MATHEMATICS

> **Paper 9709/11**
> **Paper 11**

Key messages

- Attention needs to be paid to making sure workings are carried out at a sufficient level of accuracy to ensure the accuracy of the final answers.
- Candidates need to be careful to avoid arithmetic and sign errors.
- Candidates need to be careful to read the question in detail and answer as indicated.

General comments

The performance of candidates continues to show improvement year-on-year. However, there is room for improvement. Some questions require quite 'routine' responses and candidates can obtain improved marks by making sure that they are confident on these 'routine' procedures. One example is the process of factorisation which all candidates at this level need to employ confidently and accurately, not least because it is likely to be needed several times in any one examination paper. In general, the level of algebraic facility was an area where there is the most scope for improvement.

Most of the scripts were well presented but some candidates still need to be reminded that they should work straight down the page rather than try to fit one question beside another question on the page

Comments on specific questions

Question 1

This question was a challenge for many candidates. A significant number of candidates went straight into using the double angle formulae for sine and cosine and were unable to make any further progress. Division by $\cos 2x$ leads directly to $\tan 2x = 2$ and those candidates who got to this point usually managed to score at least 3 marks. The fourth mark for the second solution was not often scored.

Answers: 31.7°   122°

Question 2

Most candidates made a good start to this question and presented a correct unsimplified term. However, errors in simplifying the coefficient of the required term were common. The most frequently seen were neglecting to raise 2 to the power of 4 in the simplification of $(2x^3)^4$ and losing the minus sign in the simplification of $(-1/x^2)^3$.

Answer: -560

Question 3

Candidates were often able to find the area of triangle *ABC* correctly (either by using the formula $\frac{1}{2} bc \sin A$ or by finding that $AQ = \sqrt{3}$). However, some candidates then used $r = 2$ in evaluating $\frac{1}{2} r^2 \theta$ for the area of the sector. Candidates also need to be reminded that the use of this formula requires θ to be in radians (in this particular case, $\frac{1}{3}\pi$).

Answer: $\sqrt{3} - \frac{1}{2}\pi$.

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Question 4

Most candidates were able to find the value of k correctly. Following this, many candidates did not appreciate that differentiation was required, followed by use of the Chain Rule or similar.

Answers: $k = 0.0032$, 0.096.

Question 5

In part **(i)**, many candidates having reached the equation $7\sqrt{x} = 6x + 2$, did not appear to realise that this could be treated as a quadratic expression which would factorise (with or without a suitable substitution). Instead candidates attempted to square both sides of the equation. Whilst this is an acceptable method errors were made in this squaring process - most commonly in losing the $24x$ term in squaring $6x + 2$.

In part **(ii)**, there are at least three methods which can be used, but by far the shortest method is to recognise that the original equation, $6x - 7\sqrt{x} + k = 0$, is a quadratic equation in $\sqrt{x}$, and to apply '$b^2 - 4ac = 0$'.

Answers: **(i)** $\dfrac{4}{9}$ or $\dfrac{1}{4}$ **(ii)** $\dfrac{49}{24}$

Question 6

Most candidates scored well on this question, although in part **(i)** some candidates were clearly worried by the use of the letter p in the column vectors given for **u** and **v** and either made mistakes when forming the scalar product or made mistakes in simplifying the algebra to a 3-term quadratic expression. It was also necessary to know that the scalar product is zero for perpendicular vectors, and while most candidates put their scalar product equal to zero there were also substantial numbers of candidates who did not indicate that the scalar product was zero.

In part **(ii)** candidates were very familiar with finding the angle between two vectors and most candidates were able to score well. The final mark was often lost because the answer was required correct to 1 decimal place.

Answers: **(i)** $-1, -4$ **(ii)** $30.0°$

Question 7

This question was a challenge for many candidates. In part **(a)** candidates usually showed they knew what was expected, and generally attempted to use the correct formula. However, errors were made leading to a loss of marks. The common difference was often taken to be $\cos^2 x$ or $1 - \cos^2 x$ instead of $\cos^2 x - 1$. Many of the candidates who obtained the correct common difference then made a sign error and simplified the common difference to $\sin^2 x$.

In part **(b)(i)** few candidates seemed to know that the condition for convergence is $0 < r < 1$. They were then not able to use this to find a set of values for θ when $r = \frac{1}{3}\tan^2 \theta$.

Candidates were able to make progress in part **(b)(ii)** in evaluating $\dfrac{1}{1 - r}$ when $\theta = \frac{1}{6}\pi$.

Answers: **(i)** $10 - 45\sin^2 x$ **(b)(i)** $0 < \theta < \frac{1}{3}\pi$ **(ii)** $\dfrac{9}{8}$

Question 8

Part **(i)** was slightly unusual with the addition of the constant k. However, on the whole candidates coped with this quite well and a reasonable proportion of candidates also scored the marks for parts **(ii)** and **(iii)**. In part **(iv)** candidates demonstrated that many of them understood how to find the inverse function, many of them opting, successfully, to interchange the variables as the first operation. However, few candidates included the ± sign when taking the square root. Candidates need to be reminded that in some questions the correct answer is obtained by taking the – sign and therefore the ± sign needs first to be included as part of the square rooting process before the appropriate sign is chosen for the final answer. It was good to see that a good proportion of candidates understood the link between the range of f (part **(ii)**) and the domain of f^{-1} (part **(iv)**).

Answers: **(i)** $(x-2)^2 - 4 + k$ **(ii)** $f(x) \geq k - 4$ **(iii)** 2 **(iv)** $f^{-1}(x) = 2 + \sqrt{x + 4 - k}$, $x \geq k - 4$

Question 9

This question was done well with the majority of candidates scoring all 5 marks for part **(i)**. In part **(ii)** the majority of candidates found the two gradients but some stopped at this point. To secure the second mark candidates had to mention 'negative reciprocal' or an equivalent reason. For part **(iii)** many candidates had lengthy methods often involving finding the equations of *AD* and *CD* and then the intersection of these lines. These lengthy methods frequently contained errors. A shorter method is to, intuitively, use the fact that for *BC*, *x* step : *y* step = 2 : 6 and that *AD* has the same ratio since it is parallel to *BC*.

