UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **(P3)** **October/November 2012**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

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1 Find the set of values of x satisfying the inequality $3|x-1| < |2x+1|$. [4]

2 Solve the equation

$$5^{x-1} = 5^x - 5,$$

giving your answer correct to 3 significant figures. [4]

3 Solve the equation

$$\sin(\theta + 45^\circ) = 2\cos(\theta - 30^\circ),$$

giving all solutions in the interval $0^\circ < \theta < 180^\circ$. [5]

4 When $(1+ax)^{-2}$, where a is a positive constant, is expanded in ascending powers of x, the coefficients of x and x^3 are equal.

(i) Find the exact value of a. [4]

(ii) When a has this value, obtain the expansion up to and including the term in x^2, simplifying the coefficients. [3]

5 **(i)** By differentiating $\dfrac{1}{\cos x}$, show that if $y = \sec x$ then $\dfrac{dy}{dx} = \sec x \tan x$. [2]

(ii) Show that $\dfrac{1}{\sec x - \tan x} \equiv \sec x + \tan x$. [1]

(iii) Deduce that $\dfrac{1}{(\sec x - \tan x)^2} \equiv 2\sec^2 x - 1 + 2\sec x \tan x$. [2]

(iv) Hence show that $\displaystyle\int_0^{\frac{1}{4}\pi} \frac{1}{(\sec x - \tan x)^2}\,dx = \tfrac{1}{4}(8\sqrt{2} - \pi)$. [3]

6 The variables x and y are related by the differential equation

$$x\frac{dy}{dx} = 1 - y^2.$$

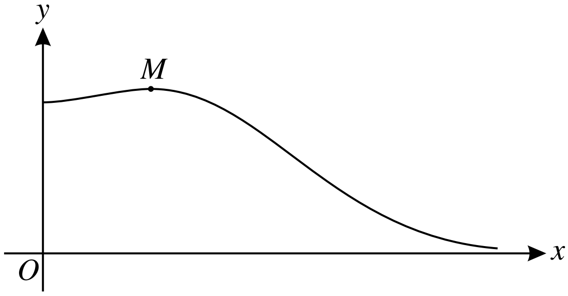
When $x = 2$, $y = 0$. Solve the differential equation, obtaining an expression for y in terms of x. [8]

7 The equation of a curve is $\ln(xy) - y^3 = 1$.

(i) Show that $\dfrac{dy}{dx} = \dfrac{y}{x(3y^3 - 1)}$. [4]

(ii) Find the coordinates of the point where the tangent to the curve is parallel to the y-axis, giving each coordinate correct to 3 significant figures. [4]

8



The diagram shows the curve $y = e^{-\frac{1}{2}x^2}\sqrt{(1 + 2x^2)}$ for $x \geqslant 0$, and its maximum point M.

(i) Find the exact value of the x-coordinate of M. [4]

(ii) The sequence of values given by the iterative formula

$$x_{n+1} = \sqrt{\left(\ln(4 + 8x_n^2)\right)},$$

with initial value $x_1 = 2$, converges to a certain value α. State an equation satisfied by α and hence show that α is the x-coordinate of a point on the curve where $y = 0.5$. [3]

(iii) Use the iterative formula to determine α correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

9 The complex number $1 + (\sqrt{2})i$ is denoted by u. The polynomial $x^4 + x^2 + 2x + 6$ is denoted by $p(x)$.

(i) Showing your working, verify that u is a root of the equation $p(x) = 0$, and write down a second complex root of the equation. [4]

(ii) Find the other two roots of the equation $p(x) = 0$. [6]

10 With respect to the origin O, the points A, B and C have position vectors given by

$$\overrightarrow{OA} = \begin{pmatrix} 3 \\ -2 \\ 4 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 2 \\ -1 \\ 7 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 1 \\ -5 \\ -3 \end{pmatrix}.$$

The plane m is parallel to $\overrightarrow{OC}$ and contains A and B.

(i) Find the equation of m, giving your answer in the form $ax + by + cz = d$. [6]

(ii) Find the length of the perpendicular from C to the line through A and B. [5]

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