Cambridge International Examinations
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MATHEMATICS **9709/12**

Paper 1 **May/June 2016**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

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Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only - often written by a ‘fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR–2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1	$f : x \mapsto 10 - 3x,\ g : x \mapsto \dfrac{10}{3-2x}$,		
	$ff(x) = 10 - 3(10 - 3x)$	**B1**	Correct unsimplified expression
	$gf(2) = \dfrac{10}{3 - 2(10 - 3(2))}\ (= -2)$	**B1**	Correct unsimplified expression with 2 in for x
	$x = 2$	**B1** [3]	
2	$f'(x) = \dfrac{8}{(5-2x)^2}$		
	$f(x) = \dfrac{8(5-2x)^{-1}}{-1} \div -2\ (+c)$	**B1** **B1**	Correct without (÷ by −2) An attempt at integration (÷ by−2)
	Uses $x = 2$, $y = 7$,	**M1**	Substitution of correct values into an integral to find c
	$c = 3$	**A1** [4]	
3	$\overrightarrow{OA} = 2\mathbf{i} - 5\mathbf{j} - 2\mathbf{k}$ and $\overrightarrow{OB} = 4\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}$.		
	$\overrightarrow{AB} = 2\mathbf{i} + \mathbf{j} + 4\mathbf{k}$ **or** $\overrightarrow{AC} = 4\mathbf{i} + 2\mathbf{j} + 8\mathbf{k}$	**B1**	
	$\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AC} = 6\mathbf{i} - 3\mathbf{j} + 6\mathbf{k}$	**M1**	correct method for $\overrightarrow{OC}$
	OR		
	$\begin{pmatrix}2\\1\\4\end{pmatrix} = \begin{pmatrix}x-4\\y+4\\z-2\end{pmatrix}$,	**B1**	
	$\overrightarrow{OC} = \begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}6\\-3\\6\end{pmatrix}$	**M1**	
	OR		
	$\overrightarrow{OB} - \overrightarrow{OA} = \overrightarrow{OC} - \overrightarrow{OB}$ $\therefore \overrightarrow{OC} = 2\overrightarrow{OB} - \overrightarrow{OA}$	**B1**	
	$= \begin{pmatrix}8\\-8\\4\end{pmatrix} - \begin{pmatrix}2\\-5\\-2\end{pmatrix} = \begin{pmatrix}6\\-3\\6\end{pmatrix}$	**M1**	
	Unit vector = (Their $\overrightarrow{OC}$) ÷ (Mod their $\overrightarrow{OC}$)	**M1**	Divides by their mod of their $\overrightarrow{OC}$
	$= (6\mathbf{i} - 3\mathbf{j} + 6\mathbf{k}) \div 9$	**A1** [4]	Correct unsimplified expression

4 (i)	$\left(x-\frac{2}{x}\right)^6$ Term is ${}_6C_3 \times (-2)^3 = (-)160$ -160	 **B1** **B1** [2]	 ±160 seen anywhere
(ii)	$\left(2+\frac{3}{x^2}\right)\left(x-\frac{2}{x}\right)^6$ Term in $x^2 = {}_6C_2(-2)^2 x^2$ $= 60\ (x^2)$ Term independent of x: $= 2 \times (\text{their}-160) + 3 \times (\text{their } 60)$ -140	 **B1** **B1** **M1** **A1** [4]	 ±60 seen anywhere Using 2 products correctly
5 (i)	$\tan\left(\frac{\pi}{3}\right) = \frac{AC}{2x}$ or $\cos\left(\frac{\pi}{3}\right)\left(=\sin\frac{\pi}{6}\right) = \frac{2x}{AB}$ $\rightarrow AC = 2\sqrt{3}x$ or $AB = 4x$ $AM = \sqrt{13\boldsymbol{x}^2}, \sqrt{13}\boldsymbol{x}, 3.61\boldsymbol{x}$	**B1** **M1A1** [3]	Either trig ratio Complete method.
(ii)	$\tan(M\hat{A}C) = \frac{x}{\text{Their } AC}$ $\theta = \frac{1}{6}\pi - \tan^{-1}\frac{1}{2\sqrt{3}}$ **AG**	**M1** **A1** [2]	"Their AC" must be f(x), $\left(M\hat{A}C\right) \neq \theta$. Justifies $\frac{\pi}{6}$ and links MAC & θ
6 (i)	$PT = r\tan\alpha$ $QT = OT - OQ = \frac{r}{\cos\alpha} - r$ or $\sqrt{r^2 + r^2\tan^2\alpha} - r$ Perimeter = sum of the 3 parts including $r\alpha$	**B1** **B1** **B1** [3]	
(ii)	Area of triangle = $\frac{1}{2} \times 10 \times 10\tan\frac{\pi}{3}$ Area of sector = $\frac{1}{2} \times 10^2 \times \frac{1}{3}\pi$ Shaded region has area 34 (2sf)	**M1** **M1** **A1** [3]	Correct formula used, $50\sqrt{3}, 86.6$ Correct formula used, $\frac{50\pi}{3}$, 52.36