Answers: **(i)** $y = -2x + 6$, $B = (3, 0)$ **(ii)** $m_1 = -1/3, m_2 = 3$, $m_1 m_2 = -1$ **(iii)** $D = (-1, 8), |AD| = \sqrt{40}$

Question 10

In part **(i)** candidates had to differentiate and put the result less than 5. The 5 had to be incorporated into the left hand side before candidates attempted to factorise or, otherwise, solve. Most candidates were able to score some marks here – although, in some cases the inequality signs were incorrect in the final solution.

In part **(ii)** a common mistake was to use the two solutions obtained from part **(i)**. The required *x* values are obtained by equating the derivative to zero. Many candidates neglected to find the corresponding *y* values which were specifically requested. Many candidates simply stated without any adequate explanation which point was a maximum and which was a minimum and this is not sufficient. Candidates need to show which process they have used to determine the nature of the stationary points.

Answers: **(i)** $-\dfrac{2}{3} < x < 2$ **(ii)** $\dfrac{4}{27}$ (max), 0 (min)

Question 11

Most candidates achieved the mark for part **(i)**. In part **(ii)** the majority of candidates were able to find the indefinite integral correctly – although a number of candidates wrote that the integral of 1 with respect to *y* was *x*. When finding the area of the shaded region using the indefinite integral already obtained, a number of candidates applied limits which were values of *x* rather than *y*. Some candidates integrated the original function with respect to *x* and then subtracted the area under the straight line. Although this is a correct method the wording of the question (Hence …) indicated that the method required was to integrate the given *y* function. In part **(iii)** the majority of candidates knew that they had to find $\int x^2 \, dy$, but mistakes were made in squaring the *y* function, in integrating the three terms, or in applying *x* limits instead of *y* limits.

Answers: **(ii)** $-\dfrac{4}{y} - y$, 1 **(iii)** $\dfrac{5\pi}{3}$

MATHEMATICS

Paper 9709/12
Paper 12

Key messages

- Candidates need to be careful to ensure that sufficient working is seen in the answers to questions.
- Candidates need to be careful to read the questions in detail and answer as indicated.
- Candidates need to take care to avoid arithmetic and sign errors.

General comments

There were many excellent responses to this paper and few poor scripts. Candidates were confident on the majority of questions, though **Question 7** and parts **(ii)** and **(iii)** of **Question 8** were challenging to candidates. In general the standard of presentation was acceptable, and in many cases excellent. However there are still many candidates who are omitting essential working (even in questions asking for "show" or "prove") and in not reading the questions carefully. This was particularly the case in **Question 9** where "the gradient of the line *BA"* was often taken as "the gradient of the curve at *B*".

Comments on specific questions

Question 1

Although the majority of solutions were correct, there were many candidates who integrated $36(x - 3)^{-2}$ as $-36(2x - 3)^{-1}$, not realising the need to divide by 2, the differential of the bracket. Only a small minority of candidates did not use the correct formula for the volume of revolution, though a few misread "volume" as "area". Use of π and limits were generally accurate.

Answer: 12π.

Question 2

(i) The differentiation of $4\sqrt{x}$ and $\dfrac{2}{\sqrt{x}}$ was generally correct, though many candidates incorrectly assumed that $\dfrac{2}{\sqrt{x}}$ was the same as $2x^{\frac{1}{2}}$ or $2x^{-1}$.

(ii) Most candidates recognised the need to use the chain rule, but less than a half obtained a correct answer. Common errors were to incorrectly substitute $x = 4$ into the differential or to assume that the rate of change of the *y*-coordinate was equal to the gradient divided by 0.12 instead of being multiplied by 0.12

Answers: **(i)** $y = \dfrac{2}{\sqrt{x}} - x^{-1.5}$ **(ii)** 0.105

Question 3

Use of the binomial expansion was generally accurate and most candidates obtained the coefficients of x^3 in the two expansions as $10a^2$ and -160 respectively. It was however common to see the answer of +160 rather than −160. The majority were able to proceed correctly to find the value of *a*, though some candidates did not realise that "x^3" could be cancelled on each side of the equation.

Answer: $a = 5$

Question 4

Whilst many candidates had little difficulty in finding the equation of the perpendicular bisector and proceeding to find the coordinates of *C* and *D* before calculating the distance *CD*, at least a third of all candidates did not realise the need either to find the mid-point of *A* and *B*, or to find a line perpendicular to *AB*. It was very common to see candidates either using the line *AB* or using a line perpendicular to *AB* through either *A* or *B*. Virtually all candidates realised the need to set *x* and *y* to 0 and the use of the distance formula was accurate.

Answer: *CD* = 2.5.

Question 5

(i)   This part was very well answered with the majority of candidates using trigonometric identities confidently.

(ii)   Only about a third of all attempts realised the need to use part **(i)** to obtain a quadratic equation in tan *x*. Most of the other candidates were unable to make any progress in reducing the given equation to an equation in one variable.

Answers: **(i)** Proof **(ii)** *x* = 45° or 116.6°

Question 6

(i)   Most candidates divided triangle *AOB* into two 90° triangles and used simple trigonometry with an angle of 1.2 radians. Answers were usually correct, though early approximation often led to 15.0 instead of 14.9. Others used the cosine rule and were usually successful. A few weaker candidates assumed that *AB* referred to an arc, rather than a straight line.

(ii)   Use of the formula $s = r\theta$ was generally accurate, though the angle of the major arc *AB* was often taken as $\pi - 2.4$ rather than $2\pi - 2.4$. Some candidates misinterpreted perimeter and only offered the arc length whilst other candidates did not realise that use of $s = r\theta$ required the angle to be in radians and not degrees.

(iii)   This part of the question was also well answered with accurate use of the formulae $A = \dfrac{1}{2}r^2\theta$ and $A = \dfrac{1}{2}r^2\sin\theta$. Again, use of the incorrect angle was the main reason for incorrect answers.

