generally less successful.

Answers: **(a)** $-2/7$ **(b)** 91

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Question 4

The substitution $h = 18 - r$ led quickly to the given result in part **(i)** however a lot of candidates did not see this.

In part **(ii)** the correct differentiation and solution of $dV/dr = 0$ were frequently followed by the calculation and correct use of the second derivative to reach the given description. Those who investigated the change in sign of dV/dr around the stationary value and gave sufficient detail of their method also gained full credit.

Most of the correct values of r in part **(ii)** led to a correct answer to part **(iii)**

Answers: **(ii)** $r = 12$ **(ii)** 288π

Question 5

Although the simplest method for part **(i)** using $\cos BAC = 8/10$ was often seen so were many more circuitous routes. There was no need to work in degrees and then convert to radians and occasionally this led to inaccuracy in the four decimal place result.

In part **(ii)** the candidates who saw that the required area was 2 × sector *ABE (*or *CBD)* – Area $\triangle ABC$ were more successful than those who saw the area as the sum of the areas of two segments and $\triangle BDE$. While the formula for the area of a sector was often quoted and used correctly the formula for the area of a segment was rarely used entirely correctly.

Answer: **(ii)** 16.4

Question 6

It was expected that the solution to part **(i)** would be obtained from the gradient and mid-point of *AB*, however, a number of solutions involved finding the gradient of the curve by differentiation and substituting the *x*-coordinate of the mid-point to find the gradient of *AB*. Although not a well-known property of the gradient of a chord and its parallel tangent, this was accepted. The gradient relationship of perpendicular lines was well understood. As the answer was given, the initial form of the straight line equation had to be rearranged to this answer for full marks.

Every attempt to answer part **(ii)** sensibly involved equating y or $6y$ rather than x. Those who equated y and involved fractional coefficients were less successful than those who equated $6y$ and obtained integer coefficients. The distance formula, when used, was nearly always correctly quoted. The use of rounded fractions made it very difficult to reach the required form.

Answers: **(ii)** $\sqrt{125/4}$

Question 7

In part **(a)** the value of a was often reached through comparison with $y = \sin x$ or the substitution of $(0,-2)$. Some went on to find b but for most the obvious methods of finding the amplitude or substituting $(\pi/2, 1)$ proved to be elusive.

The correct expansion of the brackets in part **(b)(i)** followed by elimination led to a 3 term equation which, using the trigonometric identity, could be expressed as the given result. Any other approach quickly ground to a halt. Management of the calculation on single lines of working was a feature of the better solutions.

Most attempts at part **(b)(ii)** used the given result from part **(ii)** and arrived at the correct values of $\cos\theta$. Few completely correct solutions were seen as the negative solution for θ was often omitted. As the range of θ was given in degrees it was expected that all solutions would be expressed in degrees.

Answers: **(a)** $a = -2$, $b = 3$ **(b)(ii)** $0°$, $\pm 109.5°$

Question 8

In part **(a)** the use of a sketch diagram showing PQR and O enabled some candidates to simply quote the required solution while others found $\overrightarrow{PQ}$, $\overrightarrow{QR}$ or $\overrightarrow{PR}$ and then worked out $\overrightarrow{OR}$. When no sketch was seen it was very rare to see a completed solution.

A correct equation in part **(b)** was seen more often from the magnitude than the perpendicular property. Those who appreciated the scalar product of the two vectors was zero made better progress than those who substituted into the formula for the scalar product and eventually noted $\cos\theta = 0$. When two equations were found the elimination of a (or b) was usually carried out successfully to form a quadratic equation. For the final mark it was necessary to pair the values of a and b correctly.

Answers: **(a)** $2q - p$ **(b)** $a = 9$, $b = -18$ *or* $a = -18$, $b = 9$

Question 9

Part **(i)** produced the most successful answers for this paper with many completely correct solutions. Occasionally un-simplified answers were seen which lost a mark.

The change of subject required in part **(ii)** was often completed correctly. Those who quoted the domain correctly often did so with no working but this still gained full credit. Those who linked the domain of f^{-1} to the range of f gained the available method mark even with an incorrect answer.

The candidates who were able to attempt parts **(i)** and **(ii)** were usually able to combine f and g in the correct order and rearrange to obtain and solve the required quadratic equation. The final mark was seldom gained as few candidates looked back to note the domain of both functions could not include negative values.

Answers: **(i)** $4x - 9$ **(ii)** $\sqrt{\dfrac{1}{x} + 9}$, $x > 0$ **(iii)** $x = \dfrac{7}{2}$

Question 10

Although the method for part **(i)** involved a straightforward integration the brackets distracted some candidates who attempted to integrate the function as though it was the $(ax + b)^n$ type. A negative or positive answer was acceptable from a correct method.

In part **(ii)** the candidates who realised that the function must be squared prior to integration generally produced complete solutions. Some attempts to square the function missed out the term in x^4.

The use of πx^2 in part **(iii)**, although appreciated by some candidates, was not always followed by a correct expression for x^2. The limits used in the first two parts were sometimes used again. However, some candidates produced completely correct methods to find the volume successfully.

Answers: **(i)** $\dfrac{2}{5}$ **(ii)** $\dfrac{8\pi}{45}$ **(iii)** $\dfrac{\pi}{3}$

MATHEMATICS

> **Paper 9709/12**
> **Paper 12**

Key messages

It is very important in 'show that' questions such as **Question 9(i)**, that all of the steps are clearly demonstrated and that the requested statement is shown and not what might be thought of as an equivalent statement.

General comments

The paper seemed to be well received by the candidates and many good and excellent scripts were seen. The paper seemed to work well with a number of questions being straightforward, including some of the longer questions towards the end of the paper, giving all candidates the opportunity to show what they had learned and understood but also some which provided more of a challenge. The vast majority of candidates appeared to have sufficient time to complete the paper and the standard of presentation was generally good with candidates setting their work out in a clear readable fashion in the spaces provided. A few candidates wrote their answers in pencil and then overwrote them in pen which made them very difficult to read.

Comments on specific questions

Question 1

The question proved to be a reasonably accessible start to the paper with many fully correct answers seen. Many candidates wrote out a full expansion whilst others, perhaps more sensibly, concentrated just on the required term. A significant number ignored the minus sign throughout. A much smaller number thought that $(\frac{1}{x^2})^3 = \frac{1}{x^5}$ and so were unable to find a term independent of x.

Answer: –84

Question 2

Many fully correct solutions were seen in part **(i)** with a variety of approaches taken to finding the point of intersection. Those equating the function to its inverse were usually as successful as those equating the function or the inverse to x. A significant number of candidates though, only found the x coordinate at the point of intersection. Drawing the required lines in part **(ii)** proved much more of a challenge for many. Some weaker candidates drew curves, others plotted points but not sufficiently accurately and many of these then failed to use a ruler and so straight lines were not obtained. It is worth students knowing that when asked to draw a sketch of a line then the points where it intersects the axes would normally be expected to be included and labelled. Most candidates were aware that the relationship between the function and its inverse is that of reflection in the line $y=x$ but many sketches failed to show this clearly, especially the point of intersection found in **(i).**

Answer: **(i)** $\dfrac{4-x}{5}, \left(\dfrac{2}{3}, \dfrac{2}{3}\right)$

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Question 3

This question proved to be more of a challenge for students than might have been expected. The importance of reading the question carefully and understanding the scenario cannot be over emphasised. In part **(i)** many thought that an arithmetic rather than geometric progression was required and some weaker candidates assumed the same amount of interest would be applied each year. Some candidates who realised that a geometric series was needed used 0.05 or 105 as the common ratio and obtained unrealistic answers. Many others had the wrong value of $n–1$ with 9,11 and 15 common answers, or used the sum formula instead of the nth term one. Some candidates failed to round their answer to the nearest 100. In part **(ii)** the majority of candidates decided to equate the given expression with the standard formula for the sum of an arithmetic progression but then often did not know how to proceed and wasted time trying to rearrange and solve the equation or made invalid assumptions such as $2a = 7$. Those who realised that the expression was an identity and therefore true for all values of n were usually successful. Equating coefficients or substituting two different values of n were equally successful approaches, but some assumed that the sum of the first two terms was actually the second term.

Answers: **(a)** 16300, **(b)** $a = 5$, $d = 3$

Question 4

The most important part of this question was finding the radius of the circle with centre B. This could be found after realising that the triangle BOC was a right angled isosceles one. Those candidates who realised this were often able to obtain full marks particularly in part **(i)** but those who didn't were unable to progress successfully with the question. Assuming that the radius was simply 6 was a common mistake among weaker candidates as was the use of 45° rather than $\dfrac{\pi}{4}$ radians in the formula $r\theta$. Similar errors were also seen in part **(ii)** as well as some loss of marks due to premature approximation.

Answers: **(i)** 6.66, **(ii)** 1.75

Question 5

The vast majority of candidates were able to make some progress in attempting to show the given result in part **(i)** but the use of $2x$ in the question proved a significant challenge, especially for weaker candidates. Those who replaced $2x$ with θ, solved the resulting simpler equation and then substituted back in for $2x$, were almost always successful and the use of this approach would perhaps benefit weaker candidates. Expressions such as $\dfrac{\sin^2}{\cos^2}2x$ and $\dfrac{\sin^2\theta}{\cos^2\theta}2x$ were quite common as was the omission of the cos from the $\cos2x$ part of the original expression. In part **(ii)** many candidates could obtain the 2 required values of $\cos2x$ with only the weakest ones then dividing by 2 before doing the inverse cosine of them. It was a little more common to see the value of -1 dismissed as having no solutions or the answer 120° not included.

Answer: **(ii)** 60°, 90°, 120°

Question 6

Many fully correct solutions were seen in part **(a)** of this question although in part **(ii)** a number of candidates were not in radian mode when their answers were evaluated. Part **(b)** in contrast proved to be perhaps the most challenging part of a question on the paper. Many candidates attempted to solve
$c + d\sin x = -4$ and $c + d\sin x = 10$ simultaneously or simply replaced x with -4 or 10. Those who used the fact that the maximum and minimum values of $\sin x$ are 1 and -1 were usually successful either by considering the range and amplitude or by solving the subsequent equations simultaneously.

Answers: **(a)(i)** $a = 5$, $b = -2$, **(ii)** 6.92, **(b)** $c = 3$, $d = \pm7$

Question7

Part **(i)** was found to be straightforward for many of the candidates and fully correct solutions were common. A significant number though failed to understand the situation described and perhaps the use of a sketch would have helped. A surprisingly large number of candidates simplified $\left(\dfrac{2+4}{2}\right)$ to be 4 and this shows that care needs to be taken with this type of calculation, especially if a calculator is used. In part **(ii)** many fully correct solutions were seen but algebraic mistakes with the discriminant were quite common as was the use of x in the final answer rather than m as requested.

Answers: **(i)** $\dfrac{7}{3}$, **(ii)** $-10 < m < 2$

Question 8

Many fully correct solutions were seen for this question which proved to be very accessible for all but the weakest of candidates. In part **(i)** the vast majority of candidates realised that no differentiation was required as $\dfrac{dy}{dx}$ was already given in the question. In part **(ii)** candidates seem well practised in the requirements for determining the nature of the stationary points. In part **(iii)** very many candidates knew that integration was required although weaker ones quite often used the equation of a straight line sometimes even after integration.

Answers: **(i)** $1, 4$, **(ii)** $-2x + 5, x = 1, minimum, x = 4, maximum,$ **(iii)** $y = -\dfrac{x^3}{3} + \dfrac{5x^2}{2} - 4x + 8$

Question 9

In part **(i)** almost all candidates realised that the scalar product was needed in order to confirm that the required angle was 90°. Weaker candidates often used the two given vectors and were therefore unable to make progress. A very common error was to simply assert that the scalar product was 0 rather than showing this clearly as the result of the 3 component products. Another very common mistake was to stop when the scalar product was shown to be 0 rather than going on to explain that this proved that the given angle was 90°. In part **(ii)** candidates were often much less certain of the required approach and there was confusion in some about the meaning of the term 'position vector'. Many candidates did not appreciate the fact that because **OA** was parallel to it, **CB** could be obtained by multiplying each component by 3 and instead attempted to work with the magnitudes of the vectors. In part **(iii)** some candidates seemed unaware of the formula for the area of a trapezium and also the need to use the magnitudes of the vectors. Those who split the trapezium into a triangle and a rectangle were usually equally successful.

Answer: **(ii)** $7\mathbf{j} + 4\mathbf{k}$, **(iii)** $6\sqrt{29}$

Question 10

This question proved to be popular with candidates and many fully correct solutions, particularly to part **(i)**, were seen. Almost all candidates realised that differentiation was required in order to find the gradient of the curve in part **(i).** This was usually carried out successfully with relatively few failing to multiply by 5. The great majority then worked out the negative reciprocal in order to find the gradient and then subsequently, the equation of the normal. In part **(ii)** most knew that integration would be needed but some used the formula for the area of a sector. A good number of candidates were confused over the required limits and others tried to work out the area as a single calculation rather than splitting it into the area under the curve and the area under the line. Most who did split it up realised that the area under the line was a triangle but those who integrated were usually equally successful.

Answers: **(i)** $y - 3 = -\dfrac{6}{5}(x - 2)$, **(ii)** 7.35

MATHEMATICS

Key message

In previous reports it has often been necessary to remind candidates to insert ± when taking a square root. It was good to see fewer occasions in this examination when the ± was forgotten. Sometimes, however, it becomes necessary to reject one or more solutions once the initial conditions have been re-examined and this was the case in both parts of **Question 6**. On the other hand, some candidates reject solutions which should be retained. This seems to happen most frequently with a solution of zero and it was not uncommon in both **Questions 2** and **5** to see a solution of zero obtained and then mistakenly rejected.

General comments

The paper was generally well received by candidates and many very good scripts were seen. Most candidates seemed to have sufficient time to finish the paper. It is pleasing to be able to report that candidates have again taken notice of comments in previous reports and, although there is still room for improvement, were prepared to show more working in their progress to the answer. It is also pleasing to report that candidates have become more aware of the amount of space allowed for their working and in the great majority of cases were able to fit their working into the given space without the need to resort to supplementary sheets.

Comments on specific questions

Question 1

Although there were a few cases of an incorrect formula being used, the vast majority of candidates were able to construct a correct quadratic equation and to write down the initial two solutions correctly. Some candidates employed the inequality > at an early stage, but most used = and attempted to adjust the solutions appropriately at the end. In this question it is necessary to consider the context carefully before giving the final answer. One root of the quadratic equation is negative (–29.2) and because the *least* possible value of n was required, some candidates chose this negative solution as the basis of their answer. A negative answer is not appropriate in the context of the question. The other root of the quadratic is 34.2 and the context requires an integer answer. A significant number of candidates corrected the solution to the nearest integer (34), not appreciating that this would mean that the sum of the terms would now be *less* than 3000.

Answer: 35

Question 2

This question was quite well done although a number of candidates did not realise they needed to multiply throughout by *x* in order to obtain a quadratic equation. Those that successfully negotiated this first step usually found the determinant correctly – although not always with the correct inequality. A common error at this stage was to lose the solution zero. A significant number of those candidates obtaining two solutions for *a* made mistakes in presenting the final answer with some having $0 < a < \frac{8}{9}$ and others translating $a(9a - 8) > 0$ into $a > 0$, $a > \frac{8}{9}$.

Answer: $a < 0$, $a > \frac{8}{9}$

Question 3

Part **(i)** was well done with most candidates scoring both marks. Those who scored only one mark invariably had lost the minus sign. Part **(ii)** was found more challenging with a minority of candidates not able to make headway and to distinguish which combinations of terms were required.. A common mistake following this was to equate the two terms rather than to add the terms and equate to zero.

Answers: **(i)** -4320 **(ii)** $a = 2$

Question 4

This question was generally well done with many candidates scoring full marks for the whole question. Most candidates realised they had to make the derivative negative and most were able to differentiate accurately. Some candidates lost a mark at the end by leaving their answer in terms of *x* when the question required the largest value of *k*.

Answer: Largest value of *k* is $\frac{5}{2}$

Question 5

Most candidates were successful in part **(i)** – some with elegant solutions achieved in a few lines of working others requiring rather more lines to achieve the required result. In part **(ii)** practically all candidates achieved the right solutions for cosθ but by no means all candidates obtained the 4 correct values of θ. Some lost 0^0, some gave answers to 3 significant figures rather than to 1 decimal place (required for answers in degrees) and some included, incorrectly, 180^0.

Answer: **(ii)** 0^0, 360^0, 143.1^0, 216.9^0

Question 6

The algebraic manipulation in part **(i)** was straightforward and most candidates achieved this comfortably. The majority of candidates, however, either forgot to insert ± or deliberately chose the positive root. A check of the domain of f(*x*) would have revealed that it was necessary to choose the negative value. In part **(ii)** practically all candidates found gf(*x*) correctly and equated this to 5. Some candidates made algebraic errors in simplifying this equation which should have yielded three roots. However, as in part **(i)**, the domain of f(*x*) is critical and only one of the roots satisfies this.

Answers: **(i)** $-\sqrt{\frac{2}{x} + 1}$ **(ii)** $-\sqrt{2}$

Question 7

It was intended that part **(i)** should provide a simple lead-in to the subsequent parts and the answer was given to facilitate this. It was surprising, therefore, that a significant number of candidates failed to show that angle $ABP = 0.6435$ radians. Part **(ii)** was done very well and most candidates were able to apply the formula for the area of a sector accurately. Part **(iii)** was more searching but the structure of the question was helpful to candidates and there are a number of different methods that can be used to reach the correct answer.

Answers: **(ii)** Sector $BAP = 8.04$, sector $DAQ = 7.07$ **(iii)** 6.11

Question 8

After the variable y was eliminated, candidates chose to use one of two main methods in order to tackle part **(i)**. One method is to use a dummy variable (e.g. u, where $u = \sqrt{x}$) to transform the equation to a quadratic equation in u. This can then easily be solved for u and the solutions squared to find solutions for x. Some candidates replaced x by x^2, but this usually led to confusion and candidates usually forgot to square the solutions obtained. The other method is to isolate the term $3\sqrt{x}$ on one side of the equation and then square both sides of the equation. A common error was to square individual terms. In part **(ii)**, there were many well-presented and correct answers with integration usually accurately done. The area under the straight line was more often done by integration rather than finding the area of the trapezium. Sometimes the arithmetic of substituting the limits was not shown clearly. A significant number of candidates spent unnecessary time in also finding the y-coordinates, which were not required.

