UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS **9709/33**

Paper 3 Pure Mathematics 3 **(P3)** **October/November 2012**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

1 Solve the equation

$$\ln(x + 5) = 1 + \ln x,$$

giving your answer in terms of e. [3]

2 **(i)** Express $24 \sin\theta - 7 \cos\theta$ in the form $R \sin(\theta - \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. Give the value of α correct to 2 decimal places. [3]

(ii) Hence find the smallest positive value of θ satisfying the equation

$$24 \sin\theta - 7 \cos\theta = 17.$$ [2]

3 The parametric equations of a curve are

$$x = \frac{4t}{2t + 3}, \quad y = 2 \ln(2t + 3).$$

(i) Express $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t, simplifying your answer. [4]

(ii) Find the gradient of the curve at the point for which $x = 1$. [2]

4 The variables x and y are related by the differential equation

$$(x^2 + 4)\frac{\mathrm{d}y}{\mathrm{d}x} = 6xy.$$

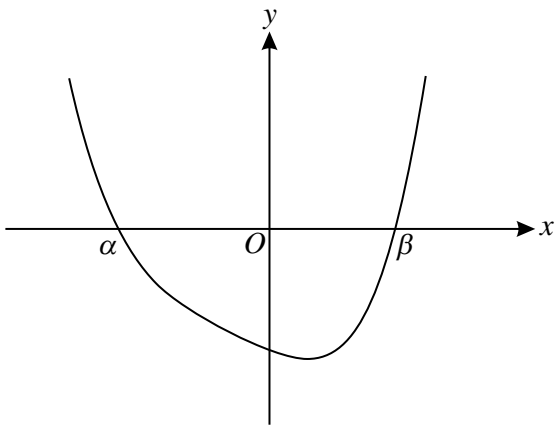
It is given that $y = 32$ when $x = 0$. Find an expression for y in terms of x. [6]

5 The expression $\mathrm{f}(x)$ is defined by $\mathrm{f}(x) = 3x\mathrm{e}^{-2x}$.

(i) Find the exact value of $\mathrm{f}'\left(-\frac{1}{2}\right)$. [3]

(ii) Find the exact value of $\int_{-\frac{1}{2}}^{0} \mathrm{f}(x)\,\mathrm{d}x$. [5]

6



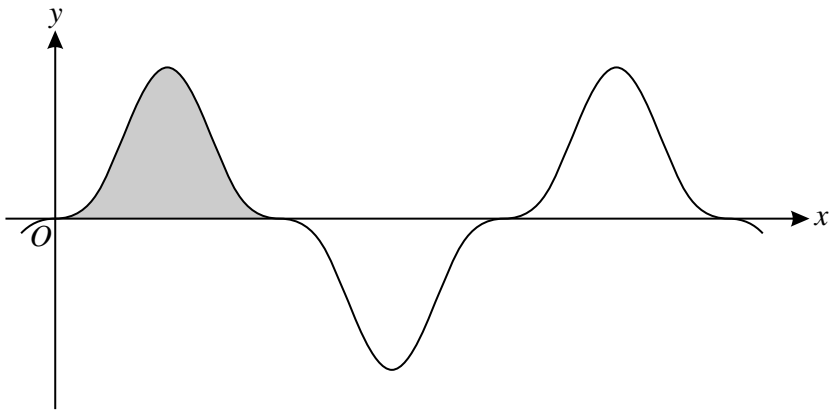
The diagram shows the curve $y = x^4 + 2x^3 + 2x^2 - 4x - 16$, which crosses the x-axis at the points $(\alpha, 0)$ and $(\beta, 0)$ where $\alpha < \beta$. It is given that α is an integer.

(i) Find the value of α. [2]

(ii) Show that β satisfies the equation $x = \sqrt[3]{(8 - 2x)}$. [3]

(iii) Use an iteration process based on the equation in part **(ii)** to find the value of β correct to 2 decimal places. Show the result of each iteration to 4 decimal places. [3]

7



The diagram shows part of the curve $y = \sin^3 2x \cos^3 2x$. The shaded region shown is bounded by the curve and the x-axis and its exact area is denoted by A.

(i) Use the substitution $u = \sin 2x$ in a suitable integral to find the value of A. [6]

(ii) Given that $\int_0^{k\pi} |\sin^3 2x \cos^3 2x| \, dx = 40A$, find the value of the constant k. [2]

[Questions 8, 9 and 10 are printed on the next page.]

8 Two lines have equations

$$\mathbf{r} = \begin{pmatrix} 5 \\ 1 \\ -4 \end{pmatrix} + s\begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix} \quad \text{and} \quad \mathbf{r} = \begin{pmatrix} p \\ 4 \\ -2 \end{pmatrix} + t\begin{pmatrix} 2 \\ 5 \\ -4 \end{pmatrix},$$

where p is a constant. It is given that the lines intersect.

(i) Find the value of p and determine the coordinates of the point of intersection. [5]

(ii) Find the equation of the plane containing the two lines, giving your answer in the form $ax + by + cz = d$, where a, b, c and d are integers. [5]

9 **(i)** Express $\dfrac{9 - 7x + 8x^2}{(3 - x)(1 + x^2)}$ in partial fractions. [5]

(ii) Hence obtain the expansion of $\dfrac{9 - 7x + 8x^2}{(3 - x)(1 + x^2)}$ in ascending powers of x, up to and including the term in x^3. [5]

10 **(a)** Without using a calculator, solve the equation $\mathrm{i}w^2 = (2 - 2\mathrm{i})^2$. [3]

(b) **(i)** Sketch an Argand diagram showing the region R consisting of points representing the complex numbers z where

$$|z - 4 - 4\mathrm{i}| \leqslant 2.$$ [2]

(ii) For the complex numbers represented by points in the region R, it is given that

$$p \leqslant |z| \leqslant q \quad \text{and} \quad \alpha \leqslant \arg z \leqslant \beta.$$

Find the values of p, q, α and β, giving your answers correct to 3 significant figures. [6]