Answers: **(i)** 14.9 cm **(ii)** 46.0 cm **(iii)** 146 cm²

Question 7

(a)   This part was poorly answered and it was rare to see a correct answer to this challenging question. Candidates did not recognise that setting *n* = 1 in the given formula would lead directly to *a*. Setting *n* to 2 leads to $S_2 = 20$, from which the value of *d* (=2) can be deduced. A few candidates preferred to set *n* to two different values and to then use the formula for S_n to obtain two simultaneous equations for *a* and *d*. The majority of candidates however equated the given formula for S_n with the general formula for the sum of an arithmetic progression, but did not appreciate that comparing coefficients of *n*² and then *n* would lead directly to *d* and then to *a*.

(b)   This part was again a challenge. Often the equation used was *ar* = 9 − *a* instead of *ar* = *a* − 9, and many candidates interpreted the sum of the second and third terms as being $S_2 + S_3$ instead of $ar + ar^2$. Only a small proportion realised the need to eliminate either *a* or *r* from two simultaneous equations. Fully correct answers were rare.

Answers: **(a)** *a* = 9, *d* = 2 **(b)** *a* = 27

Question 8

Answers to the three parts were very variable. Part **(i)** was very well answered, parts **(ii)** and **(iii)** less so. A common error was to interpret $3\mathbf{i} - 4\mathbf{k}$ as the column vector $\begin{pmatrix} 3 \\ -4 \\ 0 \end{pmatrix}$.

(i) Apart from the occasional arithmetic error, the vast majority of candidates showed an excellent understanding of scalar product and obtained a correct answer.

(ii) Many candidates experienced difficulty in appreciating the concept of a unit vector. More did not appreciate that a vector of magnitude *K* units in a given direction is equal to *K* times the unit vector in that direction.

(iii) Candidates were accurate in using $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$, but again a large number were unable to proceed to the unit vector.

Answers: **(i)** 31.0° **(ii)** $9\mathbf{i} - 12\mathbf{k}$, $4\mathbf{i} + 6\mathbf{j} - 12\mathbf{k}$ **(iii)** $\dfrac{-5\mathbf{i} + 6\mathbf{j}}{\sqrt{61}}$

Question 9

(i) Most candidates realised the need to differentiate to find the gradient of the curve and to equate this to 0 to find the *x*- coordinate of *A*. Substitution of $x = 4$ into the quadratic $x^2 + 8x - 10$ to find the *y* coordinate gave many problems and the incorrect value $y = 38$ was seen as often as the correct answer of y = 6. Finding the coordinates of *B* proved challenging for the majority of candidates, most of whom took the gradient of the curve as 2 instead of the gradient of the line *AB* as 2.

(ii) The majority of candidates realised the need to integrate and the integration and use of limits was extremely well done.

Answers: **(i)** *A* (4, 6), *B* (2, 2) **(ii)** $9\dfrac{1}{3}$

Question 10

(i) This part was well answered with most candidates obtaining correct expressions for both f $^{-1}$(*x*) and g $^{-1}$(*x*). The most common error was a simple algebraic slip in transferring data from one side of an equation to the other. A few candidates incorrectly left the answer in terms of *y* instead of *x*. Only a small proportion of candidates were able to state that the value of *x* at which g $^{-1}$(*x*) was not defined was the value of *x* for which the denominator was 0.

(ii) Most of the sketch graphs were well drawn and indeed many contained far more data and accuracy than was required for a sketch. Many candidates however did not realise that the lines $y = $ f(*x*) and $y = $ f $^{-1}$(*x*) have the line $y = x$ as a line of symmetry.

(iii) Most candidates were able to obtain a correct expression for fg(*x*), though a small proportion did not include the "+5". A significant proportion did not cancel the "+5" on both sides of the equation and thereby caused themselves an unnecessary amount of algebraic manipulation. Use of the discriminant $b^2 - 4ac$ was well known, though the solution of the subsequent quadratic inequality "$9k^2 - 64k < 0$" was rarely correct. Many candidates automatically cancelled *k* and then ignored it. Others obtained the end values of *k* as 0 and $\dfrac{64}{9}$, but then incorrectly assumed $k < 0$ and $k < \dfrac{64}{9}$.

Answers: **(i)** f $^{-1}$(*x*) = ½(*x* − 5), g $^{-1}$(*x*) = $\dfrac{8}{x} + 3$ $\quad x = 0$ **(ii)** Sketch **(iii)** $0 < k < \dfrac{64}{9}$

MATHEMATICS

| Paper 9709/13 |
| Paper 13 |

Key messages

- Candidates need to ensure that sufficient working is seen for "Show that" questions.
- Candidates need to show working to obtain the full marks available for each question.
- Any errors which occur in using a calculator to integrate will mean that no credit can be awarded if no working is shown.
- Candidates need to be careful to read the questions in detail and answer as indicated.

General comments

Many candidates performed well in this paper, with a range of marks obtained. There was evidence of formulae used in wrong contexts - the most common relating to volume of revolution in **Question 5** and the misuse of the formula for segment area in **Question 8**. Candidates would be well advised to read questions carefully. Common errors included not noticing minus signs (**Question 3, Question 9**); completely missing sin in **Question 4** i.e., trying trying to solve $2x + 3\cos 2x = 0$; and not grasping the details in **Question 6**. In all these cases a little more care would have been more profitable. The question paper reminds candidates of "the need for clear presentation" in their answers. Candidates need to use as much space as needed to present clear, readable answers.

Comments on specific questions

Question 1

(i) Several different approaches were used, with most involving the trigonometrical identities $\tan \theta = \sin \theta / \cos \theta$ and $\sin^2 \theta + \cos^2 \theta = 1$. Some candidates worked from the left and arrived at the result on the right; some worked from the right to the left; and some worked from both sides arriving at an identity though not all commented on this thereby leaving the question unfinished.

(ii) Few candidates correctly answered this part. Some candidates offered solutions which, though technically correct, ignored the instruction to use the result from **(i)**. Others had long answers in spite of the marks available for this part. Candidates were expected to state that the right hand side is positive therefore the left hand side is positive and then that $\tan^2 \theta > \sin^2 \theta$. Between 0° and 90° both tan and sin are positive and so $\tan \theta > \sin \theta$ as required.