Answers: **(i)** $x = \dfrac{1}{4}$, 1 **(ii)** $\dfrac{1}{16}$

Question 9

Part **(i)** was done well with many candidates scoring full marks; only a very few were unfamiliar with calculating the magnitude of a vector. There were, inevitably, a few slips in the arithmetic, particularly with subtraction of negative values. Also, it cannot be emphasised enough how important it is for candidates to take care in transcribing vectors accurately from the question paper to their working. With magnitudes of vectors, candidates should note that if the final answer is $\sqrt{n}$ we would expect this to be evaluated exactly if this is a square number as was the case here. Leaving answers as $\sqrt{729}$ or $\sqrt{324}$ is not acceptable whilst an answer such as $\sqrt{260}$, for example, is permitted. In contrast to part **(i)**, part **(ii)** was not done well, what most candidates did not appreciate was that the ratio $\dfrac{2}{3}$ of $|CD| : |BC|$, which most candidates found in part **(i)**, applied in part **(ii)** also, and was in fact intended as a helpful lead-in to part **(ii)**. Instead, candidates resorted to an algebraic approach in order to find this ratio, with most losing their way in a mass of algebra. A major weakness seen quite often was the failure to appreciate that the position of the origin was not specified in the question; many attempts prematurely came to nothing on the assumption that B, C, D <u>and the origin</u> were collinear. In addition, candidates should be aware that solutions which involve unlabelled numerical vectors are difficult for both candidate and Examiner to follow. Some candidates were clearly confused by the position vector of D and were happy to arrive at **CD** as their answer rather than **OD**. Others were unclear as to the nature of the second possible position and gave only one solution. There were, however, some excellent all-correct solutions and many of these were both succinct and explained well, although a few spoilt otherwise perfect answers with occasional arithmetic lapses.

Answers: **(i)** $|\mathbf{AB}| = 27$, $|\mathbf{BC}| = 18$, $|\mathbf{CD}| = 12$ **(ii)** $\mathbf{OD} = \begin{pmatrix} 10 \\ -7 \\ 7 \end{pmatrix} \begin{pmatrix} -6 \\ 1 \\ -9 \end{pmatrix}$

Question 10

Part **(i)** was done quite well with most candidates understanding what was required. Some candidates did not simplify the expression for the second derivative $\left[2a\left(-\dfrac{b}{a} \right) + b \right]$ before concluding that it was negative and that the stationary point was a maximum. In part **(ii)**, having obtained the equation $a + b = 9$, many candidates did not appreciate that f $'(-2) = 0$ and therefore did not obtain the second equation in a and b and were hence unable to find the values of a and b. These candidates were able to integrate $ax^2 + bx$ for a mark but without the values of a and b could not evaluate the constant of integration (where they had remembered it) and were unable to make further progress. Many candidates did, however, produce fully correct solutions.

Answers: **(i)** $x = -\dfrac{b}{a}$, Maximum **(ii)** $f(x) = x^3 + 3x^2 - 7$

Question 11

Almost all candidates answered part **(i)** successfully. Whilst there is an alternative method for part **(ii)**, almost all candidates chose to equate the derivative to $\frac{1}{2}$, solve for x, find the corresponding value for y and hence find the equation of the line through this point with gradient $\frac{1}{2}$. A large proportion of candidates achieved the correct answer for this part. Part **(iii)** was considerably more challenging and a variety of different correct methods were seen. By far the most popular approach was to realise that the gradient of the perpendicular to the two lines was -2 and then to choose either of the points $(0, 0)$ or $(2, 1)$ on the tangent, or any of the points $(1, 0)$, $(3,1)$ or $(5,2)$ on the line *AB*. The equation of a perpendicular through one of these points was then derived and the coordinates of the point of intersection with the other line found. It was then a simple matter to find the distance between the two points. The use of trigonometry was less popular but equally successful.

Answers: **(i)** $y = \dfrac{1}{2}x - \dfrac{1}{2}$ **(ii)** $y = \dfrac{1}{2}x$ **(iii)** 0.45

MATHEMATICS

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| **Paper 9709/21** |
| **Paper 21** |
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Key messages

Candidates are reminded of the importance of reading each question carefully to ensure that they fully understand the demands of the question. Of equal importance is the need to ensure that they give their solutions in the required form, ensuring that the correct level of accuracy has been used.

General comments

It was evident that some candidates were not fully prepared for the examination. It is clear that the topic of logarithms is one which many candidates have difficulty with as evidenced by their performances in **Questions 1**, **3** and **4(b)**. There were many cases when the actual process of both algebraic and arithmetic simplification was poor leading to an unnecessary loss of marks. Candidates should also check that their calculators are in the correct mode when dealing with questions involving trigonometry and calculus.

Comments on specific questions

Question 1

It was evident that many candidates were not familiar with the rules governing logarithmic manipulation. Very often the left hand side of the given equation was split into 4 separate logarithms; this approach yielded no marks at all. Candidates were also unable to progress far if they did not recognise that 1 could be written as $\ln \mathrm{e}$. Algebraic manipulation was also poor, with some candidates obtaining e in terms of *x*.

Answer: $x = \dfrac{2\mathrm{e} - 1}{3 - \mathrm{e}}$

Question 2

Many candidates performed well, recognising the need for the use of the double angle formula which then resulted in a simple cubic equation in $\cos\theta$. The only thing that stopped many candidates obtaining full marks was that solutions in each quadrant were given.

Answer: 42.5°, 317.5°

Question 3

Most candidates recognised that they need to solve each of the given inequalities to start with. The inequality involving the two moduli was done a lot better than the inequality in index form. Most candidates were able to obtain the critical value of 1.83 from using moduli correctly, with fewer candidates obtaining the critical value 8.35 from the use of logarithms. Provided the correct critical values were obtained, most candidates were able to provide a correct solution. Even though candidates were asked to give their values of *a* and of *b* correct to 3 significant figures (both in the question and in the rubric on the first page), there were some which failed to do so, thus losing any associated accuracy marks.

Answer: 1.83 < x < 8.35

Question 4

(a) Unless candidates recognised the need to use the identity $1 - \sin^2\theta \equiv \cos\theta$ in order to simplify the denominator of the given trigonometric fraction, no progress could be made. As a result of this, many solutions were completely incorrect. It was assumed that candidates would simplify the denominator and rewrite the fraction as $4\sec^2\theta + \tan\theta$ and then make use of the appropriate trigonometric identity to rewrite this as $5\sec^2\theta - 1$, in preparation for integration.

(b) Provided candidates recognised that the integral was a logarithmic function, then often some progress could be made. Very often the subsequent manipulation of the logarithms was done incorrectly, leaving few candidates with correct solutions. For those candidates who did not recognise the integral as a logarithm, then no marks were available.

Answer: **(a)** $5\tan\theta - \theta + c$ **(b)** 21

Question 5

(i) Many candidates were able to provide a completely correct solution for this question. There were, however, examples of poor simplification of 2 correct equations which lead to incorrect solutions. Most candidates were able to use the factor theorem correctly, but surprisingly, fewer appeared to be familiar with the remainder theorem. Attempts involving algebraic long division were seldom successful mainly due to poor algebraic simplification. It was intended that candidates make use of both the factor and remainder theorem in this part of the question.

(ii) Most candidates attempted to find a quadratic factor by either algebraic long division, synthetic division or just by observation. Occasionally once a correct quadratic factor was obtained, further factorisation was not attempted. The inclusion of the word 'completely' in the question was intended to help candidates recognise that linear factors were intended. The use of synthetic division is acceptable provided candidates recognise that it has its limitations. In this question, it failed to yield the '12' with many candidates giving incorrect factors of $(x + 2)\left(x + \dfrac{5}{2}\right)\left(x + \dfrac{1}{6}\right)$, a result which was often seen from candidates who had attempted to use their calculator to solve the equation and obtain factors from the roots. Candidates should always check in questions of this type, that their factors do give the original equation when multiplied out.

For those candidates who did not have the correct values from part **(i)** a method mark was available for a correct procedure.

Answer: **(i)** $a = 12$, $b = 40$ **(ii)** $(x + 2)\,(2x + 5)\,(6x + 1)$

Question 6

(i) Of the candidates that attempted this question, most were able to find $\dfrac{dx}{dt}$ correctly. However when it came to finding $\dfrac{dy}{dt}$, many failed to recognise that they needed to differentiate a product. Most realised that they had to equate their $\dfrac{dy}{dx}$ to zero in order to find the value of the parameter at this point, but often failed to carry on to find the coordinates as requested. Hence the need to read the question carefully and ensure that all the demands have been met.

(ii) Many candidates did not understand what was required of them. The intention was for a candidates to find the value of the parameter at the point where the curve crosses the *x*-axis, substitute this value into their derivative and than use the property of perpendicular lines. Only the very well prepared candidates made progress with this part of the question.

Answer: **(i)** $(3.16, -0.92)$ **(ii)** $-\dfrac{8}{5}$

Question 7

(i) Again, many candidates did not understand the question or misunderstood what they were supposed to be doing. As a result, few correct solutions were seen. Many candidates obtained a correct first derivative and found its value when $x = 0$. This gave them a value for m, but most did not realise that that had to equate $-m$ with their original first derivative in order to obtain the given equation.

(ii) This part was rarely attempted and it was even rarer to see a correct attempt and justification. It was expected that candidates form a function, for example, $p(x) = a \sin \dfrac{x}{2} + b - x$, using their values of a and b. By substituting $x = -4.5$ into this function and then $x = -4$ it can be seen that a change of sign occurs and hence the correct conclusion may be made. It did not help some candidates that their calculator was in degree mode rather than radian mode. Any question involving trigonometry and calculus implies, for this paper, that angles are in radians.

(iii) For those candidates that attempted to use an iterative formula, problems occurred if their calculator was in the incorrect mode – an all too often occurrence. Many used an incorrect equation, not making use of the equation found in part **(i)** correctly.

Answer: **(i)** $x = \dfrac{5}{4} \sin \dfrac{x}{2} - 3$ **(iii)** -4.11

MATHEMATICS

| Paper 9709/22 |
| Paper 22 |

Key messages

Candidates are reminded of the necessity of ensuring that a question has been answered in full. They should also ensure that working is done to the required level of accuracy (at least 4 significant figures) and that final answers should be to 3 significant figures unless otherwise requested. Calculators are a useful tool but must not replace working which should be shown, particularly when either an exact answer is needed or a question uses the word 'hence' which requires use of work from a previous part of the question.

General comments

Most candidates appeared to have sufficient time to attempt all eight questions with varying degrees of success. In most cases candidates had sufficient room on their examination paper to write their solutions. Those candidates that needed extra room made use of the blank page in the booklet. Having a given space for candidates to write their solutions appeared to make candidates more conscious of the need to set their work out in a clear and logical fashion.

Comments on specific questions

Question 1

Most candidates chose to square both sides of the modulus equation. Problems occurred with the resulting quadratic equation which although in terms of x also contained terms in a. Many candidates appeared to be unable to solve quadratic equations of this type, often making errors with the terms involving a, or using completely spurious methods which resulted in solutions in terms of both a and x.

Candidates dealing with two separate linear equations obtained from the modulus equation usually had more success with obtaining the correct solutions.

Answer: $6a$, $\dfrac{4a}{3}$

Question 2

An application of logarithms to each side of the given equation, together with the application of the power law for logarithms meant that most candidates were able to obtain the first two marks. Some candidates were careless with the use of brackets, writing $x + 4\log 3 = 2x\log 5$, rather than the required $(x+4)\log 3 = 2x\log 5$. This then caused problems with the simplification to obtain a value for x. Use of logarithms to base 10 was just as common as logarithms to base e, with some candidates making use of either base 3 or base 5 logarithms which was equally successful.

Answer: 2.07

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Question 3

(i) Too many candidates were unable to sketch the graph of $y = x^3$ and were thus unable to gain marks for this part of the question. Of those that did sketch the graph of $y = x^3$, successfully, most sketched a straight line in the correct position to show the graph of $y = 11 - 2x$. It was expected that candidates make a comment about there being only one point of intersection, thus implying only one solution of the given equation. Many candidates lost the final mark as no appropriate comment was made.

(ii) Questions involving the iteration process are usually done successfully by the majority of candidates and this was no exception. Most chose a sensible starting point for their iterations and carried out the appropriate number of iterations, usually making use of the 'Answer' function on their calculator. There are still some candidates that do not make use of their calculator in this way, which tends to lead to more errors being made not to mention the extra time needed for the process. Errors tended to be usually when candidates did not give their final answer to 4 significant figures and occasionally when the iterations were not given to the required level of accuracy.

Answer: **(ii)** 1.926

Question 4

Most candidates realised that they needed to use the quotient rule to differentiate the given equation. This was usually done successfully, with the occasional error, the most common being $\frac{d}{dx}\left(e^{4x}\right) = e^{4x}$. With an expression for $\frac{dy}{dx}$, most candidates made the substitution of zero to obtain a gradient which was then used to form an equation of the tangent at the point where $\left(0, \frac{1}{3}\right)$. This question was a prime example of candidates not ensuring that they had completed the full demands of the question, with many not giving their final answer in the required form, thus losing the final mark.

Answer: $10x - 9y + 3 = 0$

Question 5

This was the question that was probably one of the most demanding on the paper. Most candidates were able to make the statement $\ln y = \ln K - 2x \ln a$ and also find the gradient of the straight line correctly. Two approaches were possible with most candidates opting to use the gradient in an attempt to find the value of *a*. Unfortunately, many candidates equated the gradient to $\ln a$ rather than to $-2\ln a$. Errors in substituting a value for *a*, in order to find *K*, were often made with logarithms being used inappropriately in the majority of incorrect cases. Some candidates chose to use the coordinates given and the equation $\ln y = \ln K - 2x \ln a$ in order to solve a pair of simultaneous equations. Although not as common as the first approach, this method tended to be more successful, provided the coordinates were used correctly, as the error involving the gradient in the first approach was not a problem. Too many candidates did not work to the required level of accuracy throughout the question and often values for *K* were inaccurate even though a correct approach had been used.

Answer: $a = 1.3$, $K = 8.4$

Question 6

(i) The great majority of candidates were able to score at least 4 marks for this part of the question. The factor theorem was usually applied in one form or another and most candidates obtained the quotient $6x^2 + x - 35$, usually by algebraic long division, but occasionally by observation. It should be noted that synthetic division is not regarded as a valid method for showing $(x + 2)$ is a factor of the given cubic equation. Many candidates thought that the final answer was $(x + 2)(6x^2 + x - 35)$, not recognising that the quadratic quotient could be factorised. It was also obvious that some candidates had made use of the equation solver on their calculator and work 'backwards' to the

factors $(x+2)\left(x+\dfrac{5}{2}\right)\left(x-\dfrac{7}{3}\right)$ which are not the factors of the given cubic equation. Candidates should be judicious about the use of their calculators.

(ii) Candidates should take note of the word 'Deduce', which implies, together with the mark allocation of 2 marks, that the solutions are obtained by using the preceding information. Many candidates chose to start the question again, usually with little success. However there were a significant number of candidates who did recognise the link between the 2 equations and give the correct solutions.

Answer: **(i)** $(x+2)(2x+5)(3x-7)$ **(ii)** $-\dfrac{1}{2},\ -\dfrac{2}{5},\ \dfrac{3}{7}$

Question 7

(a) Most candidates realised that they needed to expand the brackets before making an attempt at integration. Unfortunately, it was evident that some candidates thought that integrating each term in the brackets was the way forward and thus failed to obtain any marks for this part of the question. Fewer candidates realised that the use of the appropriate double angle formula was needed before attempting integration. Of those candidates that did make use of the correct approach, many then 'forgot' that the variable involved was θ and integrated the constant term as $2x$ rather than 2θ.

(b)(i) Most solutions were in terms of logarithms with errors usually being in the multiples of the logarithmic functions.

(ii) Many candidates applied the limits to their integral in terms of logarithms correctly. However it was evident that many were unsure of how to simplify their logarithms to a single logarithm.

Answer: **(a)** $\dfrac{1}{2}\sin 2\theta - \sin\theta - 2\theta + c$ **(b)(i)** $2\ln(2x+1) + \dfrac{1}{2}\ln 2x$ or $2\ln(2x+1) + \dfrac{1}{2}\ln x$ **(ii)** $\ln 18$

Question 8

(i) Most candidates were able to find $\dfrac{dx}{dt}$, but very few candidates were able to find $\dfrac{dy}{dt}$ correctly, being unable to differentiate the powers of the trigonometric functions. Most realised that

$\dfrac{dy}{dx} = \dfrac{dy}{dt}\bigg/ \dfrac{dx}{dt}$ and that use of the double angle formula for $\sin 2t$ was needed, but were unable to

obtain the given result. Candidates should be aware that when an answer is given, it is inadvisable to try to contrive the given result from clearly incorrect work. By doing this, they are sometimes failing to obtain method marks that they would otherwise have obtained.

(ii) Usually done well, with most candidates making use of the given result from part **(i)**. The great majority of candidates obtained $\tan t = \dfrac{3}{2}$ but often made errors in calculating the cartesian coordinates.

(iii) Many were able to identify the correct value of the parameter and go on to attempt the gradient of the normal. Many failed to realise the importance of the word 'exact' and resorted to using their calculator, thus failing to gain the last 2 marks. Some candidates also gave the value of the grsdient of the tangent rather than the normal.

Answer: **(ii)** $(2.38, 2.66)$ **(iii)** $\dfrac{4\sqrt{2}}{3}$

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MATHEMATICS

+---------------------------+
| **Paper 9709/23** |
| **Paper 23** |
+---------------------------+

Key messages

Candidates are reminded of the importance of reading each question carefully to ensure that they fully understand the demands of the question. Of equal importance is the need to ensure that they give their solutions in the required form, ensuring that the correct level of accuracy has been used.

General comments

It was evident that some candidates were not fully prepared for the examination. It is clear that the topic of logarithms is one which many candidates have difficulty with as evidenced by their performances in **Questions 1**, **3** and **4(b)**. There were many cases when the actual process of both algebraic and arithmetic simplification was poor leading to an unnecessary loss of marks. Candidates should also check that their calculators are in the correct mode when dealing with questions involving trigonometry and calculus.

Comments on specific questions

Question 1

It was evident that many candidates were not familiar with the rules governing logarithmic manipulation. Very often the left hand side of the given equation was split into 4 separate logarithms; this approach yielded no marks at all. Candidates were also unable to progress far if they did not recognise that 1 could be written as $\ln e$. Algebraic manipulation was also poor, with some candidates obtaining e in terms of x.

Answer: $x = \dfrac{2e - 1}{3 - e}$

Question 2

Many candidates performed well, recognising the need for the use of the double angle formula which then resulted in a simple cubic equation in $\cos \theta$. The only thing that stopped many candidates obtaining full marks was that solutions in each quadrant were given.

Answer: 42.5°, 317.5°

Question 3

Most candidates recognised that they need to solve each of the given inequalities to start with. The inequality involving the two moduli was done a lot better than the inequality in index form. Most candidates were able to obtain the critical value of 1.83 from using moduli correctly, with fewer candidates obtaining the critical value 8.35 from the use of logarithms. Provided the correct critical values were obtained, most candidates were able to provide a correct solution. Even though candidates were asked to give their values of *a* and of *b* correct to 3 significant figures (both in the question and in the rubric on the first page), there were some which failed to do so, thus losing any associated accuracy marks.

Answer: 1.83 < x < 8.35

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Question 4

(a) Unless candidates recognised the need to use the identity $1 - \sin^2 \theta \equiv \cos \theta$ in order to simplify the denominator of the given trigonometric fraction, no progress could be made. As a result of this, many solutions were completely incorrect. It was assumed that candidates would simplify the denominator and rewrite the fraction as $4\sec^2 \theta + \tan \theta$ and then make use of the appropriate trigonometric identity to rewrite this as $5\sec^2 \theta - 1$, in preparation for integration.

