Cambridge
International
A Level

## MATHEMATICS

9709/53
Paper 5 Mechanics 2 (M2)
May/June 2016
1 hour 15 minutes

## Additional Materials: Answer Booklet/Paper

 Graph Paper List of Formulae (MF9)
## READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet. Write your Centre number, candidate number and name on all the work you hand in.

Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO NOT WRITE IN ANY BARCODES.

Answer all the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
Where a numerical value for the acceleration due to gravity is needed, use $10 \mathrm{~m} \mathrm{~s}^{-2}$.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question. The total number of marks for this paper is 50 .
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

1 A small ball is projected with speed $16 \mathrm{~m} \mathrm{~s}^{-1}$ at an angle of $45^{\circ}$ above the horizontal from a point on horizontal ground. Calculate the period of time, before the ball lands, for which the speed of the ball is less than $12 \mathrm{~m} \mathrm{~s}^{-1}$.

2


A uniform wire has the shape of a semicircular arc, with diameter $A B$ of length 0.8 m . The wire is attached to a vertical wall by a smooth hinge at $A$. The wire is held in equilibrium with $A B$ inclined at $70^{\circ}$ to the upward vertical by a light string attached to $B$. The other end of the string is attached to the point $C$ on the wall 0.8 m vertically above $A$. The tension in the string is 15 N (see diagram).
(i) Show that the horizontal distance of the centre of mass of the wire from the wall is 0.463 m , correct to 3 significant figures.
(ii) Calculate the weight of the wire.

3 A particle $P$ of mass 0.4 kg is released from rest at a point $O$ on a smooth plane inclined at $30^{\circ}$ to the horizontal. When the displacement of $P$ from $O$ is $x \mathrm{~m}$ down the plane, the velocity of $P$ is $v \mathrm{~m} \mathrm{~s}^{-1}$. A force of magnitude $0.8 \mathrm{e}^{-x} \mathrm{~N}$ acts on $P$ up the plane along the line of greatest slope through $O$.
(i) Show that $v \frac{\mathrm{~d} v}{\mathrm{~d} x}=5-2 \mathrm{e}^{-x}$.
(ii) Find $v$ when $x=0.6$.


A uniform solid cone has base radius 0.4 m and height 4.4 m . A uniform solid cylinder has radius 0.4 m and weight equal to the weight of the cone. An object is formed by attaching the cylinder to the cone so that the base of the cone and a circular face of the cylinder are in contact and their circumferences coincide. The object rests in equilibrium with its circular base on a plane inclined at an angle of $20^{\circ}$ to the horizontal (see diagram).
(i) Calculate the least possible value of the coefficient of friction between the plane and the object.
(ii) Calculate the greatest possible height of the cylinder.

5 A particle is projected at an angle of $\theta^{\circ}$ below the horizontal from a point at the top of a vertical cliff 26 m high. The particle strikes horizontal ground at a distance 8 m from the foot of the cliff 2 s after the instant of projection. Find
(i) the speed of projection of the particle and the value of $\theta$,
(ii) the direction of motion of the particle immediately before it strikes the ground.

6


A light inextensible string passes through a small smooth bead $B$ of mass 0.4 kg . One end of the string is attached to a fixed point $A 0.4 \mathrm{~m}$ above a fixed point $O$ on a smooth horizontal surface. The other end of the string is attached to a fixed point $C$ which is vertically below $A$ and 0.3 m above the surface. The bead moves with constant speed on the surface in a circle with centre $O$ and radius 0.3 m (see diagram).
(i) Given that the tension in the string is 2 N , calculate
(a) the angular speed of the bead,
(b) the magnitude of the contact force exerted on the bead by the surface.
(ii) Given instead that the bead is about to lose contact with the surface, calculate the speed of the bead.

7 A particle $P$ is attached to one end of a light elastic string of natural length 1.2 m and modulus of elasticity 12 N . The other end of the string is attached to a fixed point $O$ on a smooth plane inclined at an angle of $30^{\circ}$ to the horizontal. $P$ rests in equilibrium on the plane, 1.6 m from $O$.
(i) Calculate the mass of $P$.

A particle $Q$, with mass equal to the mass of $P$, is projected up the plane along a line of greatest slope. When $Q$ strikes $P$ the two particles coalesce. The combined particle remains attached to the string and moves up the plane, coming to instantaneous rest after moving 0.2 m .
(ii) Show that the initial kinetic energy of the combined particle is 1 J .

The combined particle subsequently moves down the plane.
(iii) Calculate the greatest speed of the combined particle in the subsequent motion.