Answers: **(i)** Proof **(ii)** Explanation

Question 2

(i) By far the majority of candidates managed to find $\overrightarrow{AB}$ correctly although there were some mistakes resulting from using $\overrightarrow{OA} - \overrightarrow{OB}$ or $\overrightarrow{OA} + \overrightarrow{OB}$. Having obtained their $\overrightarrow{AB}$ a significant minority did not find the modulus and some of those who did find the modulus, did not combine this correctly with the original vector to give the unit vector.

(ii) This part was very well answered. The only exceptions were candidates who had 90 in their calculations rather than cos 90°. Only a few candidates did not put 'dot product = 0'.

Answers: **(i)** $\dfrac{1}{7}\begin{pmatrix} 2 \\ 3 \\ -6 \end{pmatrix}$ **(ii)** $p = 5$

Question 3

Some candidates appeared to not understand the question. They just expanded the second bracket. Of those that made progress certain mistakes were quite common - sign errors in the expansion of the first bracket; $15ax^2$ instead of $15a^2x^2$; too few terms involved in finding b. Other candidates did take the question to a correct conclusion though not always by the most direct route (some had all 21 terms in the expansion and picked out the necessary ones from that).

Answers: $a = \frac{1}{2}$, $b = -4\frac{1}{4}$

Question 4

(i) There were several examples of the use of incorrect trig formulae, but the most common mistakes included sin $2x$/cos $2x$ = tan x and sin $2x$ = 2sin x. Once past this stage most candidates found a basic angle, often called α but sometimes called x, and thus solutions for $2x$ and subsequently for x. Not all found solutions for $2x$ up to 720° and so had just two solutions for x. Others did not halve their answers for $2x$ and some doubled them.

(ii) There should have been 4 solutions in **(i)** and most candidates realised that there would be three times as many here.

Answers: **(i)** x = 54.2° or 144.2° or 234.2° or 324.2°   **(ii)** 12 solutions

Question 5

This was a straightforward question if approached methodically. Some candidates took no account of the question being volume about the y-axis; some intended to use x^2 but forgot to square; a significant minority thought the integral of $((8/y^2)-2)^2$ was one third of this bracket cubed with or without some sort of correcting factor; limits used were often mistaken with 0 to 2 and 0 to 6 being common.

Answer: $6\frac{2}{3}\pi$

Question 6

(i) Whilst some candidates seemed to mix up expressions for nth term and sum of n terms, a large number seemed to misunderstand the question and referred to the 9th term and never to the sum of the first 9 terms. (The use of the alternative sum of n terms = $n/2$ (a + 1) was quite common, giving the 9th term first and then the common difference).

(ii) Some candidates worked only in letters never substituting 12 for a, nor their value of d from **(i)**. Having established a as being the same in both A.P. and G.P. it was expected that candidates would use d from part **(i)** to find the 9th term of the A.P. and from this find the 2nd term, and thus the common ratio, for the G.P.. To then find the 3rd term of the G.P. and equate this to the expression for the nth term of the A.P. was the final step. Few candidates made progress with this part.

Answers: **(i)** $d = \frac{3}{4}$ **(ii)** $r = 1\frac{1}{2}$, $n = 21$

Question 7

(i) A variety of methods were applied here - some of which were effectively based on knowing what the answer was and then providing information that would lead them there. The only valid solutions were based on finding the coordinates of A and differentiating the curve equation. Candidates could then find the gradient of the curve and thus of the tangent at that point and then the equation of the tangent. The differentiation itself was fairly well done with the two common mistake being that the -2 stayed rather than being differentiated to become zero, and candidates forgot to multiply the -10 by 2 from the differentiation of (2x+1).

(ii) Again many candidates misunderstood the question finding their C before they found the equation of the tangent. Simple use of Pythagoras Theorem applied to points A and C correctly gave the required answer but mislearnt formulae were quite common.

Answers: **(i)** Proof   **(ii)** $AC = 2.56$

Question 8

(i) Many candidates did not use the fact that angle *OBX* is 90°. However this information along with *OB* = *r* and *OX* = 2*r* would take them to the fact that cos *AOB* = *r*/2*r* = ½ and thus the required result. Many other methods were offered including the use of *BX* = √3 with sin or tan, but often this result for *BX* was quoted not found. Indeed many of these other methods were based on assumptions - the most simplistic being 'angle *X* is 30° therefore angle *O* is 60° i.e. *π*/3' - and thus earned no marks.

(ii) The only common mistake was to state that '*P* shaded = *P* triangle - *P* sector'. The correct method required finding *BX* and arc length *AB* and adding to *r*.

(iii) Most candidates made good attempts at the area of the sector and of triangle *OXB*. Some forgot to then subtract, or subtracted the wrong way round. There were some candidates who used the formula for the area of a segment.

Answers: **(i)** Proof   **(ii)** $r + \frac{1}{3}\pi r + r\sqrt{3}$ **(iii)** $\frac{1}{2}r^2\sqrt{3} - \frac{1}{6}r^2\pi$.

Question 9

(i) Having been given an expression for the differential, two integrations were needed with a constant of integration to be found at each stage from the information given in the question. Many candidates had no constant at the first stage and of those that did most substituted (2,12) even though they had d*y*/d*x* not *y*. The mention of maximum was infrequently used. Some candidates, having reached an expression for d*y*/d*x*, substituted *x*-2 and used the resultant as *m* in *y*=*mx*+*c* even though trying to find the equation of a curve.

(ii) This part was often omitted. Among those who attempted it the layout was poor but not penalised as long as they were multiplying the value of d*y*/d*x* at *x*=3 by 0.05. Some candidates reached the correct answer for d*y*/d*t*, but did not state that the y-coordinate was decreasing.