(b) Provided candidates recognised that the integral was a logarithmic function, then often some progress could be made. Very often the subsequent manipulation of the logarithms was done incorrectly, leaving few candidates with correct solutions. For those candidates who did not recognise the integral as a logarithm, then no marks were available.

Answer: **(a)** $5 \tan \theta - \theta + c$ **(b)** 21

Question 5

(i) Many candidates were able to provide a completely correct solution for this question. There were, however, examples of poor simplification of 2 correct equations which lead to incorrect solutions. Most candidates were able to use the factor theorem correctly, but surprisingly, fewer appeared to be familiar with the remainder theorem. Attempts involving algebraic long division were seldom successful mainly due to poor algebraic simplification. It was intended that candidates make use of both the factor and remainder theorem in this part of the question.

(ii) Most candidates attempted to find a quadratic factor by either algebraic long division, synthetic division or just by observation. Occasionally once a correct quadratic factor was obtained, further factorisation was not attempted. The inclusion of the word 'completely' in the question was intended to help candidates recognise that linear factors were intended. The use of synthetic division is acceptable provided candidates recognise that it has its limitations. In this question, it failed to yield the '12' with many candidates giving incorrect factors of $\left(x + 2\right)\left(x + \dfrac{5}{2}\right)\left(x + \dfrac{1}{6}\right)$, a result which was often seen from candidates who had attempted to use their calculator to solve the equation and obtain factors from the roots. Candidates should always check in questions of this type, that their factors do give the original equation when multiplied out.

For those candidates who did not have the correct values from part **(i)** a method mark was available for a correct procedure.

Answer: **(i)** $a = 12$, $b = 40$ **(ii)** $(x + 2)(2x + 5)(6x + 1)$

Question 6

(i) Of the candidates that attempted this question, most were able to find $\dfrac{dx}{dt}$ correctly. However when it came to finding $\dfrac{dy}{dt}$, many failed to recognise that they needed to differentiate a product. Most realised that they had to equate their $\dfrac{dy}{dx}$ to zero in order to find the value of the parameter at this point, but often failed to carry on to find the coordinates as requested. Hence the need to read the question carefully and ensure that all the demands have been met.

(ii) Many candidates did not understand what was required of them. The intention was for a candidates to find the value of the parameter at the point where the curve crosses the *x*-axis, substitute this value into their derivative and than use the property of perpendicular lines. Only the very well prepared candidates made progress with this part of the question.

Answer: **(i)** $(3.16, -0.92)$ **(ii)** $-\dfrac{8}{5}$

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Question 7

(i) Again, many candidates did not understand the question or misunderstood what they were supposed to be doing. As a result, few correct solutions were seen. Many candidates obtained a correct first derivative and found its value when $x = 0$. This gave them a value for m, but most did not realise that that had to equate $-m$ with their original first derivative in order to obtain the given equation.

(ii) This part was rarely attempted and it was even rarer to see a correct attempt and justification. It was expected that candidates form a function, for example, $p(x) = a\sin\dfrac{x}{2} + b - x$, using their values of a and b. By substituting $x = -4.5$ into this function and then $x = -4$ it can be seen that a change of sign occurs and hence the correct conclusion may be made. It did not help some candidates that their calculator was in degree mode rather than radian mode. Any question involving trigonometry and calculus implies, for this paper, that angles are in radians.

(iii) For those candidates that attempted to use an iterative formula, problems occurred if their calculator was in the incorrect mode – an all too often occurrence. Many used an incorrect equation, not making use of the equation found in part **(i)** correctly.

Answer: **(i)** $x = \dfrac{5}{4}\sin\dfrac{x}{2} - 3$ **(iii)** -4.11

MATHEMATICS

Key messages

There are some extremely important points that candidates need to address:

(i)   They must use the method asked for in the question (**Questions 2 and 3**). Those in **Question 2** who ignored the requested approach and used the given equation, together with values from their table/graph often couldn't complete the question or had arithmetical or algebraic errors. In **Question 3** some candidates did not use the iterative approach asked for.

(ii) They must note carefully what form the final answer should take (**Questions 2**, **7(a) and 9(i)**). **Question 2** requests answers to 2 s.f. In **Question 7(a)** answers containing √25 or √9 are not acceptable. In **Question 9(i)** answers where integer calculation is obvious are not allowed, for example those of the form $(1/2)(4 \pm \sqrt{12})$.

General comments

The standard of work on this paper varied considerably although all questions were accessible to well-prepared candidates. The questions or parts of questions that were generally done well were **Questions 3(i)** and **(ii)** (numerical iteration), **Question 5(i)** (implicit differentiation), **Question 7(a)** (complex numbers), **Questions 8(i)** and **(ii)** (partial fractions), **Question 9(i)** (product rule) and **Questions 10(i)**, **(ii)**, **(iii)** (vectors, line and plane)). Those that were done less well were **Question 1** (polynomial division), **Question 2** (logarithms), **Question 4(ii)** (graph of 2sec2x), **Question 5(ii)** (tangent to curve parallel to x axis), **Question 6** (differential equation), **Question 7(b)** (Argand diagram) and **Question 9(ii)** (integration by parts).

In general the presentation of the work was good, though there were some rather untidy scripts. Candidates should bear in mind that scripts will be scanned for marking and they should use a **black** pen, reasonable sized lettering and symbols and present their work clearly.

It was pleasing to see that candidates are aware of the need to show sufficient working in their solutions. Previous reports mentioned this in the context of solving a quadratic equation and substituting limits into an integral. These points will be discussed in detail below.

Where answers are given after the comments on individual questions, it should be understood that the form given is not necessarily the only 'correct answer'.

Comments on specific questions

Question 1

This question was generally not well done. Weaker candidates stopped after reaching a quotient of x^2 or even attempted to divide the wrong way round. Many candidates made careless errors with the signs, in particular when subtracting negative numbers.

Answer: Quotient $x^2 - 2x + 5$ Remainder $-12x + 5$

Question 2

This was another question that many candidates found difficult. The most successful candidates answered the question by obtaining a correct equation connecting ln y with ln C and ln a and then using their straight line on the graph to evaluate these quantities. A poor choice for their line did cost them accuracy marks later. Those who ignored the requested approach and used the given equation, together with values from their table/graph often couldn't complete the question or had arithmetical or algebraic errors. Many candidates ignored the request for answers to 2 s.f.

Answer: $C = 3.7$, $a = 1.5$

Question 3

(i)   In general this part was well done, although there were some inaccuracies. A few candidates failed to complete the argument correctly with correct calculated values.

(ii)   Many successful attempts were seen, although some candidates did not use the iterative approach asked for in the question. Instead they attempted some algebra to show divergence and received no credit for this. Some candidates forgot to round their final answer for α to 2 d.p. Candidates who used an initial value other than $x_1 = 2.5$, were deemed to have misread the question. In some cases, this led fortuitously to convergence as there is no other root nearby. Clearly this will not always be the case.

Answer: **(ii)** *A* 2.43 convergent sequence
            B 2.8750, 5.5879 divergent sequence

Question 4

(i)   Most candidates gained the first two marks. A large proportion of candidates found it difficult to progress beyond the first three marks since they did not recognise that $\sec^2 x$ could be expressed as $1/\cos^2 x$.

(ii)   Some candidates omitted this question. Even though the question said "hence", there were some candidates who tried to use the graphs of $\tan(45 + x)$ and $\tan(45 - x)$ to sketch $2\sec 2x$. Other candidates started with $\cos 2x$ but failed to introduce the vertical stretch, or omitted the asymptote, or they worked in radians rather than degrees. Some candidates plotted values, but they still needed to display the asymptotic behaviour around $x = 45°$.

Question 5

(i)   Some good work was seen, however candidates needed to show that the derivative of 10 is zero. Too often the zero was omitted so the candidate did not produce a correct equation to rearrange. Since the answer was given, every line of detail was required. It was also necessary to show clear bracketing of the two terms in dy/dx.

(ii)   Some candidates were unsure whether to set to zero the numerator or the denominator of the right hand side of the answer given in **(i)**. Sign errors were common in $8x^3 + y^3 = 0$. Many candidates could not find the cube root of $-8x^3$ (or a similar expression if working in terms of y). Unfortunately a few of the candidates who succeeded in finding the correct equation failed to realise that there were two solutions.

Answer: (1, −2), (−1, 2)

Question 6

Almost all candidates knew that they had to separate variables and could do that correctly. However, some could not integrate one or both sides. Those who successfully integrated both sides usually went on to produce an excellent solution. A common error was failing to write $1/\cos^2 y$ as $\sec^2 y$, or not realising it integrated to $\tan y$. A significant number of candidates thought that the integral of $\tan x$ was $\sec^2 x$. In many cases, the 4 moved from numerator to denominator or vice versa.

Answer: $\tan y = 4\ln \sec x + 1$, 0.587

Question 7

(a) Some candidates made little progress as they tried to square root $u = 8 - 15i$. This is a legitimate approach but it requires a polar form of the complex number and it is extremely unlikely to lead to an exact answer. To obtain an exact answer requires the square root of u to be set to $a + ib$, as stated in the question, and then squared and equated to the numerical expression $8 - 15i$. Many candidates were able to do this successfully, with the correct signs, reaching a quadratic equation in either a^2 or b^2. Unfortunately some candidates forgot to square root to find a and b, while others failed to simplify their answer, leaving $\sqrt{25}$ or $\sqrt{9}$ in it. Simplification of final answers will also be discussed in **Question 9(i)**.

(b) The circle was more usually correct than the line. Most candidates had a circle of radius of 2, but often the centre was incorrect. The line was often drawn through the origin or the point $-i$ or, even if both circle and line were correct, the candidate's shading often extended over all the circle below the line.

Answer. **(a)** $\pm (1/\sqrt{2}) (5 - 3i)$

Question 8

(i) This was extremely well done and many candidates obtained correct values for all three coefficients. However, a considerable number of fortuitously correct answers were seen from candidates who, instead of multiplying the given form of f(x) by $(x + 2)(2x - 1)$ and equating coefficients, simply divided the given f(x) out and obtained $2 + (3x - 4)/((x + 2)(2x - 1))$. This correctly gave them $A = 2$. They then produced an incorrect expression such as

$(3x - 4)/((x + 2)(2x - 1)) = 2 + B/(x + 2) + C/(2x - 1)$
or $(4x^2 + 9x - 8x)/((x + 2)(2x - 1)) = B/(x + 2) + C/(2x - 1)$.

However, by using the 'cover up rule' the correct values of B and C are obtained.

(ii) There were some errors in copying the result of **(i)** to use in **(ii)**, in integrating $1/(2x - 1)$ and substituting in the limits. A surprising number of candidates did not seem to know how to apply the laws of logarithms correctly or to realise they should use them. They needed to show all the steps in the working to obtain the given answer, for example $2\ln6 - 2\ln3 = 2\ln2 = \ln4 = \ln(16)^{1/2} = (1/2)\ln 16$

Answer. **(i)** $A = 2$, $B = 2$, $C = -1$

Question 9

(i) The product rule was almost always used correctly but errors were seen in finding du/dx and dv/dx. Some candidates had no idea how to remove the exponential terms or they made algebraic slips that prevented them from finding a quadratic equation. Others had arithmetic errors in their quadratic equation. Having successfully obtained two solutions some candidates incorrectly rejected one of them, while others wasted time finding the y–coordinates when the question clearly required x-coordinates only. Here the question of acceptable final answers arises once more. Answers where integer calculation is obvious are not allowed, for example $(1/2)(4 \pm \sqrt{12})$.

(ii) The majority of candidates realised they needed to use integration by parts twice. Not all were successful, particularly in integrating the terms in $e^{-x/2}$. Most of the errors were caused by losing track of negative signs and poor layout and notation. Since the answer was given, candidates needed to show all the steps in their integration, substitution and manipulation of terms leading convincingly to the final answer. The working was difficult to follow if candidates substituted values into earlier terms of their integration while still performing the later integration steps. This continual jumping between substitution and integration meant that terms were often omitted in the working.

Answer. **(i)** $2 + \sqrt{3}$, $2 - \sqrt{3}$

Question 10

(i)   Many algebraic errors were seen in the setting up and solving of the equations. When checking the 3rd equation for consistency using the values obtained, candidates should evaluate each side of the equation completely. Here, as in **Question 1**, poor arithmetic and algebra work meant many candidates did not obtain correct values.

(ii)   The majority of candidates produced correct answers to this part of the question. Some did not find an acute angle, or incorrectly used sine instead of cosine, or made arithmetical errors in calculating the scalar product. However, candidates appeared to have noted from previous reports the importance of showing how they calculate the scalar product, including the magnitudes of the vectors, so they were able to collect method marks. Some candidates gave the obtuse angle as their final answer, or calculated the acute angle incorrectly. At this stage, they may have been short of time but a simple sketch would have illustrated clearly the correct step required.

(iii)   Most candidates used the vector product method or scalar product, while a few found a two-parameter equation of the plane. Not all candidates realised that they should be using the direction vectors. Arithmetical errors and copying errors were frequent in calculating the components of the normal vector or in substituting the point $(3, -2, -1)$ into the scalar product.

Answer: **(ii)** $45°$ or $\pi/4$ **(iii)** $2x - 2y + z = 9$

MATHEMATICS

Paper 9709/32
Paper 32

Key messages

- Work on the algebraic skills.
- Use mathematical notation appropriately – particularly brackets.
- Set the work out clearly and legibly, and do not write a replacement solution over the original version.
- When checking work, make sure that the response does what the question asked for.

General comments

The candidates found most aspects of this paper accessible, and most candidates offered responses to most questions. The quality of the work varied considerably, resulting in some very high scores, and several scores in single figures. Many candidates seemed unfamiliar with the trapezium rule and consequently did not make a strong start. The best work was seen in **Question 3** (trigonometric equation), **Question 7** (complex numbers), **Question 8(i)** (partial fractions) and **Question 10(i)** (angle between two vectors). The candidates did not do well in **Question 1** (trapezium rule), **Question 5** (differential equation), **Question 6(ii)** (tangent parallel to the *x* axis) and **Question 9** (integration and iteration).

For many candidates, clarity of presentation of solutions is an issue. The use, or non-use, of brackets led to problems, particularly in **Question 4(i)** and **Question 6(i)**. Many candidates write over their original solution when they are making a correction – this, together with attempts to erase the original working, or to write over pencil working with a pen, leads to work which is often illegible when scanned.

A significant issue in this paper was the level of algebraic skills. Candidates had difficulty with the addition of fractions, cancelling fractions and multiplication of brackets. These skills should be second nature at this level, and candidates would benefit from spending time developing them.

Candidates should be aware that when they are working towards a given solution (as in **Question 6(i)** and **Question 9(i)**) they need to show full and sufficient working to support their final answer.

When a question specifies the level of accuracy required then full marks will not be awarded for a solution which does not comply. **Question 1(i)** and **Question 9(iii)** both made particular requests regarding accuracy. **Question 4(i)** asked for exact answers, so decimal approximations were not acceptable.

Where numerical and other answers are given after the comments on individual questions that follow it should be understood that alternative forms are often acceptable and that the form given is not necessarily the only 'correct answer'.

Comments on specific questions

Question 1

(i) There were several fully correct solutions to this question, but some candidates did not appear to be prepared to answer a question on the trapezium rule. Some made incorrect attempts to integrate. Some candidates made use of the formula for the trapezium rule, but they substituted values for *x*, not *y*. Some candidates only considered part of the interval, and some did not interpret the request to use two intervals correctly. A few candidates lost marks through not working to sufficient accuracy, or incorrect rounding of their final answer.

(ii)   Many candidates gave a response which filled the space available, but made no reference to the diagram, or to the effect of approximating the curve by two straight lines.

Answer: **(i)** 2.42

Question 2

Most candidates scored some marks in this question. Some were not sure how to deal with logarithms to base 2, but the majority did express $2\log_2 x$ as $\log_2 x^2$. Many candidates got as far as $x^2 = 8(x+1)$ but often made algebraic errors in the process of rearranging the terms to solve the quadratic equation. A common error at the very end of the solution was the acceptance of a negative value of x as a possible solution.

Answer: 8.90

Question 3

The majority of candidates scored the first mark for correct use of the expansions of $\tan(\theta \pm 60)$ although there were some sign errors in the expansions. Some candidates did not recognise the topic at all and used the incorrect formulae $\tan(\theta \pm 60°) = \tan\theta \pm \tan 60°$ and were consequently unable to access any of the marks. It was common to see addition of fractions by adding the numerators, and the incorrect multiplication $(1 - \sqrt{3}\tan\theta)(1 + \sqrt{3}\tan\theta) = 1 + 3\tan^2\theta$. Those candidates who left it until the end of their working to substitute $\tan 60° = \sqrt{3}$ were more likely to make errors in simplifying the algebra. Candidates who reached the correct equation $\tan^2\theta = \dfrac{1}{11}$ did not always reach the correct conclusion – some appeared to be evaluating $\tan^{-1}\left(\dfrac{1}{11}\right)$, possibly due to errors in using their calculators.

Answer: 16.8°

Question 4

(i)   The vast majority of candidates used the quotient rule to answer this question, but a few used the product rule or rewrote the equation as $y = 2\sec x - \tan x$ before differentiating. The initial differentiation was often correct, but a small number of candidates misquoted the quotient rule or did not include the denominator.
Most candidates set their derivative equal to zero and attempted to solve for x. There were several errors in the process of simplifying their expression: many did not recognise $-\cos^2 x - \sin^2 x$ as –1, and the incorrect cancelling $\dfrac{-\cos^2 x + 2\sin x - \sin^2 x}{\cos^2 x} = -1 + 2\sin x - \sin^2 x$ was a common error. In solving their equation, some candidates gave answers as decimals or in degrees, and some did not find the value of y.

(ii)   The response to this question was generally poor. Only a few candidates followed the simpler method of evaluating y or y' on either side of the turning point and many of these candidates failed to give numerical values to support their argument, thus gaining no marks. The majority of candidates attempted to find the second derivative. Even when this was done correctly, most either gave up before substituting for x, gave no numerical value for the second derivative (simply stating positive/negative) or evaluated incorrectly.

Answers: **(i)** $x = \dfrac{\pi}{6}$, $y = \sqrt{3}$ **(ii)** Minimum

Question 5

The majority of candidates made a good start with correct separation of the variables, and most went on to complete $\int \frac{1}{y}\,dy = \ln y$ but scored no further marks. The candidates tried a variety of approaches to

$\int \frac{x+2}{x+1}\,dx$, most of them incorrect – the most common response was $(x+2)\ln(x+1)$, and there were some protracted attempts to use integration by parts. Some candidates succeeded in using integration by substitution. Only a minority recognised the potential to use partial fractions to split the fraction into two easily integrated terms. Those candidates who completed the integration correctly usually reached a correct expression for y.

Answer: $y = (x+1)e^{x-1}$

Question 6

(i)   There were several fully correct responses to this part of the question. Most candidates recognised the need to use implicit differentiation and the product rule, and they usually worked through the solution correctly. The most common errors were sign errors and transcription errors in rearranging the terms. There was some uncertainty about what to do with $2a^4$; occasionally $8a^3$ appeared in the solution. A few candidates attempted to work back from the given answer, but none were successful.

