CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level

MARK SCHEME for the May/June 2013 series

9709 MATHEMATICS

9709/21 Paper 2, maximum raw mark 50

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the May/June 2013 series for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level components and some Ordinary Level components.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a "fortuitous" answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR–2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1 Either: State or imply non-modular equation $(2^x - 7)^2 = 1^2$, or corresponding pair of equations — M1
Obtain $2^x = 8$ and $2^x = 6$ — A1
State answer 3 — B1
Use logarithmic method to solve an equation of the form $2^x = k$, where $k > 0$ — M1
State answer 2.58 — A1

Or: State or imply one value for 2^x, e.g. 8, by solving an equation or by inspection — B1
State answer 3 — B1
State second value for 2^x — B1
Use logarithmic method to solve an equation of the form $2^x = k$, where $k > 0$ — M1
State answer 2.58 — A1 [5]

2 Use $2 \ln x = \ln(x^2)$ — M1
Use law for addition or subtraction of logarithms — M1
Obtain correct quadratic equation in x — A1
Make reasonable solution attempt at a 3-term quadratic (dependent on previous M marks) — DM1
State $x = \frac{3}{5}$ and no other solutions — A1 [5]

3 **(i)** Either
Use $\sin 2x = 2\sin x \cos x$ to convert integrand to $k \sin^2 2x$ — M1
Use $\cos 4x = 1 - 2\sin^2 2x$ — M1
State correct expression $\frac{1}{2} - \frac{1}{2}\cos 4x$ or equivalent — A1

Or
Use $\cos^2 x = \frac{1 - \cos 2x}{2}$ and/or $x = \frac{1 - \cos 2x}{2}$ to obtain an equation in $\cos 2x$ only — M1
Use $\cos^2 2x = \frac{1 + \cos 4x}{2}$ — M1
State correct expression $\frac{1}{2} - \frac{1}{2}\cos 4x$ or equivalent — A1 [3]

(ii) State correct integral $\frac{3}{2}x - \frac{3}{8}\sin 4x$, or equivalent — B1
Attempt to substitute limits, using exact values — M1
Obtain given answer correctly — A1 [3]

4 **(i)** Substitute $x = -\frac{3}{2}$, equate to zero — M1
Substitute $x = -1$ and equate to 8 — M1
Obtain a correct equation in any form — A1
Solve a relevant pair of equations for a or for b — M1
Obtain $a = 2$ and $b = -6$ — A1 [5]

(ii) Attempt either division by $2x + 3$ and reach a partial quotient of $x^2 + kx$, use of an identity or observation M1

Obtain quotient $x^2 - 4x + 3$

Obtain linear factors $x - 1$ and $x - 3$ A1

[Condone omission of repetition that $2x + 3$ is a factor.] A1

[If linear factors $x - 1$, $x - 3$ obtained by remainder theorem or inspection, award B2 + B1.] [3]

5 **(i)** Use product rule to differentiate y M1

Obtain correct derivative in any form A1

Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$ M1

Obtain given answer correctly A1 [4]

(ii) Substitute $t = 0$ in $\frac{dy}{dx}$ and both parametric equations B1

Obtain $\frac{dy}{dx} = 2$ and coordinates (1, 0) B1

Form equation of the normal at their point, using negative reciprocal of their $\frac{dy}{dx}$ M1

State correct equation of normal $y = -\frac{1}{2}x + \frac{1}{2}$ or equivalent A1 [4]

6 **(i)** Make a recognisable sketch of a relevant graph, e.g. $y = \cot x$ or $y = 4x - 2$ B1

Sketch a second relevant graph and justify the given statement B1 [2]

(ii) Consider sign of $4x - 2 - \cot x$ at $x = 0.7$ and $x = 0.9$, or equivalent M1

Complete the argument correctly with appropriate calculations A1 [2]

(iii) Show that given equation is equivalent to $x = \frac{1 + 2\tan x}{4\tan x}$, or vice versa B1 [1]

(iv) Use the iterative formula correctly at least once M1

Obtain final answer 0.76 A1

Show sufficient iterations to justify its accuracy to 2 d.p. or show there is a sign change in the interval (0.755, 0.765) B1 [3]

7 **(i)** State $R = \sqrt{29}$ B1

Use trig formula to find α M1

Obtain $\alpha = 21.80^0$ with no errors seen A1 [3]

(ii) Carry out evaluation of $\sin^{-1}\left(\frac{4}{R}\right)$ $\left(\approx 47.97^0\right)$ M1

Carry out correct method for one correct answer M1

Obtain one correct answer e.g. 13.1^o A1

Carry out correct method for a further answer M1

Obtain remaining 3 answers 55.1^0, 193.1^0, 235.1^0 and no others in the range A1 [5]

(iii) Greatest value of $10 \sin 2\theta + 4 \cos 2\theta = 2\sqrt{29}$ M1

$\frac{1}{116}$ A1 [2]