Answers: **(i)** $y = \dfrac{-2x^3}{3} + 8x + \dfrac{4}{3}$ **(ii)** $\dfrac{dy}{dt} = -0.5$ units per second, decreasing

Question 10

(i) The common approach adopted in this part was to solve the two equations simultaneously to obtain the coordinates of *A* and *B*. This was well done in most cases although some incorrect algebra was seen. The majority of candidates found the gradient of *AB* and of a line perpendicular to this. Several candidates did not use the midpoint of *AB* and found the equation of a perpendicular, but not of a bisector. Many candidates found the equation of *AB* without, seemingly, realising this was the original line equation.

(ii) Some candidates held on to k = 8 as in **(i)** which did not allow progress. The use of b^2-4*ac* was common amongst the remaining candidates although their work often had their *a*, *b*, *c* mixed up. The majority reached k^2=48. Some candidates forgot to square root, others settled on $k > \pm \sqrt{48}$, while still others offered *k* as being between the two values, not outside them. A recurring error is the lack of care in the use of inequalities e.g. k ≥ √48 is incorrect, k > √48 is correct.

Answers: **(i)** *y* = 2*x* - 6   **(ii)** *k* < - √48 and *k* > √48

Question 11

Some candidates did not attempt this question. Those candidates that did the majority performed well on parts **(i)** to **(iii)**, but part **(iv)** caused problems.

(i) The methods adopted varied, but the *x*-value was commonly correct; the *y*-value was sometimes forgotten. Some candidates seemed not to understand 'the nature' while others avoided this.

(ii) The answer is simply $k = 2$ but again there was indiscriminate use of inequality signs.

(iii) Some candidates had problems with signs, but this aside the idea of making *x* the subject and dealing, in turn, with the 8, the square root and the 2 was understood by most. Candidates who expanded the bracket often made errors in the constant and in signs, but could still get to an answer with more complicated algebra.

(iv) Most candidates ignored the restricted domain of function g as compared to f and effectively sketched f and its reflection in y=x. The other common fault was to put numbers on axes and then draw a line labelled y=x which clearly was not. Some candidates used differing scales on the axes so that the idea of reflection is complicated.

Answers: **(i)** (2,8), maximum   **(ii)** $k = 2$ **(iii)** $g^{-1}(x) = 2 + \sqrt{(8 - x)}$ **(iv)** Sketch.

MATHEMATICS

| Paper 9709/21 |
| Paper 21 |

Key messages

- Candidates need to show full working to obtain the marks available for each question.
- Candidates need to be careful to eliminate arithmetic and numerical errors in their answers.
- Candidates need to make sure they do not make approximations during their working, as this can impact on the accuracy of the final result.
- Candidates need to be careful to read questions in detail and answer as indicated.

General comments

The paper produced results covering the entire range of available marks. The great majority of candidates were able to attempt the questions set and show what they had learned. Some candidates achieved full marks. Most candidates were able to show a reasonable understanding of the syllabus aims and their application. Candidates should be reminded of the importance of showing all their working clearly to ensure that method marks can be gained in the event that algebraic or arithmetic slips are made. Marks were lost by some candidates due to not working to the correct level of accuracy. Candidates should work to at least 4 significant figures during the question and give their final answer to 3 significant figures unless told otherwise or working in degrees. Candidates need to read the questions carefully and make sure that they give their answers in the correct form required. Many candidates do not appear to understand the meaning of the phrase 'exact value'.

Comments on specific questions

Question 1

The great majority of candidates were able to make a good start by obtaining at least one of the two critical values needed, either by squaring and thus obtaining a quadratic equation, or by obtaining two separate, easily solvable cubic equations.

Answers: 1 and 3

Question 2

This question was a challenge for many candidates who did not appreciate the significance of the vertical axis representing ln y. Provided candidates realised that the best approach was to re-write the equation $y = A(b^x)$ in the form ln $A + x$ ln b, marks were usually obtainable. Substitution into the original equation of the line proved to be less successful in most cases, as candidates did not realise that the exponential function had to be introduced. Common mistakes included the statement that $A = 2.14$, followed by incorrect substitution leading to $b = 1.2$.

Answers: A = 0.85, *b* = 1.6

Question 3

(i) Most candidates were able to obtain the correct result by finding the value of p(2) and equating it to zero. Others used synthetic division with equal success. Algebraic long division was attempted by some candidates with less success due mainly to sign errors in the process adopted.

(ii)(a) Different approaches were used in the attempt to factorise p(x), with synthetic division and algebraic long division being the most commonly used techniques. As most candidates had the correct value for a, completely correct solutions were common. Those candidates that chose to use their calculator to solve the equation p(x) = 0 scored marks providing they realised that factors were needed.

(b) This part of the question rarely caused any problems with most candidates finding the value of p(1) by substitution.

Answers: **(i)** 2 **(ii) (a)** $(x-2)(2x+3)(x-1)$ **(b)** 6

Question 4

(i) Provided the correct identity was used, the great majority of candidates were able to find the value of tan θ from the resulting quadratic equation with equal roots. Many candidates did not stop at this point and went on to solve the resulting equation. Candidates are encouraged to check that they are giving their answers in the correct form.

(ii)(a) and **(b)** Many candidates did not appreciate the request for the exact value and chose to use their calculators together with the value of θ obtained from solving the equation obtained in the first part of the question. When asked for exact values, decimal responses are not sufficient. Other problems arose with candidates making sign errors when using the formula both tan $(\theta - 45°)$ and tan 2θ.

Answers: **(i)** 6 **(ii)(a)** $\dfrac{5}{7}$ **(b)** $-\dfrac{12}{35}$

Question 5

(i) Provided candidates were able to differentiate exponential functions correctly, they were able to obtain marks. Most were able to employ a correct method of equating $\dfrac{dy}{dx}$ to zero with responses of $x = 2 \ln 3$ being fairly common. However, having found a solution for x, many candidates thought that this part of the question was finished, and did not give the answer in the required form.

(ii) Realising that they needed to integrate in order to find the given area, this part was not a problem for most candidates. However, they needed to be able to deal with the exponential function correctly when integrating in order to gain marks. Many candidates did not appreciate the word 'exact' and used their calculators to evaluate their answers. There were also mistakes related to the fact that when the lower limit of the integral is zero, it is often incorrectly assumed that the second bracket, when evaluating, is also zero.