(ii)   Most candidates recognised the need to set the derivative equal to zero, but very often the possibility of $y = 0$ was overlooked, and $x^2 = y^2$ led only to $x = y$. Those candidates who considered $y = 0$ often gave (0, 0) as one of their points, rather than rejecting the possibility. The great majority of those candidates who formed an equation using $x = y$ went straight from $y^4 = -a^4$ to $y = -a$ with no sign that they recognised the fallacy in their working. The small number of candidates who considered the possibility of $y = -x$ reached a correct equation in x or y, but then usually considered only one of the two roots.

Answer: **(ii)** $(a,-a)$, $(-a,a)$

Question 7

(i)   Many candidates gave a fully correct answer. Most errors were due to the argument being stated as $\frac{\pi}{3}$. Errors in the modulus were due to arithmetic errors or leaving the final answer as $\sqrt{4}$ or ± 2. A few candidates did not appear to be aware of the terms used or the processes required.

(ii)   A minority of candidates used their answers from part **(i)**, cubing the modulus and trebling the argument to give a concise and correct solution. The majority of candidates attempted to cube $1-\sqrt{3}i$. Some candidates stated the expansion of the cube directly, and often reached the correct answer. Candidates who started by squaring the number usually did better if they simplified $1-2\sqrt{3}i-3$ to $-2-2\sqrt{3}i$ before proceeding to the next step. The most common algebraic error was to claim that $\left(1-\sqrt{3}i\right)^3 = 1^3 - \left(\sqrt{3}i\right)^3$. Some candidates simply ignored the i and found the cube of $\left(1-\sqrt{3}\right)$.

(iii)   Most candidates offered an attempt to draw the Argand diagram. Several fully correct diagrams were seen. The majority of diagrams did use the same scale on the real and imaginary axes. Most diagrams included a circle, usually in the correct quadrant, but often not passing through the origin. Some circles were centred on $1 + \sqrt{3}i$, and some on $-1 + \sqrt{3}i$. The straight line was sometimes horizontal, and sometimes a diagonal passing through the origin. Some candidates drew two circles.

Answer. **(i)** $2, \ -\dfrac{\pi}{3}$.

Question 8

(i)   Many candidates scored full marks for this part of the question. The majority were able to split the fraction into partial fractions appropriately, and then use a relevant method to determine a constant. A small number of candidates only scored the first mark because they went on to compare fractions with unequal denominators. Some candidates lost marks through inaccurate algebraic manipulation or numerical errors, even though they were using a correct method. Some candidates compared coefficients to form three equations in three unknowns and simply wrote down an answer – presumably having used their calculator to solve the simultaneous equations. An error in this process results in incorrect solutions with no method shown, so no marks.

(ii)   Many candidates scored marks for the correct expansion of $(1 - x)^{-1}$. The expansions of

$\left(1 + \dfrac{2}{3}x\right)^{-1}$ and $\left(1 + \dfrac{2}{3}x\right)^{-2}$ were often correct, but the candidates often did not deal with the factors

of $\dfrac{1}{3}$ and $\dfrac{1}{9}$ correctly: the most common errors were to multiply by 3 and by 9, or to use $\dfrac{1}{3}$ in place

of $\dfrac{1}{9}$. The final answer was often incorrect due to sign errors or transcription errors in the working.

Answers: **(i)** $\dfrac{1}{1-x} - \dfrac{2}{2x+3} + \dfrac{5}{(2x+3)^2}$, **(ii)** $\dfrac{8}{9} + \dfrac{19}{27}x + \dfrac{13}{9}x^2$

Question 9

(i)   The majority of candidates recognised that they needed to start with the integral and to use integration by parts. Some candidates did not deal correctly with the fractional index, or did not

simplify $\dfrac{x^{\frac{3}{2}}}{x}$ correctly, but many did complete the integration correctly. Errors in using the limits

were common, usually due to missing brackets. In some cases it was not possible to tell whether a candidate had used the limits correctly but made a sign error, or they had possibly substituted limits and then added the two expressions rather than subtract. A minority of candidates forgot to equate their integral to 2. Those candidates who tidied up the fractions early in their working were more

likely to be successful in reaching the given answer. Many candidates formed an expression for $a^{\frac{3}{2}}$ in terms of *a* but could not see how to get from their answer to the given answer.

(ii)   Many candidates clearly understood that they were looking for a sign change in the value of a function between 2 and 4. There were many incorrect answers, partly because some candidates struggled to key this function correctly on their calculators (often not using brackets when they needed to), and partly because candidates were not considering an appropriate function.

(iii)   This part of the question was a good source of marks for many candidates. Some candidates lost at least one of the marks through not giving their answers to the required level of accuracy. Most errors were through errors in calculating the value of the function – some candidates achieved a convergent series of values, but very different to the correct values; again, the problem appears to have been due to not keying the function correctly on the calculator.

Answer: **(iii)** 3.031

Question 10

(i)   Most candidates clearly knew what they needed to do, they identified the correct vectors to use, and found the scalar product and moduli correctly. Apart from transcription and arithmetic errors, the main errors were the use of sine in place of cosine, and subtraction of the final answer from 90° to obtain the incorrect answer 17.5°.

(ii)   Candidates who substituted $y = 2$ into the two plane equations at the outset usually arrived at a concise correct solution. For many candidates, the starting point was to find the equation of the line of intersection of the two planes and then use $y = 2$ to solve for x and z. Errors in the working often led to fractional values at this stage. Only a small number of candidates who found the equation of the line recognised that the direction vector of the line gave then the normal vector for the new plane – most went on to try to find the normal by another method. Some candidates misunderstood the question and assumed that A was the point (0,2,0).

Answers: **(i)** 72.5°, **(ii)** $7x + 5y − 4z = 27$

MATHEMATICS

Key messages

There are some extremely important points that candidates need to address:

(i) They must use the method asked for in the question (**Questions 2 and 3)**. Those in **Question 2** who ignored the requested approach and used the given equation, together with values from their table/graph often couldn't complete the question or had arithmetical or algebraic errors. In **Question 3** some candidates did not use the iterative approach asked for.

(ii) They must note carefully what final answer is acceptable (**Questions 2**, **7(a) and 9(i)**. **Question2** requests answers to 2 s.f. In the other 2 questions acceptable final answers arises once more. In **Question 7(a)** answers containing $\sqrt{25}$ or $\sqrt{9}$ are not acceptable. In **Question 9(i)** answers where integer calculation is obvious are not allowed, for example those of the form $(1/2)(4 \pm \sqrt{12})$.

General comments

The standard of work on this paper varied considerably although all questions were accessible to well-prepared candidates. The questions or parts of questions that were generally done well were **Questions 3(i)** and **(ii)** (numerical iteration), **Question 5(i)** (implicit differentiation), **Question 7(a)** (complex numbers), **Questions 8(i)** and **(ii)** (partial fractions), **Question 9(i)** (product rule) and **Questions 10(i)**, **(ii)**, **(iii)** (vectors, line and plane). Those that were done less well were **Question 1** (polynomial division), **Question 2** (logarithms), **Question 4(ii)** (graph of 2sec2x), **Question 5(ii)** (tangent to curve parallel to x axis), **Question 6** (differential equation), **Question 7(b)** (Argand diagram) and **Question 9(ii)** (integration by parts).

In general the presentation of the work was good, though there were some rather untidy scripts. Candidates should bear in mind that scripts will be scanned for marking and they should use a **black** pen, reasonable sized lettering and symbols and present their work clearly.

It was pleasing to see that candidates are aware of the need to show sufficient working in their solutions. Previous reports mentioned this in the context of solving a quadratic equation and substituting limits into an integral. There are some extremely important points that candidates need to address: they must use the method asked for in the question (**Questions 2 and 3)** and note carefully what final answer is acceptable (**Questions 2**, **7(a)** and **9(i)**). These points will be discussed in detail below.

Where answers are given after the comments on individual questions, it should be understood that the form given is not necessarily the only 'correct answer'.

Comments on specific questions

Question 1

This question was generally not well done. Weaker candidates stopped after reaching a quotient of x^2 or even attempted to divide the wrong way round. Many candidates made careless errors with the signs, in particular when subtracting negative numbers. *Answer*: Quotient $x^2 - 2x + 5$ Remainder $- 12x +5$

Question 2

This was another question that many candidates found difficult. The most successful candidates answered the question by obtaining a correct equation connecting ln y with ln C and ln a and then using their straight line on the graph to evaluate these quantities. A poor choice for their line did cost them accuracy marks later. Those who ignored the requested approach and used the given equation, together with values from their table/graph often couldn't complete the question or had arithmetical or algebraic errors. Many candidates ignored the request for answers to 2 s.f.

Answer: $C = 3.7$ $a = 1.5$

Question 3

(i)   In general this part was well done, although there were some inaccuracies. A few candidates failed to complete the argument correctly with correct calculated values.

(ii)   Many successful attempts were seen, although some candidates did not use the iterative approach asked for in the question. Instead they attempted some algebra to show divergence and received no credit for this. Some candidates forgot to round their final answer for α to 2 d.p. Candidates who used an initial value other than $x_1 = 2.5$, were deemed to have misread the question. In some cases, this led fortuitously to convergence as there is no other root nearby. Clearly this will not always be the case.

Answer: **(ii)** *A* 2.43 convergent sequence
           B 2.8750, 5.5879 divergent sequence

Question 4

(i)   Most candidates gained the first two marks. A large proportion of candidates found it difficult to progress beyond the first 3 marks since they did not recognise that $\sec^2 x$ could be expressed as $1/\cos^2 x$.

(ii)   Some candidates omitted this question. Even though the question said "hence", there were some candidates who tried to use the graphs of $\tan(45 + x)$ and $\tan(45 - x)$ to sketch $2\sec 2x$. Other candidates started with $\cos 2x$ but failed to introduce the vertical stretch, or omitted the asymptote, or they worked in radians rather than degrees. Some candidates plotted values, but they still needed to display the asymptotic behaviour around $x = 45°$.

Question 5

(i)   Some good work was seen, however candidates needed to show that the derivative of 10 is zero. Too often the zero was omitted so the candidate did not produce a correct equation to rearrange. Since the answer was given, every line of detail was required. It was also necessary to show clear bracketing of the two terms in dy/dx.

(ii)   Some candidates were unsure whether to set to zero the numerator or the denominator of the right hand side of the answer given in **(i)**. Sign errors were common in $8x^3 + y^3 = 0$. Many candidates could not find the cube root of $-8x^3$ (or a similar expression if in working in terms of y). Unfortunately a few of the candidates who succeeded in finding the correct equation failed to realise that there were two solutions.

Answer: $(1, -2)$, $(-1, 2)$

Question 6

Almost all candidates knew that they had to separate variables and could do that correctly. However, some could not integrate one or both sides. Those who successfully integrated both sides usually went on to produce an excellent solution. A common error was failing to write $1/\cos^2 y$ as $\sec^2 y$, or not realising it integrated to $\tan y$. A significant number of candidates thought that the integral of $\tan x$ was $\sec^2 x$. In many cases, the 4 moved from numerator to denominator or vice versa.

Answer: $\tan y = 4\ln \sec x + 1$ 0.587

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Question 7

(a) Some candidates made little progress as they tried to square root $u = 8 - 15i$. This is a legitimate approach but it requires a polar form of the complex number and it is extremely unlikely to lead to an exact answer. To obtain an exact answer requires the square root of u to be set to $a + ib$, as stated in the question, and then squared and equated to the numerical expression $8 - 15i$. Many candidates were able to do this successfully, with the correct signs, reaching a quadratic equation in either a^2 or b^2. Unfortunately some candidates forgot to square root to find a and b, while others failed to simplify their answer, leaving $\sqrt{25}$ or $\sqrt{9}$ in it. Simplification of final answers will also be discussed in **Question 9(i)**.

(b) The circle was more usually correct than the line. Most candidates had a circle of radius of 2, but often the centre was incorrect. The line was often drawn through the origin or the point $-i$ or, even if both circle and line were correct, the candidate's shading often extended over all the circle below the line.

Answer: **(a)** $\pm (1/\sqrt{2}) (5 - 3i)$

Question 8

(i) This was extremely well done and many candidates obtained correct values for all three coefficients. However, a considerable number of fortuitously correct answers were seen from candidates who, instead of multiplying the given form of $f(x)$ by $(x + 2)(2x - 1)$ and equating coefficients, simply divided the given $f(x)$ out and obtained $2 + (3x - 4)/((x + 2)(2x - 1))$. This correctly gave them $A = 2$. They then produced an incorrect expression such as

$(3x - 4)/((x + 2)(2x - 1)) = 2 + B/(x + 2) + C/(2x - 1)$
or $(4x^2 + 9x - 8x)/((x + 2)(2x - 1)) = B/(x + 2) + C/(2x - 1)$.

However, by using the 'cover up rule' the correct values of B and C are obtained.

(ii) There were some errors in copying the result of **(i)** to use in **(ii)**, in integrating $1/(2x - 1)$ and substituting in the limits. A surprising number of candidates did not seem to know how to apply the laws of logarithms correctly or to realise they should use them. They needed to show all the steps in the working to obtain the given answer, for example $2\ln6 - 2\ln3 = 2\ln2 = \ln4 = \ln(16)^{1/2} = (1/2)\ln 16$

Answer: **(i)** $A = 2$, $B = 2$, $C = -1$

Question 9

(i) The product rule was almost always used correctly but errors were seen in finding du/dx and dv/dx. Some candidates had no idea how to remove the exponential terms or they made algebraic slips that prevented them from finding a quadratic equation. Others had arithmetic errors in their quadratic equation. Having successfully obtained two solutions some candidates incorrectly rejected one of them, while others wasted time finding the y–coordinates when the question clearly required x-coordinates only. Here the question of acceptable final answers arises once more. Answers where integer calculation is obvious are not allowed, for example $(1/2)(4 \pm \sqrt{12})$.

(ii) The majority of candidates realised they needed to use integration by parts twice. Not all were successful, particularly in integrating the terms in $e^{-x/2}$. Most of the errors were caused by losing track of negative signs and poor layout and notation. Since the answer was given, candidates needed to show all the steps in their integration, substitution and manipulation of terms leading convincingly to the final answer. The working was difficult to follow if candidates substituted values into earlier terms of their integration while still performing the later integration steps. This continual jumping between substitution and integration meant that terms were often omitted in the working.

Answer: **(i)** $2 + \sqrt{3}$, $2 - \sqrt{3}$

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Question 10

(i)   Many algebraic errors were seen in the setting up and solving of the equations. When checking the 3rd equation for consistency using the values obtained, candidates should evaluate each side of the equation completely. Here, as in **Question 1**, poor arithmetic and algebra work meant many candidates did not obtain correct values.

(ii)   The majority of candidates produced correct answers to this part of the question. Some did not find an acute angle, or incorrectly used sine instead of cosine, or made arithmetical errors in calculating the scalar product. However, candidates appeared to have noted from previous reports the importance of showing how they calculate the scalar product, including the magnitudes of the vectors, so they were able to collect method marks. Some candidates gave the obtuse angle as their final answer, or calculated the acute angle incorrectly. At this stage, they may have been short of time but a simple sketch would have illustrated clearly the correct step required.

(iii)   Most candidates used the vector product method or scalar product, while a few found a two-parameter equation of the plane. Not all candidates realised that they should be using the direction vectors. Arithmetical errors and copying errors were frequent in calculating the components of the normal vector or in substituting the point $(3, -2, -1)$ into the scalar product.

Answer: **(ii)** 45° or $\pi/4$ **(iii)** $2x - 2y + z = 9$

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Key messages

*   Non-exact numerical answers are required correct to three significant figures as stated on the question paper. Students should be reminded that if an answer is required to 3sf then their working should be performed to at least 4sf. This was often seen to affect answers to **Questions 1**, **2**, **3**, **6** and **7** in this paper.

*   When answering questions such as **Questions 5(ii)** and **5(iii)** on this paper where the acceleration is a function of time, the equations of constant acceleration cannot be used in such cases.

*   In questions such as **Questions 7(i)** and **7(ii)** where the motion takes place on an inclined plane, it is always advisable to draw a force diagram. In order to give your answer more clarity you should also state the origin of your equations e.g. applying Newton's law to particle *A* along the plane.

General comments

There were some candidates who produced very good answers on this paper but a significant number who were not well prepared for the paper. Overall a wide range of performance was seen.

Question 1 was found to be the easiest question whilst **Questions 7(i)** and **(ii)** proved to be the most challenging.

One of the rubrics on this paper is to take *g* = 10 and it has been noted that virtually all candidates are now following this instruction. In fact in some cases it is impossible to achieve a correct given answer unless this value is used.

Comments on specific questions

Question 1

Most candidates made a good attempt at this question. The 12 N force has to be resolved along the horizontal direction of motion. This is the only force acting since the plane is smooth and so there is no frictional force. Newton's second law must be used and this enables the acceleration of the block to be determined. Once this acceleration is found the distance travelled in the first 5 seconds can be determined by using any of the relevant constant acceleration equations. An error that was noticed was that there was sometimes a mix up with sine and cosine when dealing with the component of the force.

Answer: Distance travelled in the first 5 seconds is 45.3 m (to 3sf)

Question 2

(i)   Most candidates found this question straightforward. As the tractor is moving at constant speed, the net force acting on it is zero. Hence the driving force must balance the resistive force and so the driving force is 1150 N. Use is then made of the formula *P* = *Fv* where *F* is the driving force and *v* is the given constant velocity *v* = 12. From this the power output of the tractor's engine can be found.

Answer: The power output of the tractor's engine is 13 800 W = 13.8 kW

(ii)   In this question an additional force is acting and also the forces are no longer in balance. Newton's second law must be applied to the tractor. The driving force, DF, will be given by DF = P/v = 25000/12. As the tractor is going up a hill there will be a component of the weight of the tractor acting against the direction of motion and this takes the form $3700g \sin 4$. The three forces, namely driving force, weight component and resistance will produce an acceleration. Application of Newton's second law will determine this required acceleration. Some candidates forgot that the resistance force was still acting. Others did not include the weight component. Another error seen was the use of cos 4 rather than sin 4 in the term involving the weight component. Some also forgot to include the factor of g in the weight component term.

Answer: The acceleration of the tractor as it begins to climb the hill is -0.445 m s^{-2} (to 3sf)

(iii)   In this part, once again the forces are in balance since the tractor is moving at constant speed. The driving force 25000/v must balance the sum of the weight component and the resistance force. By setting up this equation, the value of the steady speed on the hill can be found. Similar errors were seen here as in **2(ii)** with terms missing, mix up of sine and cosine and not including the factor g in the weight component.

Answer: The constant speed that the tractor can maintain on the hill is 6.70 m s^{-1} (to 3sf)

Question 3

(i)   Many candidates made this part harder than it actually was. The resistance to motion is given as 640 N. As the roller coaster travels 8 m directly against this resistance and then a further 10 m directly against it, the total work done against the resistance is simply $(8 + 10) \times 640$. However many candidates worked out components of this resistive force or components of the distances in their attempt to find the total work done.