7 **(i)**	$\frac{1+\cos\theta}{1-\cos\theta}-\frac{1-\cos\theta}{1+\cos\theta} \equiv \frac{4}{\sin\theta\tan\theta}$		
	LHS = $\frac{1+2c+c^2-\left(1-2c+c^2\right)}{(1-c)(1+c)}$	**M1**	Attempt at combining fractions.
	$= \frac{4c}{1-c^2}$	**A1 A1**	A1 for numerator. A1 denominator
	$= \frac{4c}{s^2}$		Essential step for award of A1
	$= \frac{4}{ts}$ **AG**	**A1** [4]	
(ii)	$\sin\theta\left(\frac{1+\cos\theta}{1-\cos\theta}-\frac{1-\cos\theta}{1+\cos\theta}\right)=3.$		Uses part (i) to eliminate "s" correctly.
	$\rightarrow s\times\frac{4}{ts}=3$ ($\rightarrow t=\frac{4}{3}$)	**M1**	
	$\theta = 53.1°$ and $233.1°$	**A1 A1**√ [3]	√ for 180° + 1st answer.
8	A (0, 7), B (8, 3) and C ($3k$, k)		
(i)	m of AB is −½ oe.	**B1**	Using A,B or C to get an equation
	Eqn of AB is $y = -\frac{1}{2}x + 7$	**M1**	Using C or A,B in the equation
	Let $x = 3k$, $y = k$	**M1**	
	$\boldsymbol{k = 2.8}$ **oe**	**A1**	
	OR		
	$\frac{7-k}{0-3k}=\frac{3-k}{8-3k}$	**M1A1**	Using A,B & C to equate gradients
	$\rightarrow 20k=56\rightarrow k=2.8$	**DM1A1**	Simplifies to a linear or 3 term quadratic = 0.
	OR		
	$\frac{7-k}{0-3k}=\frac{7-3}{0-8}$	**M1A1**	Using A,B and C to equate gradients
	$\rightarrow 20k=56\rightarrow k=2.8$	**DM1A1** [4]	Simplifies to a linear or 3 term quadratic = 0.