Answers: **(i)** ln 9 **(ii)** 8e – 14

Question 6

(i) Many correct attempts at finding $\dfrac{dy}{dx}$ and equating it to –1, were made. Mistakes tended to be related to incorrect signs and dealing with fractional and negative indices. However, correct methods were usually used. These errors were then compounded when trying to arrange the obtained equation into the form of the given answer. Many candidates were able to obtain four marks for this question, with the algebraic manipulation required for the last two marks proving too challenging.

(ii) Most candidates used the given answer from the first part of the question and were able to provide sufficient iterations to give a result to the required level of accuracy. However, many did not give their final answer correct to 3 decimal places.

Answers: **(ii)** 0.678

Question 7

(i) Correct expansions of the trigonometric expression were common as was the use of sin $2x$ = 2sin x cos x. Problems arose when candidates attempted to use the double angle formulae for cos $2x$. Different approaches were used with the most successful being a substitution for $\sin^2 x$ and one for $\cos^2 x$, and then simplification of the resulting terms.

(ii) Most candidates appreciated the use of the word 'Hence' in the question and used the given answer from the first part of the question, even if they had been less than successful in gaining it correctly. Errors in signs and multiples were the main problems that occurred with the integration itself. Again, candidates did not realise the significance of the word 'exact' with many candidates losing the last mark available by using their calculator for evaluation.

Answers: **(ii)** $\dfrac{5\pi}{8} + \dfrac{1}{4}$

MATHEMATICS

Key messages

- Candidates need to show full working to obtain the marks available for each question.
- Candidates need to be careful to eliminate arithmetic and numerical errors in their answers.
- Candidates need to make sure they do not make approximations during their working, as this can impact on the accuracy of the final result.
- Candidates need to be careful to read questions in detail and answer as indicated.

General comments

The paper produced results covering the entire range of available marks. The great majority of candidates were able to attempt the questions set and show what they had learned. There were some candidates who achieved full marks, showing an excellent understanding of the syllabus aims and their application. Candidates should be reminded of the importance of showing all their working clearly to ensure that method marks can be gained in the event that algebraic or arithmetic slips are made. Marks were lost by some candidates due to not working to the correct level of accuracy. Candidates should work to at least 4 significant figures during the question and give their final answer to 3 significant figures unless told otherwise or working in degrees.

Comments on Specific Questions

Question 1

Most candidates were able to make a reasonable start to the question. Some gained marks by either obtaining a quadratic equation or two linear equations from the given inequality. Correct critical values were common with most mistakes occurring when it came to the appropriate range.

Answers: $x < -\dfrac{4}{3}, \ x > 2$

Question 2

This was probably the most problematic question on the paper with many candidates not realising that the first part of the question did not require them to solve the given equation for x, but was a 'step' on the way to solving the equation for x. Many candidates attempted to take logarithms straightaway and thus were unable to obtain any marks.

(i) Many candidates did recognise that the equation was a quadratic equation in terms of 5^x, solved the equation correctly and went on to obtain a correct value for x. Those that did so were able to obtain full marks for the whole question, provided $5^x = -4$ was discounted.

(ii) If candidates had not made a reasonable attempt at part **(i)**, which was meant to help them with part **(ii)**, then marks were usually not achievable. Some candidates did not give their correct solution to the required level of accuracy, with 0.68 being a common result.

Answers: **(i)** 3 **(ii)** 0.683

Question 3

(i) The great majority of candidates were able to produce the correct quotient using algebraic long division and confirming that 4 was the remainder.

(ii) Candidates should be reminded of the importance of reading the question carefully. Factors of the polynomial were required for this part, not solutions of the polynomial equated to zero. Those candidates that did solve the polynomial equated to zero using factors were not penalised, with most realising that $4x^2 + 4x - 3$ was a quadratic factor of the given polynomial. Some candidates did not use part **(i)** and chose to deal with the polynomial by searching for linear factors. Candidates did not score marks if they used their calculators to solve the polynomial, equated to zero and as a result never produced any linear factors.

Answers: **(i)** $2x - 3$ **(ii)** $(2x - 3)(2x - 1)(2x + 3)$

Question 4

(i) This trigonometry question produced correct results for many candidates with the occasional sign error for *α*. These mistakes then affected the next part of the question, as did not giving *α* to the required level of accuracy.

(ii) Many correct solutions were seen, with few candidates not recognising the correct approach to take.

(iii) Correct solutions were uncommon, with candidates not recognising the significance of the word 'state' and the fact that there was only one mark for this part. Some attempts at calculus were made together with other methods, which never resulted in the correct result.

Answers: **(i)** $15\sin(\theta - 53.13°)$ **(ii)** $68.6°, 217.7°$ **(iii)** 15

Question 5

(i) Most candidates made attempts to differentiate the given expressions with respect to *t* with varying degrees of success. There were mistakes made when dealing with the chain rule. However, in general most candidates were aware of the correct approach to take. A few however attempted to obtain the Cartesian form of the equation of the curve first.

(ii) Most candidates were aware of the correct approach to take and were able to gain marks even if they had been less than successful in the first part of the question.

Answers: **(i)** $(2e^{2t} + 2)(t + 1)$ **(ii)** $x + 4y - 4 = 0$

Question 6

(i) Provided candidates recognised the equation of the curve needed to be dealt with as a quotient (or on occasion, a product), most were able to make a reasonable attempt to equate their result to zero. Problems arose with dealing with this result algebraically, with many not realising that the numerator of the fraction was equal to zero.

(ii) Most candidates realised that the correct approach involved looking for a change of sign of $\tan 2x - 2x - 4$ or equivalent when $x = 0.6$ and $x = 0.7$. It is necessary to state the actual numerical value of the function when the different values of x are substituted in. Correct use of calculators is important in this part, as a calculator in degree mode, rather than radian mode, does not give a sign change.