Answer: The total work done against the resistance force is 11 520 J

(ii)   It was stated that energy methods must be used in this part and almost all candidates followed this approach. The result from **3(i)** must be used in this work-energy equation. As the roller coaster rises and falls, so potential energy is gained and lost and so the net potential energy gain at the bottom of the second slope must be evaluated. The initial speed of the roller coaster is given and so the initial kinetic energy can be determined. Once evaluated these terms can now be used in the work-energy equation which takes the form "Kinetic Energy loss = Potential Energy gain + Work Done against resistance forces". Some errors seen here were not evaluating both parts of the potential energy or not including all of the terms from the energy equation or failing to use the result from **3(i)** or using incorrect signs within the work-energy equation.

Answer: The speed at the bottom of the second ramp is 12.5 m s^{-1} (to 3sf)

Question 4

(i)   Most candidates were able to show this result. The answer is given in the question and so extra care must be taken to show all working in this question. The most straightforward method of approach here was to use the fact that the slope of the graph between $t = 3.5$ and $t = 6$ gives the acceleration for this part of the motion. An alternative method is to use the velocity values at $t = 3.5$ and $t = 6$ from the graph and to use the constant acceleration formulae.

Answer: The acceleration between $t = 3.5$ and $t = 6$ is -10 m s^{-2} (given)

(ii)   Most candidates again performed well on this part. The methods used were either use of the constant acceleration formulae or the use of similar triangles incorporating the information given in the diagram. Errors seen here were usually numerical.

Answer: $V = 15$

(iii)   Almost all candidates attempted this part of the question but very few reached the correct final answer. It should first be noted that the line on the graph representing the motion between $t = 3.5$ and $t = 6$ crosses the t-axis at $t = 4.5$ and that the line representing the motion between $t = 6$ and $t = 10$ crosses the t-axis at $t = 8$. It is then necessary to evaluate distance travelled and one method is to find the total area under the graph. This was the method used by the majority of candidates. The method involved finding the area of the trapezium up to $t = 4.5$, the triangle which has all of its area below the t-axis and the triangle representing the distance from $t = 8$ to $t = T$. As the information given involves distance, not displacement, the area below the t-axis is counted as positive. Once these areas are found they can be added and then equated to 100 which will produce an equation which can be solved for T. Most of the errors here were numerical, often due to not finding the correct values at which the lines cross the t-axis. Very few candidates attempted to solve this part of the question by any other method such as using the constant acceleration equations.

Answer: $T = 13.5$

Question 5

(i)   In the first 5 seconds of motion, v is given by $v = 1.5 + 0.4t$. By using simple differentiation of v the acceleration is found. Most candidates found this correctly.

Answer: Acceleration is $0.4\,\text{m s}^{-2}$

(ii)   Candidates need to know that the value of t when the particle is at instantaneous rest involved finding values of t where $v = 0$. This is clearly impossible for values of t between 0 and 5 from the given definition. Hence it was necessary that the expression given for v for $t \geqslant 5$ is set to zero and that the equation produced is then solved for t. Most candidates who attempted this question found the correct value.

Answer: $t = 10$

(iii)   It is necessary here to realise that integration of the velocity is required to find the distance travelled. Since the velocity is given for the two different time zones, the integration must be performed separately for each zone. In fact if a v-t graph is drawn, it is clear that the distance travelled in the first 5 seconds can be evaluated more simply by finding the area under a trapezium rather than by integration. The distance travelled between the times $t = 5$ to $t = 10$ can only be found by integration. This integration has to be performed and the limits of integration used must be $t = 5$ and $t = 10$. Many candidates found the correct value for the distance travelled between $t = 0$ and $t = 5$. However, the integration of v for $t \geqslant 5$ proved to be difficult for some. However, some good answers were seen for this question.

Answer: The distance travelled in the first ten seconds of motion is 18.75 m

Question 6

(i)   Most candidates made a reasonable attempt at this question. It involves two forces, one of 75 N and the other of 50 N with an angle of 60º between them. The horizontal sum (X) and vertical sum (Y) of the components of these two forces need to be evaluated. Once this has been achieved, use of Pythagoras would enable the resultant force, R, to be found as $R^2 = X^2 + Y^2$ and trigonometry would give the angle, θ, that the resultant makes with the x-axis by using $\tan \theta = Y/X$. This was the method adopted by almost all candidates. An alternative approach would be to use the cosine rule to find the resultant force and the sine rule to determine the required direction. Errors seen here were mainly numerical. The direction of this force must be specified correctly, not merely quoting the numerical value of an angle.

Answers: Magnitude of the resultant force is 109 N (to 3sf)
The direction of the resultant is 23.4° anticlockwise from the positive x-axis (to 1dp)

(ii)   In this part of the question the forces are in equilibrium and so it is necessary to resolve forces in two perpendicular directions, usually the directions of the *x* and *y* axes and to state that these components are both equal to zero. Resolving in the *x* direction produces an equation with two terms involving *F* and *α*. When resolving in the *y* direction, this produces an equation with three terms involving *F* and α. This gives two simultaneous equations in *F* and *α*. By eliminating *F* between these equations a value of α can be found and then this can be used in either equation to find *F*. This question was well done by most candidates who attempted it. One error seen was to omit one of the terms in the equation in the *y* direction. Some candidates resolved the forces but did not state that the resultant force in each direction was zero and so failed to produce the two equations.

Answers: *F* = 13.1 (to 3sf), α = 80.3 (to 1dp)

Question 7

(i)   This question involves connected particles, each on an inclined plane and as the particles are accelerating it will be necessary to apply Newton's second law to the particles, either individually or to the system. These equations will involve the tension in the string, *T*, and the acceleration of the particles, *a*. The two forces acting on particle *A* are *T* and the component of the weight of particle *A* namely 0.9*g* sin 15 and the correct combination of these forces will be equated to 0.9*a*. The forces acting on particle *B* will be 2.5, the weight component 0.4*g* sin 25 and *T* and the correct combination of these will be equated to 0.4*a*. Elimination of *T* between these equations will produce the required value of the acceleration *a*. The alternative approach is to use the system equation which will not involve *T* but involves forces 2.5, 0.9*g* sin 15 and 0.4*g* sin 25 and this combination is to be equated to (0.4 + 0.9)*a*. Not many candidates followed this through to a correct value for *a* with errors seen being leaving out some of the terms, sign errors for the forces or in the system case forgetting that the mass of the system is the sum of the two masses.

Answer: The acceleration of *B* is 1.43 m s^{-2} (to 3sf)

(ii)   In this part of the question friction is introduced to one of the planes but the system is now in equilibrium. Again it is possible to use either equilibrium equations for each particle or an equilibrium equation for the system just as in **7(i)**. Firstly the friction force acting on *B* must be evaluated as $F = \mu R$ since the system is in limiting equilibrium. The normal reaction, *R*, is found by resolving perpendicular to the plane on which *B* rests. The equilibrium equation for *A* comprises of two terms, *T* and 0.9*g* sin *θ* while the equation for *B* comprises of 4 terms, namely 2.5, 0.4*g* sin 25, *T* and *F*. Again elimination of *T* will produce a trigonometric equation from which θ can be found. The system equation could be used in a similar way to that in **7(i)** where *T* is not involved and an equation in θ is produced. Some candidates found *T* in **7(i)** but it must be remembered that *T* in **7(ii)** is different. Again the main errors seen were the omission of terms, sine and cosine errors and sign errors when combining the forces.

Answer: θ = 8.2 (to 1dp)

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Key messages

- Non-exact numerical answers are required correct to three significant figures as stated on the question paper and cases where this was not adhered to were seen in **Question 1(ii)**, **Question 2**, **Question 5** and **Question 6**. Candidates would be advised to carry out all working to at least 4sf if a final answer is required to 3sf.

- When answering questions involving an inclined plane, a force diagram could help candidates to include all relevant terms when forming a Newton's Law equation or a work/energy equation. This was particularly noticeable here in both parts of **Question 6**.

- In questions such as **Question 7** in this paper, where velocity is given as a cubic function of time, then calculus must be used and it is not possible to apply the equations of constant acceleration.

General comments

The paper was generally well done by many candidates although as usual a wide range of marks was seen.

The presentation of the work was good in most cases and as the papers are now scanned, it is important to write answers clearly using black/dark blue pen.

In **Question 5** the sine of an angle which is used in the question was given. Also in **Question 6** the tangent of an angle is given. In both of these questions it was not necessary to calculate the angle itself as the sine and cosine required could be evaluated exactly. However, many candidates often proceeded to find the relevant angle to 1 decimal place and immediately lost accuracy and in some cases lost marks.

Question 1 was found to be the easiest question whilst **Question 4** and both parts of **Question 6** proved to be the most challenging.

One of the rubrics on this paper is to take $g = 10$ and it has been noted that virtually all candidates are now following this instruction. In fact in some cases such as **Question 1(i)** it is impossible to achieve a correct given answer unless this value is used.

Comments on specific questions

Question 1

(i)   Almost all candidates scored this mark. Since the answer was given then it is particularly important to show any working. In this case it was vital to show that the force acting was $0.2g \sin 20 = 2 \sin 20$. This is the case where the given answer cannot be achieved unless $g = 10$ is used. One error that was seen was that candidates assumed that the particle was in the limiting friction case and found the value of μ as 0.364 from use of $F = \mu R$ but this was not the case given.

Answer: Force = 0.684 N (to 3sf, answer given)

(ii)   Some candidates wrongly attempted to use the same frictional force as had been given in **1(i)** but now the particle is in motion and so the normal reaction, R, has to be calculated by resolving perpendicular to the plane and then the equation $F = \mu R$ must be used to find the friction force. Both the 0.9 N force and the weight component, $0.2g \sin 20$, act down the plane with the friction force F opposing the motion. It is necessary to write down Newton's second law of motion applied to the particle and this involves a combination of the three forces and this is equated to $0.2a$ in order to find the acceleration. Some candidates lost marks by omitting one or other of the forces and in some cases mixing up the use of the sine and cosine of the angles.

Answer: The acceleration is 2.28 m s^{-2} (to 3sf)

Question 2

Almost all candidates attempted to resolve forces horizontally and vertically in their attempt to solve this problem. A few candidates used Lami's theorem. The horizontal equation involved two terms, the components of the tensions T N and 120 N. Most found this equation correctly. The vertical equation involved three terms, the vertical components of the two tensions and the 150 N weight force. Together these equations comprise a pair of simultaneous equations in T and θ and can be solved either by first finding $\tan \theta$ and then using this value to determine T or by using Pythagoras to find T and substituting to find θ. Most candidates made a good attempt at this question with many finding the correct results. Those who used Lami's theorem found that the angles involved were not simple and it required good trigonometric skill to reach the results with this method.

Answers: $T = 107$ (to 3sf) and $\theta = 37.5$ (to 1dp)

Question 3

(i)   Most candidates made a good attempt at this question. Generally use was made of the constant acceleration formulae to determine expressions for the distances AB and AC in terms of the acceleration of the car. The fact that AB and BC were of equal length was then used to show the given result. Once again with a given answer, candidates must be careful to include all of the detail of their working. One error that was often seen was that candidates did not answer the question as it was asked, but found AB and BC before proving the given result.

Answers: $AB = 70 + 12.5a$ and $AC = 112 + 32a$ where a is the acceleration of the car. $a = 4 \text{ m s}^{-2}$ (given)

(ii)   Again most candidates found this part very straightforward. Even those who had failed to find AB and AC correctly were able to score marks here by using the given information. The most straightforward method was to use the equation $v = u + at$ with the given values. Some candidates chose to find the distance AC or BC and then used $v^2 = u^2 + 2as$ to find the required speed of the car. This is a perfectly correct method but often lead to numerical errors within these more complicated calculations.

Answer: The speed of the car is 46 m s^{-1}

Question 4

(i)   Most candidates attempted this problem by using one of the constant acceleration equations. One method was to use the equation $s = ut + \frac{1}{2}at^2$ for the vertical motion with $u = 12$ and $a = -10$ and find the times at which $s = 0$. This leads to a quadratic equation, one solution being the trivial solution $t = 0$ and the other giving the required time. An alternative approach is to use the equation $v = u + at$ with $v = 0$, $u = 12$ and $a = -10$ and determine the time at which the particle reaches its highest point. The time found here should then be doubled to obtain the required answer. Many candidates who used the second method forgot to double their answer and lost marks because this is not a complete method for this question.

Answer: The time taken for P to return to the ground is 2.4 s .

(ii) Candidates found this to be the most difficult question on this paper. Many candidates showed lots of working but did not find any relevant times which helped to solve the problem. It was necessary to use the answer in **4(i)** to determine how long both particles were moving upwards at the same time. Since the second particle did not start to move until $t = 1$ the two particles were moving upwards at the same time from time $t = 1$ until the first particle reached its highest point. It was then necessary to find when the second particle reached its highest point which was after 1 second of motion which is at $t = 2$. This is the time at which the second particle starts to move downwards and so both particles will be moving in the downward direction from this time until the first particle reaches the ground. The key times which are needed in this question are $t = 1$, $t = 1.2$, $t = 2$ and $t = 2.4$

Answer: $1 < t < 1.2$ (both moving upwards) and $2 < t < 2.4$ (both moving downwards)

Question 5

(i) There are two possible approaches to this question. As the cyclist is moving at constant speed the net force acting on the cyclist is zero. The driving force must be equal to the sum of the weight component of the cycle down the hill and the resistive force. Once this driving force has been calculated, the formula $P = Fv$ can be used to determine the required power output of the cyclist. An alternative approach would use the formula for power as $P = $ Work Done $\div$ time taken. The work done by the driving force is comprised of the increase in potential energy as the cyclist moves the 25m up the hill plus the work done against resistance. The time, T, taken can be found simply by using $T = 25/4$ since there is no acceleration. Use of the formula now gives the required power. Most candidates chose the first of these methods and most performed well on this question. Some wrong signs were seen when combining the two forces and similar wrong signs also when finding the work done in the alternative approach. Some cases were seen where candidates were mixing up work done and force, and hence obtaining equations which were dimensionally inconsistent. Kinetic energy does not play a part in this question since the speed remains constant and hence so too does the kinetic energy.

Answer: The power output of the cyclist is 224 W

(ii) Many candidates did not perform particularly well on this part of the question. As the cyclist is now accelerating, the forces acting will vary and it cannot be assumed that the motion has constant acceleration. In order to solve this problem it must be approached by using the work-energy principle. Since the power, P, remains constant for the ten seconds of motion, the work done by the cyclist during this period of time will be 10P. This work done will be used to overcome the work done against resistive forces and also to increase the kinetic energy. There is no change in the potential energy. The problem will be solved by using the equation, "WD by the cyclist = WD against resistance + KE increase", and this will give the required speed after 10 seconds. Many candidates assumed the force acting was constant and attempted to use the constant acceleration equations which do not apply in this case.

Answer: The speed of the cyclist after 10 seconds is 6.48 m s^{-1} (to 3sf)

Question 6

(i) There are two alternative approaches to this problem. As the particles are in equilibrium, this balance could be stated by an equation of equilibrium for each particle. Applying equilibrium to particle Q shows a balance between the weight of the particle and the tension in the string. For particle P the frictional force can be expressed as $F = \mu R$ since it is in limiting equilibrium. R must be found by resolving forces perpendicular to the plane. This frictional force acting on particle P is in balance with the sum of the tension in the string and the component of the weight acting down the plane. Combining these two equations for P and Q will enable the tension, T, to be eliminated and give an equation for the required coefficient of friction. Alternatively an equation of equilibrium for the whole system can be used which will not involve the tension in the string. In this case the friction force is acting up the plane and is directly balanced by the weight component of P plus the weight of Q. Most candidates applied the first of these methods and generally performed well. Again it is a given answer and care must be taken to show all working. An error that was seen was candidates mixing up sine and cosine when taking components of the forces. As the tangent of the angle is given in the question, it is not necessary to evaluate the angle itself since sine and cosine

of the angle can easily be found. However, many candidates did find the angle and hence lost accuracy.

Answer: Coefficient of friction is 4/3 (given)

(ii) In this part of the question an extra force is applied to the system to cause motion but the method of approach is similar to **6(i)**. Newton's second law of motion can be applied either to P and Q separately or to the whole system. Both methods were used by candidates. The friction term F can be found from **6(i)**. It is important to realise that the tension, T, in the string is different from its value in **6(i)**. When applying Newton's second law to P and Q separately, the Q equation has 3 terms, namely mg, T and $2.5m$, while the P equation has 5 terms, namely 10, F, T, $mg \sin \theta$ and $2.5m$. When F is evaluated and T eliminated between these equations the required value of m can be found. If the system equation is used, then the terms involved are 10, mg, $mg \sin \theta$, F and $5m$. Again when F is evaluated this equation gives the required value of m. Very few candidates followed this through to the correct answer. In both methods many wrong signs were seen and also terms were missed out. In the system case many forgot that the "ma" term was $5m$ and not $2.5m$ since the system is comprised of both particles. Another error often seen was that the tension, T, was wrongly assumed to be $10m$, its value in **6(i)**.

Answer: $m = 0.327$ (to 3sf)

Question 7

(i) In order to find the times at which the particle is at instantaneous rest it is necessary to set $v = 0$. The given expression for v is a cubic but when this is set equal to zero, a factor of t can be taken leaving the problem as the solution of a quadratic equation. This can either be solved using the formula or by factorising. It is useful to take out the factor $-0.01t$ from the expression which leads to a simpler quadratic equation which can easily be factorised. The two positive values were asked for and so these are the two solutions of the quadratic equation since $t = 0$ is the trivial solution. Many used the formula directly with the decimal values as coefficients. This sometimes led to arithmetical errors. However, most candidates solved the quadratic equation correctly to find the two positive values. Some wrongly felt that they had to differentiate the given expression before setting it equal to zero.

Answer: The two positive values of t are $t = 2$ and $t = 20$

(ii) In order to find when the acceleration is greatest it is first necessary to find the acceleration itself and this is achieved by differentiating the given expression for v. Most candidates realised that this was necessary and scored well on this part. In order to find when the maximum acceleration occurs, the most straightforward approach is to differentiate again and set this value to zero. This produces a linear expression and is simply solved to give the required value of t. An alternative method is to rewrite the acceleration, which is a quadratic, using the method of completing the square and again this will produce the required value of t. Many candidates scored the first mark for finding acceleration but did not know how to proceed from there.

Answer: Acceleration is greatest at $t = 22/3 = 7.33$ s

(iii) It must first be realised that the velocity is positive between the two values of t found in **7(i)**. Most candidates realised that it was necessary to integrate the given expression for v in order to find the distance travelled. Many candidates were able to perform this integration successfully. It was then necessary to evaluate the integral between the two limits found in **7(i)**. Many scored the marks for the integration but either made numerical errors in their evaluations of the limits or used the wrong limits or in some cases evaluating at just one of the limits. Some tried to use the constant acceleration formulae which are clearly not applicable in this case.