(ii)	M(4, 5) Perpendicular gradient = 2. Perp bisector has eqn $y-5=2(x-4)$ Let $x = 3k$, $y = k$ $k=\frac{3}{5}$ oe **OR** $(0-3k)^2+(7-k)^2=(8-3k)^2+(3-k)^2$ $-14k+49=73-54k \to 40k=24 \to k=0.6$	**B1** **M1** **M1** **A1** **M1A1** **DM1A1** [4]	anywhere in (ii) Use of $m_1m_2=-1$ soi Forming eqn using their M and their "*perpendicular m*" Use of Pythagoras. Simplifies to a linear or 3 term quadratic = 0.
9 (i) (a)	$a+(n-1)d$ = 10 + 29×2 = 68	**M1** **A1** [2]	Use of nth term of an AP with a=±10, d=±2, n=30 or 29 Condone – 68 → 68
(b)	$\frac{1}{2}n(20+2(n-1))=2000$ or 0 $\to 2n^2+18n-4000=0$ oe (n=) 41	**M1** **A1** **A1** [3]	Use of S_n formula for an AP with a=±10, d=±2 and equated to either 0 or 2000. Correct 3 term quadratic = 0.
(ii)	$r = 1.1$, oe Uses $S_{30}=\frac{10(1.1^{30}-1)}{1.1-1}$ (= 1645) Percentage lost = $\frac{2000-1645}{2000}\times 100$ = 17.75	**B1** **M1** **DM1** **A1** [4]	e.g. $\frac{11}{10}$, 110% Use of S_n formula for a GP, a=±10, n=30. Fully correct method for % left with "their 1645" allow 17.7 or 17.8.
10 **(i)**	$y=\frac{8}{x}+2x.$ $\frac{dy}{dx}=-8x^{-2}+2$ $\frac{d^2y}{dx^2}=16x^{-3}$ $\int y^2 dx=-64x^{-1}$ oe$+\,32x$ oe $+\,\frac{4x^3}{3}$ **oe** $(+c)$	 **B1** **B1** **3 × B1** [5]	 unsimplified ok unsimplified ok B1 for each term – unsimplified ok

(ii)	sets $\frac{dy}{dx}$ to 0 $\to x = \pm 2$	**M1**	Sets to 0 and attempts to solve
	$\to M(2, 8)$	**A1**	Any pair of correct values A1
	Other turning point is $(-2, -8)$	**A1**	Second pair of values A1
	If $x = -2$, $\frac{d^2y}{dx^2} < 0$	**M1**	Using their $\frac{d^2y}{dx^2}$ if kx^{-3} and $x < 0$
	$\therefore$Maximum	**A1** [5]	
(iii)	Vol = $\pi \times$ [part (i)] from 1 to 2	**M1**	Evidence of using limits 1&2 in their integral of y^2 (ignore π)
	$\frac{220\pi}{3}, 73.3\pi, 230$	**A1** [2]	
11	$f : x \mapsto 6x - x^2 - 5$		
(i)	$6x - x^2 - 5 \leqslant 3$		
	$\to x^2 - 6x + 8 \geqslant 0$	**M1**	$\pm(6x - x^2 - 8) =, \leqslant, \geqslant 0$ and attempts to solve
	$\to x = 2, x = 4$	**A1**	Needs both values whether =2, <2, >2
	$x \leqslant 2, x \geqslant 4$ condone < and/or >	**A1** [3]	Accept all recognisable notation.
(ii)	Equate $mx + c$ and $6x - x^2 - 5$	**M1**	Equates, sets to 0.
	Use of "$b^2 - 4ac$"	**DM1**	Use of discriminant with values of $a.b.c$ independent of x.
	$4c = m^2 - 12m + 16$. **AG**	**A1**	= (0) must appear before last line.
	OR		
	$\frac{dy}{dx} = 6 - 2x = m \to x = \left(\frac{6-m}{2}\right)$	**M1**	Equates $\frac{dy}{dx}$ to m and rearrange
	$m\left(\frac{6-m}{2}\right) + c = 6\left(\frac{6-m}{2}\right) - \left(\frac{6-m}{2}\right)^2 - 5$	**M1**	Equates $mx + c$ and $6x - x^2 - 5$ and substitutes for x
	$4c = m^2 - 12m + 16$. **AG**	**A1** [3]	
(iii)	$6x - x^2 - 5 = 4 - (x - 3)^2$	**B1 B1** [2]	4 B1 – $(x - 3)^2$ B1
(iv)	$k = 3$.	**B1**√ [1]	√ for "b".
(v)	$g^{-1}(x) = \sqrt{4 - x} + 3$	**M1 A1** [2]	Correct order of operations. $\pm\sqrt{4-x} + 3$ M1A0 $\sqrt{x-4} + 3$ M1A0 $\sqrt{4-y} + 3$ M1A0