(iii) This part was usually done well by most provided calculators were in the correct mode. Most candidates gave their working to the required level of accuracy during their working, but often did not give their final answer to 3 significant figures as requested.

Answers: **(i)** $\dfrac{2(x+2)\cos 2x - \sin 2x}{(x+2)^2}$ **(iii)** 0.694

Question 7

(i) The use of the identity $\tan^2 x + 1 = \sec^2 x$ was recognised and used by most candidates. However, problems arose when using the double angle formulae with many being unable to obtain the given result. Most realised that they needed to use the given result for the integration required but many did not appreciate that the 'exact value' implies a non-decimal answer is required. Those candidates that chose to use their calculators were unable to gain marks as their answers were not exact.

(ii) This part of the question was fairly demanding, needing candidates to make a deductive conclusion using part **(i)**. Many realised that they needed to integrate (tan x + cos x)^2 with respect to x as part of the process of finding the required volume. Problems arose when the squaring process was done incorrectly, with the $2\sin x$ term being omitted. If this was the case, then candidates were unable to continue with any measure of success. Of those that did 'square correctly', many deduced that all they had to do was integrate $2\sin x$ with respect to x, using the correct limits together with their answer for part **(i)** and multiply by π. Again problems with the wording 'exact volume' caused some candidates to lose marks, as in the first part of the question.

Answers: **(i)** $\dfrac{5}{4} - \dfrac{\pi}{8}$ **(ii)** $\pi\left(\dfrac{5}{4} - \dfrac{\pi}{8}\right) + \pi\left(2 - \sqrt{2}\right)$

MATHEMATICS

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Paper 9709/23
Paper 23
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Key messages

- Candidates need to show full working to obtain the marks available for each question.
- Candidates need to be careful to eliminate arithmetic and numerical errors in their answers.
- Candidates need to make sure they do not make approximations during their working, as this can impact on the accuracy of the final result.
- Candidates need to be careful to read questions in detail and answer as indicated.

General comments

The paper produced results covering the entire range of available marks. The great majority of candidates were able to attempt the questions set and show what they had learned. Some candidates achieved full marks. Most candidates were able to show a reasonable understanding of the syllabus aims and their application. Candidates should be reminded of the importance of showing all their working clearly to ensure that method marks can be gained in the event that algebraic or arithmetic slips are made. Marks were lost by some candidates due to not working to the correct level of accuracy. Candidates should work to at least 4 significant figures during the question and give their final answer to 3 significant figures unless told otherwise or working in degrees. Candidates need to read the questions carefully and make sure that they give their answers in the correct form required. Many candidates do not appear to understand the meaning of the phrase 'exact value'.

Comments on specific questions

Question 1

The great majority of candidates were able to make a good start by obtaining at least one of the two critical values needed, either by squaring and thus obtaining a quadratic equation, or by obtaining two separate, easily solvable cubic equations.

Answers: 1 and 3

Question 2

This question was a challenge for many candidates who did not appreciate the significance of the vertical axis representing ln y. Provided candidates realised that the best approach was to re-write the equation $y = A(b^x)$ in the form ln $A + x$ ln b, marks were usually obtainable. Substitution into the original equation of the line proved to be less successful in most cases, as candidates did not realise that the exponential function had to be introduced. Common mistakes included the statement that $A = 2.14$, followed by incorrect substitution leading to $b = 1.2$.

Answers: A = 0.85, *b* = 1.6

Question 3

(i) Most candidates were able to obtain the correct result by finding the value of p(2) and equating it to zero. Others used synthetic division with equal success. Algebraic long division was attempted by some candidates with less success due mainly to sign errors in the process adopted.

(ii)(a) Different approaches were used in the attempt to factorise p(x), with synthetic division and algebraic long division being the most commonly used techniques. As most candidates had the correct value for a, completely correct solutions were common. Those candidates that chose to use their calculator to solve the equation p(x) = 0 scored marks providing they realised that factors were needed.

(b) This part of the question rarely caused any problems with most candidates finding the value of p(1) by substitution.

Answers: **(i)** 2 **(ii) (a)** $(x - 2)(2x + 3)(x - 1)$ **(b)** 6

Question 4

(i) Provided the correct identity was used, the great majority of candidates were able to find the value of $\tan\theta$ from the resulting quadratic equation with equal roots. Many candidates did not stop at this point and went on to solve the resulting equation. Candidates are encouraged to check that they are giving their answers in the correct form.

(ii)(a) and (b) Many candidates did not appreciate the request for the exact value and chose to use their calculators together with the value of θ obtained from solving the equation obtained in the first part of the question. When asked for exact values, decimal responses are not sufficient. Other problems arose with candidates making sign errors when using the formula both $\tan(\theta - 45°)$ and $\tan 2\theta$.

Answers: **(i)** 6 **(ii)(a)** $\dfrac{5}{7}$ **(b)** $-\dfrac{12}{35}$

Question 5

(i) Provided candidates were able to differentiate exponential functions correctly, they were able to obtain marks. Most were able to employ a correct method of equating $\dfrac{dy}{dx}$ to zero with responses of $x = 2 \ln 3$ being fairly common. However, having found a solution for x, many candidates thought that this part of the question was finished, and did not give the answer in the required form.

(ii) Realising that they needed to integrate in order to find the given area, this part was not a problem for most candidates. However, they needed to be able to deal with the exponential function correctly when integrating in order to gain marks. Many candidates did not appreciate the word 'exact' and used their calculators to evaluate their answers. There were also mistakes related to the fact that when the lower limit of the integral is zero, it is often incorrectly assumed that the second bracket, when evaluating, is also zero.

Answers: **(i)** $\ln 9$ **(ii)** $8e - 14$

Question 6

(i) Many correct attempts at finding $\dfrac{dy}{dx}$ and equating it to –1, were made. Mistakes tended to be related to incorrect signs and dealing with fractional and negative indices. However, correct methods were usually used. These errors were then compounded when trying to arrange the obtained equation into the form of the given answer. Many candidates were able to obtain four marks for this question, with the algebraic manipulation required for the last two marks proving too challenging.