Answer: Distance travelled while the velocity is positive is $2673/25 = 106.92$ m

MATHEMATICS

Paper 9709/43
Paper 43

Key messages

* Non-exact numerical answers are required correct to three significant figures as stated on the question paper (e.g. **Question 3(i)** and **Question 7(i)**).
* When answering questions involving an inclined plane, a force diagram could help candidates to include all terms in forming a Newton's Law equation or a work/energy equation (e.g. **Question 2(i)** and **(ii)**, **Question 3(i)** and **Question 7(i)** and **(ii)**).

General comments

The examination allowed candidates at all levels to show what they knew, whilst differentiating well between even the stronger candidates. **Question 1** was found to be the easiest question whilst **Question 7(i)** and **(ii)** proved to be the most challenging.

Comments on specific questions

Question 1

This was a straightforward question for the majority of candidates who realised that the horizontal component of the resultant was zero. Nevertheless, there was a significant minority who attempted mistakenly to find F using perpendicular components and Pythagoras' theorem either in the form $\sqrt{(25^2 + (5\sqrt{3})^2)}$ or in the form $\sqrt{((25 - F)^2 + (5\sqrt{3})^2)}$.

Answer: 25

Question 2

This question was often answered successfully. Whilst almost all candidates attempted to apply $P = Fv$, the understanding of F as the driving force was not always demonstrated.

(i) It was common to see $\dfrac{P}{10} = 1480$, $P = 14800$ W, suggesting either the application of $P = Rv$, or the resolution of forces parallel to the plane with a missing component of weight.

(ii) The main errors were either to omit the component of weight when applying Newton's second law, $\left(\dfrac{P}{15} - 1480 = 7850 \times 0.8\right)$, or to apply $P = mav$, ($P = 7850 \times 0.8 \times 15$), suggesting a mistaken belief that $P = Fv$ and $F = ma$ refer to the same force F.

In answering both parts of the question there were occasional sign errors when resolving. Candidates could benefit from checking the direction of all forces involved on a force diagram. A small number of candidates believed that $P = \dfrac{F}{v}$ rather than $P = Fv$.

Answers: **(i)** 55 900 W **(ii)** 54 800 W

Question 3

(i) Candidates often applied Newton's second law and $F = \mu R$ as expected but then frequently gave an answer of $0.6\,\text{ms}^{-2}$ or $0.06\,\text{ms}^{-2}$ instead of rounding correct to three significant figures as required. Some candidates found difficulty working with an unknown mass, and a few used G or W without relating these to the mass of the particle. This led to algebraic expressions for the acceleration such as $(G\sin 25^\circ - 0.4\,G\cos 25^\circ)/m$ instead of a numerical value. A small number of candidates oversimplified the situation by leaving out the component of weight ($0.4\,mg\cos 25^\circ = ma$); or made a sign error ($mg\sin 25^\circ + 0.4\,mg\cos 25^\circ = ma$); or omitted g from one or both of $mg4\sin 25^\circ$ and $0.4\,mg\cos 25^\circ$.

(ii) Most candidates were able to use their answer from part **(i)**, successfully substituting into one or more constant acceleration formulae to find the distance travelled.

Answers: **(i)** $0.601\,\text{ms}^{-2}$ **(ii)** $2.70\,\text{m}$

Question 4

(i) A majority of candidates obtained the correct acceleration ($1.25\,\text{ms}^{-2}$) for the system, most commonly by setting up and solving two equations for T and a. Erroneous solutions included those which assumed $a = g$ and also those with errors in solving the correct simultaneous equations. Those who attempted a single equation in a sometimes mistakenly solved $a = \dfrac{(0.45g - 0.35g)}{0.45}$ instead of using the combined mass. Those who attempted an energy approach for one particle invariably omitted to consider the work done by the tension. The application of $v^2 = u^2 + 2as$ was straightforward for most although 0.36, 1 and 1.36 were all occasionally used instead of 0.64 for s.

(ii) Most candidates found the distance travelled by A with the string slack to the maximum height, with only a few using the acceleration found in part **(i)** rather than $a = -g$. Some candidates misinterpreted the overall distance required, concluding e.g. $0.08 \times 2 + 0.64 = 0.8$ (the total distance travelled by particle A) or $0.08 + 0.64 = 0.72$ (the total distance travelled by particle A to its maximum height).

Answers: **(i)** $1.26\,\text{ms}^{-1}$ **(ii)** $0.16\,\text{m}$

Question 5

(i) Candidates were expected to integrate $a(t)$ to obtain $v(t)$ and to use the two conditions provided to obtain $k = 0.1$. Fully correct solutions were often seen. The two main sources of error were firstly, the omission of a constant of integration leading to $k \neq 0.1$ and secondly, incorrect integration. Some included $k^2/2$ or kt in their integration attempt whilst a few overcomplicated the situation by attempting, for example, integration by parts. There were a few attempts to use a constant acceleration formula $v = u + at$, $\left(a = \dfrac{(0.1 - 0.4)}{1}\right)$ alongside the variable $a = k(3t^2 - 12t + 2)$ in a mistaken attempt to find k.

(ii) Although some candidates were unable to show that $k = 0.1$ in part **(i)**, they were still able to use this value in part **(ii)** to obtain a fully correct solution. Others who omitted the constant of integration in part **(i)** used a correct method in part **(ii)** but formed an expression for displacement with '$0.4t$' missing.

(iii) In order to **verify** that the particle was again at the origin, candidates were expected to show the substitution of $t = 2$ leading to a zero displacement. Alternatively, a few opted for a more complicated method of equating their expression for displacement to zero and solving the resulting quartic equation. Unfortunately those who did not have a correct expression in part **(ii)** were unable to show the required result.

Answers: **(i)** Answer Given **(ii)** $s = 0.025t^4 - 0.2t^3 + 0.1t^2 + 0.4t$ **(iii)** Answer Given

Question 6

(i)   Candidates found the distance travelled by particle P, often correctly as 42 m, either from an area on the graph or from the use of constant acceleration formulae. Whilst the question asked for the displacement of P from O, this was very frequently either overlooked or misunderstood, providing the distance instead.

(ii)   The velocity of P was often found correctly. A variety of methods was seen including the use of similar triangles, coordinate geometry or constant acceleration formulae.

(iii)   Whilst candidates often knew how to form and solve an equation in T, their previous values from parts **(i)** and **(ii)** sometimes led to an inaccurate or even impossible T value. A few candidates used a valid method to find the time interval from 10 to T, and added this to 10 rather than directly forming an equation in T.

(iv)   Candidates usually knew how to find V, either using the graph and the gradient property or using $v = u + at$. Many found the distance travelled by Q using their value of T correctly. To find the distance between the particles required an understanding of the significance of the areas above and below the t axis. A common incorrect answer was 17 m (from 66.5 − 49.5) suggesting that the the particles were travelling in the same direction at all times. A few candidates found the difference in distances travelled only after $t = 10$.

Answers: **(i)** 42 m **(ii)** 3 ms^{-1} **(iii)** 15 **(iv)** 7 ms^{-1} and 101 m

Question 7

Although this question proved to be more challenging, most candidates attempted both parts. Accurate solutions in both parts depended on including all relevant forces acting in the appropriate directions.

(i)   Many solutions showed an oversimplified situation in which the force T N was considered only when resolving parallel to the plane. Thus, it was common to see an answer of $T = 0.497$ or $T = 1.57$ following from $T\cos 15^{\circ} \pm 0.3 \times 0.2\,g\cos 30^{\circ} = 0.2\,g\sin 30^{\circ}$. Some candidates included all forces but with the frictional force acting down instead of up the plane to give the 'least' value of T. Thus, it was common to see $T = 1.46$ following from $T\cos 15^{\circ} - 0.3(0.2\,g\cos 30^{\circ} - T\sin 15^{\circ}) = 0.2\,g\sin 30^{\circ}$.

(ii)   Although most candidates attempted an energy method as required, fully correct solutions were in a minority. Some oversimplified the situation to KE gain = PE loss. Some duplicated the potential energy by including the work done by the component of weight down the plane as well. Some omitted the work done by either the 0.25 N force or by the frictional force, and a few used the frictional force from part **(i)**. For those who included all the appropriate terms in their work/energy equation, only the best had correct signs throughout leading to $v = 2.63$ ms^{-1}. The few who used a non-energy method were sometimes able to achieve a correct answer but could only gain limited credit.

Answers: **(i)** 0.541 **(ii)** 2.63 ms^{-1}

MATHEMATICS

> **Paper 9709/51**
> **Paper 51**

Key messages

Candidates should be reminded that an answer should be given to three significant figures. This means that they should work to at least four significant figures.

Occasionally candidates use an incorrect formula. A formula booklet is provided so it is recommended that, when using a formula, they refer to the booklet.

General comments

The paper appeared to be the same standard as the one set last November.

Candidates should be reminded of the need for clear presentation in their working, as per the front cover instructions. The work of a few candidates was rather untidy and sometimes was difficult to read.

Only a handful of candidates gave answers to two significant figures.

It is pleasing to note that most candidates now use $g = 10$ as requested.

The easier questions proved to be **1**, **2**, **4(i)** and **7(i)**.

The harder questions proved to be **5(i)**, **5(ii)** and **6(iii)**.

Comments on specific questions

Question 1

This question was generally well done. Most candidates realised that it was necessary to use Newton's Second Law horizontally, and also to resolve vertically, in order to find the normal reaction and the friction force respectively.

Answer: 0.556

Question 2

This question was quite well done. By using horizontal and vertical motion, u_x and u_y, the initial horizontal and vertical velocities could be found. If θ was the required angle, then $\tan\theta = u_y / u_x$, would give the correct answer. A few candidates found s_x and s_y, the horizontal and vertical distances. They then said $\tan\theta = s_y / s_x$ which was an incorrect method.

Answer: 77.3°

Question 3

(i)   This part of the question was generally well done. Candidates needed to resolve vertically and to also use Newton's Second Law horizontally.

(ii)   This part was not as well done. Too many candidates failed to realise that the reaction was zero.

Answers: **(i)** 4.70 N **(ii)** 1.07

Question 4

(i)   This part of the question was generally well done. Unfortunately a number of candidates did not obey the instructions given in the question. They just used the trajectory equation and substituted the known values into the equation.

(ii)   Most candidates substituted $y = 5$ into the given equation in **part (i)**. Two values of x were found but these values were not subtracted in order to find the required distance.

Answers: **(i)** $y = \dfrac{x}{\sqrt{3}} - \dfrac{4x^2}{375}$ **(ii)** 32.5 m

Question 5

(i)   This part of the question was not particularly well done. The first point that candidates needed to recognise was that zero acceleration occurred at the equilibrium position. The extension at this point was required. The candidates had to then set up a five term energy equation. Too often only four terms were seen.

(ii)   This part of the question proved to be far too difficult for many candidates. A three term energy equation was required starting at the point where the velocity was that found in **part (i)** up to the point where the velocity was zero.

Answers: **(i)** $5\,\text{m}s^{-1}$ **(ii)** 1.2 m

Question 6

Too many candidates tried to involve volumes in their moment equation. This was not necessary as the weights were given.
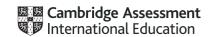
(i)   This part of the question could be solved by taking moments about O.

(ii)   Again it was necessary to take moments about O.

(iii)   Very few candidates were able to answer this part of the question. To do so candidates were required to take moments about the point of contact of the hemisphere and the rough surface. The required equation was $30 \times 0.075\sin 60 = P \times 0.4\sin 60$.

Answers: **(i)** 0.3 m **(ii)** 0.075 m **(iii)** 5.625

Question 7

(i)   This part of the question was usually well done. Candidates were required to find the friction force and then to use Newton's Second Law parallel to the plane.

(ii)   The value of x had to be calculated and this could be done by equating the acceleration to zero. Candidates had then to integrate the equation found in **part (i)**, the correct limits could then be substituted. Quite a number of candidates got $v = \text{f}(x)$ and not $\dfrac{v^2}{2} = \text{f}(x)$.

(iii)   This part of the question was generally well done. Quite a number of candidates got $a = -7.16$ but failed to state that the magnitude was 7.16. so they lost the last mark.

Answers: **(i)** $v\dfrac{\text{d}v}{\text{d}x} = 5\sqrt{3} - 1.5 - 3x$ **(ii)** 2.3867 , $4.13\,\text{m}s^{-1}$ **(iii)** 7.16

MATHEMATICS

Key messages

Candidates should be reminded that an answer should be given to three significant figures. This means that they should work to at least four significant figures.

Occasionally candidates use an incorrect formula. A formula booklet is provided so it is recommended that, when using a formula, they refer to the booklet.

General comments

Candidates should be reminded of the need for clear presentation in their working, as per the front cover instructions. The work of a few candidates was rather untidy and sometimes was difficult to read.

Most candidates now use $g = 10$ as instructed.

The easier questions proved to be **3(i)**, **4(i)** and **6(I)**.

The harder questions proved to be **2**, **5(i)**, **5(ii)** and **7**.

Comments on specific questions

Question 1

This question was reasonably well done. Some candidates were unable to separate the variables correctly and did not know that the integral of e^v was e^v. Some candidates just assumed $c=0$ which was not the case.

Answer: $2.4\,\text{m}s^{-1}$

Question 2

This question proved to be too difficult for many candidates. Too many candidates tried to use volumes of figures. If the weight of the cone was W then that of the cylinder would be $60 - W$. When moments were taken about the base the resulting equation was $\dfrac{0.6}{4} \times W + 0.3(60 - W) = 0.25 \times 60$. From this, W could be calculated.

Answer: $20\,\text{N}$

Question 3

(i) This part of the question was usually well done.

(ii) Some candidates were unable to integrate $\dfrac{v}{\left(5 - 0.5v^2\right)}$ and so failed to score any marks on this part of the question. Of those who did progress correctly some failed to reach a final expression for v in terms of x.

Answers: **(i)** $v\dfrac{dv}{dx} = 5 - 0.5v^2$ **(ii)** $v = \sqrt{\left(10 - 10e^{-x}\right)}$

Question 4

(i) This part of the question was generally well done. Some candidates lost the last mark as they got an answer of 0.8999 … They found the angle the string made with the vertical which resulted in a slight inaccuracy. The answer of 0.9 was stated in the question and so it must be exact.

(ii) Too many candidates failed to find the new extension when the particle came to instantaneous rest. Some candidates only had 3 terms in their energy equation and not 4 as required.

Answers: **(i)** 0.9 **(ii)** 7.54 ms^{-1}

Question 5

Many candidates found this question very difficult. It contained a lot of trigonometry which was not used correctly.

(i) *OG* was often found correctly. A few candidates used the wrong formula. Very few candidates were able to calculate *AG*. *AG* could be found by using the cosine formula on triangle *OGA*.

(ii) This part of the question was only correctly done by a few candidates. The angle *BAG* had first to be found. Triangle *GAX*, where *X* is the mid-point of *BG*, could be used to find the angle which is equal to angle *BAG*. Once this was done then by taking moments about *A* the weight could be found.

Answers: **(i)** 0.572 m **(ii)** 3.55 N

Question 6

(i) This part of the question was generally well done. Candidates needed to find two expressions for the tension. These two expressions were then equated and the resulting equation had to be solved.

(ii) Again candidates were required to set up two equations. quite a number of candidates used EE = 2KE instead of KE = 2EE for the first equation. The second equation needed two values for the tension to be equated.

Answers: **(i)** 0.12(0) **(ii)** 0.4 and 5.66

Question 7

This question proved rather difficult for many candidates.

(i) If *V* and θ were the initial speed and the angle of projection respectively, then by using horizontal and vertical motion, two equations could be found. By using Pythagoras's theorem and trigonometry the values of *V* and θ could be calculated.

(ii) The best method for this part of the question was to set up an energy equation. Alternatively, two equations for the vertical velocity at the ground could be found and then equated.

(iii) Candidates found it very difficult to find the time for the first part of the flight. The second stage time was proved easier to find by many candidates.

Answers: **(i)** 32.9 ms^{-1}, 61.7° to the horizontal **(ii)** 18 **(iii)** 8.87 s

MATHEMATICS

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Paper 9709/53
Paper 53
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Key messages

Candidates should be reminded that an answer should be given to three significant figures. This means that they should work to at least four significant figures.

Occasionally candidates use an incorrect formula. A formula booklet is provided so it is recommended that, when using a formula, they refer to the booklet.

General comments

The paper appeared to be the same standard as the one set last November.

Candidates should be reminded of the need for clear presentation in their working, as per the front cover instructions. The work of a few candidates was rather untidy and sometimes was difficult to read.

Only a handful of candidates gave answers to two significant figures.

It is pleasing to note that most candidates now use $g = 10$ as requested.

The easier questions proved to be **1**, **2**, **4(i)** and **7(i)**.

The harder questions proved to be **5(i)**, **5(ii)** and **6(iii)**.

Comments on specific questions

Question 1

This question was generally well done. Most candidates realised that it was necessary to use Newton's Second Law horizontally, and also to resolve vertically, in order to find the normal reaction and the friction force respectively.

Answer: 0.556

Question 2

This question was quite well done. By using horizontal and vertical motion, u_x and u_y, the initial horizontal and vertical velocities could be found. If θ was the required angle, then $\tan\theta = u_y / u_x$, would give the correct answer. A few candidates found s_x and s_y, the horizontal and vertical distances. They then said $\tan\theta = s_y / s_x$ which was an incorrect method.

Answer: 77.3°

Question 3

(i)   This part of the question was generally well done. Candidates needed to resolve vertically and to also use Newton's Second Law horizontally.

(ii)   This part was not as well done. Too many candidates failed to realise that the reaction was zero.

Answers: **(i)** 4.70 N **(ii)** 1.07

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Question 4

(i)   This part of the question was generally well done. Unfortunately a number of candidates did not obey the instructions given in the question. They just used the trajectory equation and substituted the known values into the equation.

(ii)   Most candidates substituted $y = 5$ into the given equation in **part (i)**. Two values of x were found but these values were not subtracted in order to find the required distance.

Answers: **(i)** $y = \dfrac{x}{\sqrt{3}} - \dfrac{4x^2}{375}$ **(ii)** 32.5 m

Question 5

(i)   This part of the question was not particularly well done. The first point that candidates needed to recognise was that zero acceleration occurred at the equilibrium position. The extension at this point was required. The candidates had to then set up a five term energy equation. Too often only four terms were seen.

(ii)   This part of the question proved to be far too difficult for many candidates. A three term energy equation was required starting at the point where the velocity was that found in **part (i)** up to the point where the velocity was zero.

Answers: **(i)** $5 \, \text{m} s^{-1}$ **(ii)** 1.2 m

Question 6

Too many candidates tried to involve volumes in their moment equation. This was not necessary as the weights were given.
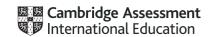
(i)   This part of the question could be solved by taking moments about O.

(ii)   Again it was necessary to take moments about O.

(iii)   Very few candidates were able to answer this part of the question. To do so candidates were required to take moments about the point of contact of the hemisphere and the rough surface. The required equation was $30 \times 0.075 \sin 60 = P \times 0.4 \sin 60$.

Answers: **(i)** 0.3 m **(ii)** 0.075 m **(iii)** 5.625

Question 7

(i)   This part of the question was usually well done. Candidates were required to find the friction force and then to use Newton's Second Law parallel to the plane.

(ii)   The value of *x* had to be calculated and this could be done by equating the acceleration to zero. Candidates had then to integrate the equation found in **part (i)**, the correct limits could then be substituted. Quite a number of candidates got $v = \text{f}(x)$ and not $\dfrac{v^2}{2} = \text{f}(x)$.

(iii)   This part of the question was generally well done. Quite a number of candidates got $a = -7.16$ but failed to state that the magnitude was 7.16. so they lost the last mark.

Answers: **(i)** $v\dfrac{dv}{dx} = 5\sqrt{3} - 1.5 - 3x$ **(ii)** 2.3867 , $4.13 \, \text{m} s^{-1}$ **(iii)** 7.16

MATHEMATICS

Key messages

Candidates should be aware of the need to work to at least 4 significant figures to achieve the required degree of accuracy. Efficient use of a calculator is expected, but candidates should be encouraged to show sufficient workings in all questions to communicate their reasoning.

General comments

Candidates would be well advised to read the question again after completing their solution to ensure they have included all the relevant details.

Candidates who presented their work in clear and ordered fashion often avoided careless mistakes that were seen on other papers.

It was disappointing that a significant numbers of candidates did not appear to have prepared well for this paper and had correspondingly low marks.

Question 3 was generally answered well.

Comments on specific questions

Question 1

Candidates were expected to use both properties of the probability distribution table. Good solutions clearly stated that the total of the probabilities was 1, and showed clearly how to calculate the expected value of X. This provided the 2 equations which could be solved simultaneously to arrive at the values of p and $q.$ Many candidates only produced one equation and were therefore unable to continue. Simple arithmetic and algebraic slips were noted in a number of occasions, often where the work was not presented in a systematic manner.

Answer: $p = 0.25$, $q = 0.2$

Question 2

Although this should have been a question that was accessible to all candidates, the lack of accuracy or attention to detail often led to inaccurate solutions.

(i)   Although not required specifically by the question, the most successful solutions extended the frequency table to record the cumulative frequencies required. Many other solutions generated the required values simply as a list somewhere on the page. Candidates should be aware when labelling the axes; the units need to be stated as well as 'time' to ensure there is clarity of information. As the times were recorded as discrete data, the upper boundary correction needed to be used when plotting the curve. The most accurate graphs were achieved when either the time axis 'translated' so that 5.5 s, 10.5 s etc. were on the main gridlines or the scale was 2 cm = 5 seconds which ensured a gridline was available.

A surprising number of candidates calculated the frequency density, and then either plotted their values as a curve, or constructed a histogram.

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(ii)   Although many candidates read their graphs correctly, a significant number stated the time for which 35 cars accelerated in less than *t* seconds.

Answer: **(ii)** 6.5 sec

Question 3

The best solutions recognised that this was a binomial approximation using practical data. A number of candidates did not use the context in the question and assumed that a fair die was thrown.

(i)   Many candidates recognised that to calculate the probability they needed to divide the average number by the group size. Unfortunately, some candidates only provided an answer to 2 significant figures, which is not acceptable at this level. Better candidates often stated the answer as a fraction which is acceptable and more accurate.

(ii)   This was a standard calculation using the formula *variance = npq*. Candidates are reminded that 'hence' in the question indicates that they must use information from the previous part within the question.

(iii)   The most successful solutions recognised that the probability of scoring 2 or more 4's is 1–P(0,1). When attempted most provided at least one correct binomial term, although a number also subtracted P(2). A number of solutions did attempt to sum the allowable outcomes, but many had arithmetical inaccuracies leading to an incorrect final probability.

Answers: **(i)** 0.207 **(ii)** 4.92 **(iii)** 0.848 or 0.8485

Question 4

(i)   Good solutions approached the problem systematically, calculating the total age of the people in each Art class before summing and dividing by the total number of people. However, the majority of solutions simply averaged the 2 ages ignoring the relative group sizes.

(ii)   Many candidates found this question quite involved and challenging, even though some guidance of an appropriate method was provided. Good solutions stated the formula $\sigma^2 = \dfrac{\sum x^2}{n} - \mu^2$, substituted in the provided information and rearranged to obtain the $\sum x^2$ and $\sum y^2$ as anticipated. They were then able to use the same formula to calculate the required standard deviation using the value of the mean for the group calculated in part **(i)**. Weaker solutions often combined the data without any allowance for the initial group sizes.

Answers: **(i)** 44.8 **(ii)** 8.36 or 8.37

Question 5

(i)   Many candidates did not attempt to draw the tree diagram fully. Good diagrams had clear identification of the outcome for each branch and the probabilities clearly stated. A number of good candidates replaced the *x* on their diagram with the value that they calculated in part **(ii)**. Where the original solution was not clearly visible, this did not fulfil the requirements of the question.

(ii)   Even where the tree diagram was inaccurate, many candidates were able to produce the linear equation for the probability that 'Julian getting a good night's sleep' directly from the initial information. Again, a number of candidate penalised themselves with careless arithmetical errors.

(iii)   Few successful solutions were seen. Many candidates simply calculated the probability of flying economy and getting a good night's sleep. The best solutions recognised that the required denominator could be determined directly from the information provided for part **(ii)**.

Answers: **(ii)** 0.15 **(ii)** 0.975

Question 6

(a) (i) Many candidates recognised the key word 'arrangements' and correctly used permutations to calculate the ways the 5 people could sit in the hall. As the answer here is an integer value, it is expected to be fully stated and not rounded to 3 significant figures.

(ii) Few fully correct solutions were seen. The best solutions followed a systematic approach and considered the number of ways Mary and Ahmad could sit in the front row and then the number of ways the other 3 people could sit in the remaining 7 rows. The most common error was to add these answers as most did not consider that the seating arrangements for the 3 would be possible for each of the ways that Mary and Ahmad could sit in the front row.

(b) Again few fully correct answers were seen. The most efficient method was to calculate the total number of teams of 4 and then subtract the number of teams which had both Ross and Lionel in. Most attempted to calculate the number of ways of choosing teams with either Ross or Lionel in and then adding to the number of teams without them. Poor presentation often resulted in partial solutions, or each value being calculated but not being summed to achieve a total.

Answers: **(a)(i)** 78960960 **(ii)** 1008 **(iii)** 182

Question 7

The best solutions used a sketch of the normal curve to help clarify the information within the question.

(i) Candidates who worked systematically and calculated the small pineapples first were often more successful. As the data was continuous, no continuity correction was required in the normal approximation when standardising. Most candidates stated the formula correctly, but there was some confusion when operating the tables or interpreting their answers. A few candidates did not use the property that probabilities sum to 1, with weaker candidates having the total of their probabilities being greater than 1.

(ii) Where attempted, there was a good understanding of this part. Common errors were not using the critical value as stated on the normal distribution table or inaccurate algebraic manipulation of the standardisation formula.

(iii) Many solutions showed a lack of understanding of the normal approximation and the information that the normal distribution table provides. The best solutions reduced their work by recognising the symmetry between $P(x > 610)$ and $P(x < 390)$, while many candidates repeated the work undertaken in part **(i)**. Those candidates who had sketched the scenario were often successful in identifying the area required and achieving the final answer.

Answers: **(i)** Large: 0.222, Medium: 0.663, Small: 0.115 **(ii)** 651 **(iii)** 520

MATHEMATICS

Key messages

Candidates are reminded that to achieve non-exact answers correct to 3 significant figures, all calculations should be carried out using at least 4 significant figures.

Candidates are advised to ensure that workings are submitted within their solutions to communicate their thinking, as there are occasions when a correct answer value may not be sufficient to justify full credit. The workings also enable marks to be awarded where there are errors in the process.

Candidates should be aware that when they are required to show that a statement is true, as in **Question 3(i)**, simply writing a calculation will not gain full credit unless it is clearly linked to the context.

General comments

Many candidates seemed well prepared for the examination, and there were many praiseworthy solutions seen to each of the syllabus areas.

Candidates who used diagrams to interpret the information stated in the questions were often more successful, especially when considering the normal distribution. Many candidates were able to gain credit in **Question 6** because their diagram indicated the approach that they were seeking to achieve, even if there was confusion over the details of the data provided.

Candidates who had prepared well appeared to have sufficient time to attempt all questions. However, a number of candidates failed to follow the instructions stated within questions, or to appreciate that the skills developed in Pure Mathematics 1 may be used in this component.

Comments on specific questions

Question 1

This question was focusing on interpreting the coded mean and was well attempted by the majority of candidates. The most common approach was to expand the brackets and equate to the mean, although there were often slips within the algebra. A number of candidates quoted $\text{mean} = \dfrac{\sum(x-a)}{n} + a$ and were able to solve simple substitution and rearrangement

Answer: 18

Question 2

Although this should have been a question that was accessible to all candidates, the lack of accuracy or attention to detail often led to inaccurate solutions. A significant number of candidates used scales that did not cover the full range of values and/or used non-linear scales.

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(i)   Although not required specifically by the question, the most successful solutions extended the frequency table to record the cumulative frequencies required. Many other solutions generated the required values simply as a list somewhere on the page. This may have led to the inappropriate end values being used when plotting the graph. Candidates should be aware when labelling the axes; the units need to be stated with the circumference to ensure there is clarity of information.

A surprising number of candidates calculated the frequency density, and then either plotted their values as a curve, or constructed a histogram.

(ii)   Although most candidates read their graphs correctly, many did not answer the question and simply stated the number or percentage of trees with a circumference less that 75 cm. Candidates should be aware of the need to use their graphs accurately at this level.

Answer: **(ii)** 61.4%

Question 3

Candidates who read the context in this question carefully and identified that the order of events was essential were often more successful.

(i)   Candidates were required to show that the probability that the third disc picked was blue was 1/15. Many candidates obtained the expected value by considering that the first disc was blue and the remaining discs were red, and hence gained no credit. The use of a tree diagram was often helpful, but not sufficient in itself to gain full credit unless both the calculation and identifying clearly the required branch were present. A number of good candidates inappropriately assumed that as the probability of the third disc being blue was 1, it did not need to be included in their proof.

(ii)   It was encouraging that most solutions included an appropriate probability distribution table, even if the values for the probabilities were inaccurate. Weaker candidates simply substituted the given value. Many candidates included incorrect additional values for X and occasionally probabilities that were either individually or in total greater than 1.

Answers: **(ii)**

X	1	2	3
P(X)	10/15	4/15	1/15

Question 4

Many candidates found the context of this question challenging. Candidates who presented their work in a structured manner were most successful in including all the elements of the solution. A number of candidates did not use the fact that the die was tetrahedral, and used a standard die throughout.

(i)   The majority of candidates successfully recognised that if the die lands on a 4, then the biased coin needed to be thrown 4 times, and calculated the probability appropriately. Few candidates recognised the link to combinations required to calculate the number of ways that these 4 coins could be thrown to gain 2 heads, simply stating 4 rather than $^4C_2$. A surprising number of candidates did not multiply their result by the probability of throwing a 4, even when they had previously stated it.

(ii)   This part was answered well by the majority of candidates. Again, a small number of candidates did not include the probability of gaining a 3 on the die in their calculation.

(iii)   Although this was an extension of **(ii)**, a number of candidates did not attempt having gained credit previously. The best solutions took a systematic approach considering the different possible conditions. Occasional arithmetical errors were noted in handling the fractions; candidates who worked with decimals were more likely to have inaccuracies occur from premature approximations.

Answers: **(i)** 2/27 **(ii)** 1/108 **(iii)** 10/81

Question 5

Many good attempts at this question were seen, with candidates outlining their approach before attempting calculations. Although part **(ii)** was a fairly standard normal distribution question, many solutions became inaccurate as the 3 significant figure value was used from part **(i)** rather than a value accurate to at least 4 significant figures. Candidates should be aware that the efficient use of a calculator is anticipated, and this can include retaining accurate values to use in later parts of the question.

(i) The majority of candidates recognised that this was a binomial approximation. Good solutions stated that the probabilities of having 0, 1 or 2 faulty CDs needed to be calculated and subtracted from 1. A number of candidates interpreted 'more than 2' to include 2 and so lost accuracy marks. A small number of candidates attempted to use the normal approximation which could gain no credit.

(ii) Almost all candidates recognised that a normal approximation was appropriate. Good solutions used an accurate value of their probability from **(i)** to calculate the mean and variance, used an appropriate continuity correction in the normal approximation and then recognised the required probability area by having a simple sketch. The most common error was to use the probability that an individual CD was faulty rather than the box being rejected when calculating the mean and variance. This often led to z-values which could not be interpreted.

Answers: **(i)** 0.117 **(ii)** 0.726

Question 6

The best solutions included diagrams to illustrate the set of conditions that were being considered.

(a)(i) Many candidates found the context of this question challenging. More successful solutions often considered the possible arrangements for a specific first digit, and then multiplying their total by 4. The best solutions used a simple diagram to indicate how the digits could be arranged within the conditions, and then use a simple calculation to produce the total. Weaker candidates often omitted numbers in the form 3** as not being greater than 300. A few candidates introduced digits that were not present on the cards stated within the question

(ii) Although this question was really an extension of the previous part, many candidates who had made a credible attempt at **(i)** chose to omit **(ii)**. Many candidates again successfully considered the arrangements for each lead digit and then combining their totals. The best solutions considered how many different digits were available for each position in the number and then simply multiplied.

(b)(i) Almost all candidates answered this question correctly. The anticipated approach was to consider the combinations that were possible to pick 5 people from a group of 10. Some candidates considered the different combinations of boys and girls that could form a group of 5 and then summed their answers. As only 1 mark was available for this question, candidates should be expecting that a simple approach should be possible.

(ii) This question was answered well by many candidates. Again, a systematic approach ensured that all possible arrangements were considered, although a common error was to omit the team consisting of 5 boys and 0 girls. A small number of candidates incorrectly used permutations, or exchanged the multiplication and additions.

Answers: **(a)(i)** 80 **(ii)** 96 **(b)(i)** 252 **(ii)** 186

Question 7

Candidates should be reminded that a simple sketch of the normal distribution can assist both interpretation of the information and identify the required probability or value.

(i) All but the weakest candidates successfully answered this question, giving their answers to an appropriate degree of accuracy. The common errors were the introduction of a continuity correction or failing to subtract from 1 to achieve the appropriate probability. An unusual error seen quite regularly was subtracting the z-value from 1 before using the tables to find the probability.

(ii) Many candidates found this the most challenging question on the paper. The best solutions used a sketch of the normal distribution to identify the region required, which made the symmetry link with part **(i)** much clearer. The majority of candidates reworked the probability of the weight being less than 65 kilograms and then combined with the 25% from the question. Evidence of poor understanding of the normal approximation was seen, as *z*-values and probabilities were frequently added, or by using the normal distribution tables incorrectly when converting.

(iii) This question was answered well by many of the candidates who attempted. Good solutions clearly identified the 2 normal approximations for the conditions stated within the question which created a pair of simultaneous equations. At this level, candidates are expected to use the critical values which are included on the normal distribution table. Many candidates rearranged these to make σ the subject of both which were then equated and solved. Other successful alternative approaches were seen, although the more complex approach of rearrangement and replacement could lead to algebraic errors. Weaker candidates who equated the normal approximations to the stated probabilities were able to gain credit here.

Answers: **(i)** 0.385 **(ii)** 75.0 **(iii)** $\mu = 61.9$, $\sigma = 15.2$

MATHEMATICS

Key messages

Those candidates who found the neatest ways to answer some of the more challenging questions, particularly **6(ii)** and **6(iii)**, seemed to have no problem in finishing the paper in the allocated time. However, a significant number of candidates gave no response to the four parts of **Question 7** – a clear indication that they need to learn to manage their time more efficiently. Candidates also need to be encouraged to reread the question once they believe they have found the solution to ensure that they have answered the question. This may have helped the significant number of entrants who did not spot that **Question 6(ii)** asked for a probability or that **Question 6(iii)** asked for the number of selections and not a probability.

It is expected that candidates who use sophisticated calculators when working with binomial and normal distributions clearly communicate the appropriate expression prior to evaluation.

General comments

This paper proved accessible to most candidates. Coding in **Question 2** and permutations in **Question 6** produced the greatest challenges. Few candidates scored full marks in **Question 5**, mostly because of poor labelling. Even those who recognised the need for a key in **5(i)** frequently omitted to mention the word 'medals' either in the title or the key. Similarly, Box and Whisker plots in **5(ii)** were often presented without the word 'medals'.

Some candidates lost marks as a result of premature approximation and need to be aware that if their final answer is to be correct to 3 significant figures, their input numbers must be correct to at least 4sf. This was particularly apparent in **Question 7(iii)** when they were adding their binomial terms. They also need to be aware that they must use the correct statistical tables which require them to work with numbers correct to 4sf.

Comments on specific questions

Question 1

This question was well answered with most candidates scoring full marks. The best candidates found the shortest method which was to subtract the probability of no-one completing the survey from 1. Those who used a tree diagram took more space and time but generally achieved the correct answer. The candidates who attempted to sum the probabilities of one, two or three people completing the survey were generally less successful, often omitting the coefficients of the binomial terms.

Answer: 0.488

Question 2

A pleasing number of candidates correctly found $\Sigma(x - 45)$ although some misinterpreted the expression as meaning E$(x–45)$ and gave 15.9 as their answer while others gave 1218 – 45 = 1173.
Candidates were less successful in finding $\Sigma(x - 45)^2$. A common error was to mix the coded and uncoded values in the variance formula for $(X - 45)$, usually using Σx rather than their $\Sigma(x - 45)$. Very few of those who chose the much longer method of finding Σx^2 and then expanding $\Sigma(x - 45)^2$ were successful. Many who successfully calculated Σx^2 as 74529 either presented this as the final solution or made a mistake in the expansion of $\Sigma(x - 45)^2$, often omitting to multiply 45 squared by 20.

Answers: 318, 5409

Question 3

(i)   This question was very well answered with most candidates scoring full marks. Very few added extra branches after Pass at the first attempt and, of those who did, some correctly appreciated that a Pass now had a probability of 1.

(ii)   A pleasing number of candidates successfully applied the conditional probability formula and scored full marks. Many of those who did not recognise the conditional probability gave the answer as 0.0975, having interpreted the question as asking for the probability of failing the first test and gaining a certificate.

   Answer: **(ii)** 0.103

Question 4

(i)   Those who understood the wording of the question were usually successful. A surprising number of candidates misunderstood the word 'subtracting', seeming to think it meant 'dividing'. It was common to see ¼, 0, 9/4 and 9 for the values of X. A few candidates did not attempt to calculate a score at all and presented the numbers on the die in their probability distribution. Some showed the x-value 0 in their table three times, each with the probability of 1/6. A few others gave each probability of 0 as ½, meaning that the total of their probabilities came to 2.

(ii)   Most candidates were able to use the correct method to find E(X) although a few did not appreciate the requirement for the probabilities to sum to 1. A few candidates prematurely approximated 1/6 to 0.167, giving them the wrong answer.
   Finding the variance caused a few more problems with only the best candidates correctly squaring the negative score and remembering to subtract the square of E(X). A number of others divided their answers for both E(X) and Var(X) by 4 or 6.

   Answers: **(i)**

x	−3	0	5	32
Prob	1/6	1/2	1/6	1/6

   (ii) 17/3, 144

Question 5

(i)   The best candidates appreciated that all numbers in the stem from 0 to 10 needed to be present and that the leaves needed to be lined up underneath each other, without commas, and increasing from the stem. They also appreciated that a key with units is an essential part of a stem and leaf diagram and that all the leaves with the stem number 1 needed to be on a single line to highlight the shape of the graph.

(ii)   Most candidates successfully identified the median and quartiles as the 7th, 14th and 21st items, with only a few presenting the position numbers as their values. A more common error was to give the upper quartile as the mean of the 20th and 21st items of data.
   Drawing the Box Plot proved to be more challenging. Those who were successful chose a sensible scale, identified at least three points on their uniformly scaled line from 0 to 104 and added the label 'medals'. They also appreciated that the 'whiskers' do not pass through the 'box' and that they had not been asked to identify outliers.

   Answers: **(ii)** Median = 17, LQ =10, UQ = 35

Question 6

(i)   Most candidates answered this question correctly with the most frequent incorrect responses being 18C5 or 13P5.

(ii)   This proved to be the most challenging question on the paper. The strongest candidates identified the number of ways the five cars could be together as 14 × 5!, divided this answer by **(i)** to find the probability and subtracted from 1.
A surprising number of candidates started this method and then presented their answer as 1 026 480 which is the number of ways the cars can be arranged so that they are not all next to each other, rather than the probability. More careful reading of the question should have ensured that this large number could not be presented as a probability.
A few candidates attempted the daunting task of listing all the possible scenarios where the cars would not all be next to each other and summing the number of ways. None of them identified all six ways and most managed to gain only one mark if they divided their attempt by **(i)** to create a probability.

(iii)   This question was done well by the majority of candidates. The neatest solution was to subtract the number of ways of having three cars all the same colour from the number of ways of choosing three cars. However, most candidates chose to list the seven ways of combining the colours and summed them and they were generally successful although some forgot the case where there are three different colours.
A more complicated approach seen was to use probabilities. Of those who successfully calculated the probability of selecting cars of at least two different colours (41/44), very few remembered to multiply the sum of their probabilities by 12C3, the number of ways of selecting three cars.

Answers: **(i)** 1 028 160, **(ii)** 0.998, **(iii)** 205

Question 7

(i)   Most candidates knew to standardise and the majority of those gained full marks. Some were tricked by the *z*-value of 0.3333 and treated it as a probability, subtracting it from one before using the tables. A few squared or square rooted the standard deviation when standardising and some thought that a continuity correction was appropriate. A pleasing number knew to subtract the probability from one to find the correct area.

(ii)   This question was well answered by most candidates. There were still some who used the tables the wrong way round and equated their standardisation to a probability and others who obtained the correct *z*-value of 1.645 but subtracted 1 from it before equating to the standardised expression. Only a few prematurely approximated the *z*-value to 1.65.

(iii)   Those who appreciated that this was a question about the binomial distribution usually went on to obtain the correct answer although there were a few who included an extra term or who made calculation errors. Some incorrectly assumed a normal distribution either to obtain a probability of 0.53 ($np = 10$, $p = 5.3$) for the binomial terms or to equate a standardised expression to 0.3. Premature approximation occurred more in this question than any other, resulting in an incorrect final answer.

(iv)   Many candidates struggled with the wording of this question. Those who realised that the number of minutes needed to be less than zero generally achieved the correct probability although a surprising number used a 3- figure *z*-value instead of 4 when using the tables which resulted in an inaccurate final answer. Those candidates who changed P($t < 0$) to the P($t > 10.6$) risked losing marks if they did not make their reasoning clear.

Answers: **(i)** 0.370, **(ii)** 8.75, **(iii)** 0.988, **(iv)** 0.0058

MATHEMATICS

**Paper 9709/71
Paper 71**

Key messages

For a situation where a normal distribution has been applied to a significance test, two stages are required for the calculation of the probability of a Type II error. Firstly the acceptance region for the null hypothesis is required. Secondly this is used with the new parameter value. Diagrams can be of help.

Working and answers are expected to be of at least 3 significant figures. Some questions request a specific level of accuracy, which must be stated to gain full marks. This was the case for **Question 3** and for **Question 4(i)**.

General comments

Many candidates presented their work clearly and completely, showing the necessary steps in their solutions. This was particularly important in solutions to longer questions. For the significance test questions many candidates stated the essential comparisons very clearly.

Comments on specific questions

Question 1

Most candidates used the correct normal approximation N(31, 31) and standardised with mean = 31 and variance = 31. This distribution is appropriate as the Poisson parameter 31 is greater than 15. A few candidates standardised with standard deviation 31 which was incorrect.

Many candidates applied the correct continuity correction and used 40.5. Other candidates used 39.5 or omitted the correction and used 40. In order to find P ($X > 40$) the upper tail was required.

Answer: 0.0441 or 0.0440

Question 2

The number of seats sold followed the binomial distribution B(403, 1/100), where n (= 403) was greater than 50 and np (= 4.03) was less than 5. Hence the suitable approximation was the Poisson distribution P_o(4.03).

Some candidates applied this distribution correctly to find the sum of the first three Poisson terms.

Other candidates used different incorrect values for the parameter λ such as 4 or 0.04 or 0.01.

A normal distribution was not suitable. The binomial calculation was not an approximation as requested, but was allowed to score a special case mark for 0.232.

Answer: 0.234

Question 3

The standard deviation of the proportion of adults who voted was required, namely $\sqrt{}$, based on the proportion who said that they had voted, namely 153 / 200.

Some candidates used these correctly to find the value of $z = 2.335$.

Other candidates used incorrect values for the proportion, such as 0.695 or 0.835.

The value of $z = 2.335$ gave a probability of 0.9902.

Some candidates thought incorrectly that this gave the value of α to be 99%.

It was necessary to find '$2p - 1$' or '$p - (1 - p)$' or equivalent.

The question requested that the answer should be given correct to the nearest integer. This accuracy had to be seen to score full marks.

Answer: 98

Question 4

(i)   Many candidates found the correct confidence interval by using the mean 300.1 and the standard deviation ($\frac{0.9}{\sqrt{75}}$) for means of samples and the correct z value (2.576). Some candidates used an incorrect value for z (often 2.326). Other candidates used an incorrect form for the standard deviation (such as 0.9 / 75). Other candidates found only one end of the interval. Other candidates stated their answers as two separate values instead of as an interval. Two acceptable forms are in the answer below.

The question requested that the answer should be given correct to 2 decimal places. This accuracy had to be seen to score full marks.

(ii)   Many candidates correctly referred to their confidence interval (for example '300 mm mean length is contained in the interval') and stated their conclusion (for example 'claim is supported'). Both of these parts were necessary. A follow through mark was allowed here for candidates who had found an incorrect confidence interval. Many candidates did state their conclusion in an appropriate non-definite form such as 'claim supported' or 'claim justified' rather than 'claim is proved'.

Answers: **(i)** 299.83 to 300.37 or (299.83, 300.37), **(ii)** The confidence interval includes 300 so the claim is supported

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(i)   Many candidates answered this question correctly. A few candidates omitted the 'x', so did not score any marks. A few candidates multiplied by the ¼ incorrectly.

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Answers: **(i)** 7/6 or 1.17, **(ii)** 1.24 or $\sqrt{5} - 1$

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Answers: **(i)** mean = 288 variance = 1296, **(ii)** 0.653

Question 7

(i) Many candidates stated the two hypotheses correctly, using 'greater than 3.5' in H_1. Hypotheses stated in terms of 0.7 (per day) were also accepted, though $\lambda = 3.5$ (for 5 days) was required for the calculations. Some candidates omitted these. To test the result of 5 cars sold in the 5 days it was necessary to find the probability of the 'tail', $P(X \geqslant 5)$ by calculating '$1 - P(X \leqslant 4) = 0.275$'. Then the comparison for the test was $0.275 > 0.10$. Many candidates made the error of finding '$1 - P(X \leqslant 5) = 0.142$', which was the probability of '6 or more'. The comparison could still gain some marks. Other candidates found only the single term $P(X = 5) = 0.132$. This led to an invalid comparison.

The correct comparison could use the compliments. Thus $0.725 < 0.90$ was valid. Some candidates mixed these incorrectly, stating for example '$0.725 > 0.10$', which was invalid.

The conclusion was that the null hypothesis should not be rejected and hence that there was no evidence to suggest that the sales per day had increased.

Some candidates attempted the critical region method. Many of these candidates omitted some of the essential steps. It was necessary to find P(greater than or equal to 6) = 0.142 and P(greater than or equal to 7) = 0.065 and to deduce that the critical region was 7 or more. Also the result of 5 cars had to be stated as being not in the critical region and hence the conclusion.

(ii) Many candidates found the efficient method of combining the sales at the two showrooms to give a Poisson distribution with $\lambda = 3.9$ and then found $P(X = 2)$. Some candidates used incorrect values for λ such as 1.3 or 2. A few candidates incorrectly found P(0, 1, 2). Some candidates successfully used the longer method of dealing with the two showrooms separately with parameters 2.1 and 1.8 and found the probabilities of the three combinations P(2,0), P(1,1) and P(0,2) and summed these.

Answers: **(i)** no evidence to suggest that the sales per day have increased, **(ii)** 0.154

Question 8

(i) Many candidates displayed a sound understanding of the requirements needed to carry out the test, finding unbiased estimates, standardising, comparing and concluding. Some candidates stated the hypotheses correctly, using population mean or μ and 'less than 0.185 'for H_1. Other candidates incorrectly used p or x. Other candidates omitted them. Most candidates found 0.18. Many candidates found the unbiased estimate for the variance. Other candidates substituted incorrectly in their choice of variance formula, for example

$s^2 = \frac{1}{149}\left(5.01 - \frac{0.18^2}{150}\right) = 0.0336227$. A few candidates incorrectly found only the sample

$\therefore s = 0.183365$

variance. Many candidates omitted the essential 150 when standardising. This gave $z = 0.158$ instead of 1.93. The comparison could be carried out in various ways. Using the z values the main result was $-1.930 > -2.326$ or $1.93 < 2.326$, but signs had to be consistent. Using the areas or probabilities gave $0.0268 > 0.01$. The complements could be used, but a mix such as $0.9732 > 0.01$ was incorrect. Other candidates attempted the critical value method where various errors were seen in 0.185, 2.326 and 150, and in the comparison. The conclusion depended on a correct comparison method.

(ii)   Two stages were required to find this probability. Many candidates attempted only the second stage. Some credit was given for such attempts. For the first stage the acceptance region for H_0 needed to be found, using 0.185 and 150 and –2.326. The region was found to be $X > 0.179$.

For the second stage 0.17897 was standardised using 0.175 and 150 and the smaller area selected for the probability. Diagrams could help with these parts. A follow through method mark was available for selecting the appropriate area following certain errors.

Answers: **(i)** no evidence that the mean concentration with the drug is less than without the drug **(ii)** 0.0625 also answers in the range 0.0610 to 0.0628 accepted

MATHEMATICS

Key messages

It is important that candidates not only learn how to apply statistical techniques, but also understand and can explain the underlying theory. When a question asks for an explanation 'in the context of the question' text book definitions will not be accepted, the answer must relate to the situation given in the question. See comments below on **Questions 1(a)(ii)**, **3(ii)** and **6(iii)**.

General comments

In general, candidates scored well on **Questions 1** (excluding part **1(a)(i)**) and **5** whilst **Question 6**, particularly parts **(ii)** and **(iii)**, was more demanding. **Question 4**, on probability density functions, proved slightly more demanding than has been the case on this type of question in the past. Candidates were largely able to interpret the requirements of the question and to demonstrate and apply their knowledge, though as is often the case, the questions requiring explanations and understanding of statistical theories were not well answered (**Questions 1(a)(ii), 3(ii) and 6(iii)**). There was a complete range of scripts from good ones to poor ones.

Most candidates kept to the required level of accuracy, and timing did not appear to be a problem for candidates.

Detailed comments on individual questions follow. Whilst the comments indicate particular errors and misconceptions, it should be noted that there were also some good and complete answers.

Comments on specific questions

Question 1

This was a reasonably well attempted question, though part **1(a)(ii)**, which required justification of the approximation used, was poorly attempted. It is important that candidates appreciate the conditions for which a binomial distribution can be approximated to a Poisson distribution; some candidates did not give the conditions fully.

Errors in part **1(a)(i)** included omission of one term in the Poisson expression and in part **(b)** most candidates were able to find the mean, but many made errors calculating the standard deviation or left their answer as the variance; it is important for candidates to read the question carefully.

Answers: 0.721
         n large and p small
         5.6 2.37

Question 2

Many candidates were able to successfully construct the required confidence interval in part **(i)**. Common errors included an incorrect *z* value, and a confusion in the use of the formula (1420, rather than$\sqrt{1420}$, was required). It is important that the final answer is written as an interval rather than two separate answers as was occasionally seen. Part **(ii)** was not so well attempted. In order to start the solution, either the value of $\bar{x}$ or the width of the interval needed to be calculated. Many candidates incorrectly used the value 4820 from part **(i)** and hence made little progress with this part of the question. Those who made a correct start often went on to a correct final answer, though some candidates failed to give the final answer as an integer or incorrectly chose 396.

Answers: 4524 to 5115
         395

Question 3

Part **(i)** was quite well attempted, though fully correct answers were relatively few; some candidates omitted to state null and alternative hypotheses, and some calculated a biased estimate rather than an unbiased estimate of the population variance. Accuracy caused a loss of marks on this question for some candidates (mainly caused by premature approximation or truncating their calculated values). Many candidates also lost marks when calculating the *z* value by omitting $\sqrt{100}$ in the calculation. As has been stated before, it is important that the comparison of *z* values or areas is clearly shown; and if the 'rejection region' is referred to as part of the comparison then this must be clearly defined or labelled on a diagram. The comparison was unfortunately not performed well here by a good number of candidates. It is good practice for final conclusions to be written in context, and expressed in a non-definite way.

Part **(ii)** was not well attempted. It is important that candidates understand when the Central Limit theorem needs to be applied, and also be able to explain its use or non-use fully. There were cases when candidates made statements such as 'it is a normal distribution', this was not sufficient to score the marks. The statement must clearly state that 'the parent population is normally distributed' (i.e. 'it' needs to be qualified), therefore the CLT does not need to be applied.

Answers: No evidence that the mean mass is less than 1.01
         Distribution of *X* normal so CLT not used

Question 4

In part **(i)** information was given which led to two equations in '*a*' and '*k*' which could then be solved simultaneously. Whilst there were some good elegant solutions here, there were also candidates who did not present logical solutions, and as it was required to show that the value of *k* was 1/6, it was important that all relevant working was clearly shown. Marks can be withheld for lack of essential working. Many candidates made a good start to the question, but not all successfully found the two correct, relevant equations to reach the required values of *a* and *k*. Algebraic errors when dealing with indices often caused a loss of marks in solving the simultaneous equations.

In part **(ii)** many candidates successfully found the median value, though algebraic errors were again seen here in some solutions; there were candidates who correctly reached $\sqrt{m}$ =3/2 but square rooted 1.5 rather than squaring it.

Answers: *a* = 9
         2.25

Question 5

This question was well attempted with many candidates successfully finding the required probabilities. Errors included incorrect values for the mean and, more commonly, the variance. Standardising was generally done well (though continuity corrections were occasionally incorrectly applied) and a common loss of marks came from choosing the wrong tail probability.

Answers: 0.952
         0.0655

Question 6

Candidates found this question particularly demanding. A Poisson probability of P(X≤2) with λ = 5.15 was required. Common errors included using λ = 1.03, calculating P(X<2) or P(X=2). A comparison with 0.1 should have been clearly shown in order to reach a conclusion. This comparison was not always clearly shown. Again it is good practice to give a conclusion that is in context and not definite.

In part **(ii)** the full justification of the probability of a Type I error was not always given; it was important to state that P($X \leqslant 1$)= 0.0357 was less than 0.1 whilst P($X \leqslant 2$)= 0.113 was greater than 0.1 hence showing that the required probability was 0.0357. Not all candidates gave a fully justified solution, and many candidates were unable to make a reasonable start to this part of the question.

It was important in part **(iii)** that the context of the question was clearly referred to in the candidate's answer as well as making it perfectly clear what conclusion was drawn as opposed to the reality.

Answers: No evidence to believe mean number of defectives has decreased
0.0357
Mean no of defectives not actually reduced, but conclude that it is reduced

MATHEMATICS

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Paper 9709/73
Paper 73
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Key messages

For a situation where a normal distribution has been applied to a significance test, two stages are required for the calculation of the probability of a Type II error. Firstly the acceptance region for the null hypothesis is required. Secondly this is used with the new parameter value. Diagrams can be of help.

Working and answers are expected to be of at least 3 significant figures. Some questions request a specific level of accuracy, which must be stated to gain full marks. This was the case for **Question 3** and for **Question 4(i)**.

General comments

Many candidates presented their work clearly and completely, showing the necessary steps in their solutions. This was particularly important in solutions to longer questions. For the significance test questions many candidates stated the essential comparisons very clearly.

Comments on specific questions

Question 1

Most candidates used the correct normal approximation N(31, 31) and standardised with mean = 31 and variance = 31. This distribution is appropriate as the Poisson parameter 31 is greater than 15. A few candidates standardised with standard deviation 31 which was incorrect.

Many candidates applied the correct continuity correction and used 40.5. Other candidates used 39.5 or omitted the correction and used 40. In order to find P $(X > 40)$ the upper tail was required.

Answer: 0.0441 or 0.0440

Question 2

The number of seats sold followed the binomial distribution B(403, 1/100), where n (= 403) was greater than 50 and np (= 4.03) was less than 5. Hence the suitable approximation was the Poisson distribution P_o(4.03).

Some candidates applied this distribution correctly to find the sum of the first three Poisson terms.

Other candidates used different incorrect values for the parameter λ such as 4 or 0.04 or 0.01.

A normal distribution was not suitable. The binomial calculation was not an approximation as requested, but was allowed to score a special case mark for 0.232.

Answer: 0.234

Question 3

The standard deviation of the proportion of adults who voted was required, namely $\sqrt{}$, based on the proportion who said that they had voted, namely 153 / 200.

Some candidates used these correctly to find the value of $z = 2.335$.

Other candidates used incorrect values for the proportion, such as 0.695 or 0.835.

The value of $z = 2.335$ gave a probability of 0.9902.

Some candidates thought incorrectly that this gave the value of α to be 99%.

It was necessary to find '$2p - 1$' or '$p - (1 - p)$' or equivalent.

The question requested that the answer should be given correct to the nearest integer. This accuracy had to be seen to score full marks.

Answer: 98

Question 4

(i)   Many candidates found the correct confidence interval by using the mean 300.1 and the standard deviation ($\frac{0.9}{\sqrt{75}}$) for means of samples and the correct z value (2.576). Some candidates used an incorrect value for z (often 2.326). Other candidates used an incorrect form for the standard deviation (such as 0.9 / 75). Other candidates found only one end of the interval. Other candidates stated their answers as two separate values instead of as an interval. Two acceptable forms are in the answer below.

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