(ii) Most candidates used the given answer from the first part of the question and were able to provide sufficient iterations to give a result to the required level of accuracy. However, many did not give their final answer correct to 3 decimal places.

Answers: **(ii)** 0.678

Question 7

(i) Correct expansions of the trigonometric expression were common as was the use of sin 2x = 2sin x cos x. Problems arose when candidates attempted to use the double angle formulae for cos 2x. Different approaches were used with the most successful being a substitution for sin^2 x and one for cos^2 x, and then simplification of the resulting terms.

(ii) Most candidates appreciated the use of the word 'Hence' in the question and used the given answer from the first part of the question, even if they had been less than successful in gaining it correctly. Errors in signs and multiples were the main problems that occurred with the integration itself. Again, candidates did not realise the significance of the word 'exact' with many candidates losing the last mark available by using their calculator for evaluation.

Answers: **(ii)** $\dfrac{5\pi}{8} + \dfrac{1}{4}$

MATHEMATICS

Paper 9709/31

Paper 31

Key messages

- Answers should be tidy and legibly presented in the examination.
- A considerable proportion of the total marks depend on giving answers which are correct to three significant figures. In order to gain as many marks as possible all working should be either exact or correct to four significant figures.
- Candidate need to show working to obtain the full marks available for each question.

General comments

The standard of work on this paper varied considerably. No question or part of a question seemed to be too difficulty for the more able candidates, and most questions discriminated well within this group. The questions that candidates found relatively easy were **Question 2 (i)** (binomial expansion), **Question 3(i)**, **(ii)** (remainder theorem and factorisation) and **Question 4 (i)** (complex numbers). Those that they found difficult were **Question 1** (logarithms), **Question 5** (integration and differentiation) and **Question 8** (vectors).

In general the presentation of the work fell below that expected from candidates attempting this paper. When attempting a question, candidates need to be aware that it is essential that sufficient working is shown to indicate how they arrive at their answers, whether they are working towards a given answer, for example as in **Question 9** and **Question 10 (i)**, or an answer that is not given, as in **Question 10 (ii)** and **Question 10 (iii)**.

Where numerical and other answers are given after the comments on individual questions it should be understood that alternative forms are often acceptable and that the form given is not necessarily the only 'correct answer'.

Comments on specific questions

Question 1

This question proved difficult for many since, instead of removing the modulus sign and obtaining the equations $2^x = 14$ and $2^x = -6$, they tried to expand $(4 - 2^x)^2$. Few were successful with this latter approach, even fewer realised that they needed then to solve their quadratic in 2^x prior to attempting to take logs. Many only ever considered the single equation $2^x = 14$, and only a few candidates gave the answer to 3 significant figures, as requested.

Answer: 3.81

Question 2

(i) Some candidates dealt with this easily. However there were some sign errors in the x term and many incorrect coefficients for the x^2 term.

(ii) Although many knew what was required, the removal of the coefficient 4 from the bracket $(1 - 4x)^{-\frac{1}{2}}$ caused problems, resulting in final answers of 2.5, 10 or 20 being common.

Answers: **(i)** $1 + 2x + 6x^2$ **(ii)** 5

Question 3

    (i)   This question was answered well by those candidates who substituted $x = 2$ and equated to zero. However, those who opted to divide by $(x$-$2)$ often experienced problems when they reached the stage of dealing with the term $(4 - 3a)x$.

    (ii) (a)   Again most candidates were successful, although some did not progress beyond $(x - 2)(x^2 + 2x - 8)$.

        (b)   This part tended to be omitted, and from those few candidates that did attempt it usually only $\sqrt{2}$ or the unsimplified complex answer of $i\sqrt{4}$ was seen.

Answers: **(i)** 4 **(ii) (a)** $(x+4)(x-2)^2$ **(b)** $+\sqrt{2}, -\sqrt{2}, i2, -i2$

Question 4

    (i)   This part was answered well. Although many who opted not to simplify their $(1 + 2i)^2$ to $-3 + 4i$ before proceeding with their calculation often introduced some form of error.

    (ii)   Whilst most knew that the locus was a circle only the occasional candidate had it passing through the origin. In addition, too many sketches had one sign, or even both, of their values established in **(i)** changed as the coordinates of the centre of the circle.

Answer: (i) $u = \dfrac{-2}{5} + \dfrac{11}{5}i$

Question 5

    (i)   Differentiating $\tan\dfrac{1}{2}x$ proved difficult, although those that managed to reach the form $b\sec^2\dfrac{1}{2}x$ were able to continue to solve for $\cos\dfrac{1}{2}x$.

    (ii)   Whilst a few candidates managed to obtain $-16\cos\dfrac{1}{2}x$ usually the sign or the coefficient was incorrect, it was rare to see $2\ln(\cos\dfrac{1}{2}x)$.

Answer: **(ii)** $8 + 2\ln\dfrac{1}{2}$

Question 6

    (i)   The y^2 term was differentiated correctly far more often than the $-4xy$ term. The main problem with the latter being caused by the minus sign, as $-4x\mathrm{d}y/\mathrm{d}x + 4y$ or $-4y + 4x\mathrm{d}y/\mathrm{d}x$ were common. Another common error was to still see the constant term of 45 present following differentiation.

    (ii)   Again, many knew how to proceed, but following one or more of the errors mentioned in **(i)** their expression for y was not of a form from which they could recognise a contradiction.

Answer: **(i)** 12/7

Question 7

Usually the separation of variables was undertaken correctly, together with the correct integration of the y^2 term. However some candidates appeared to be differentiating this side of the equation, since $2y$ was seen. The integration of the xe^{3x} was often incorrect with one of the following wrong approaches usually being seen: the product rule for differentiation; integration by parts with the incorrect sign; and applying integration by parts but integrating x instead of e^{3x}.

Answers: **(i)** $y^3 = 6xe^{3x} - 2e^{3x} + 10$ **(ii)** 2.44

Question 8

The attempts at this question, apart from the occasional fully correct solution for both parts, were frequently incorrect. The problem throughout was that in both **(i)** and **(ii)** the vectors $\mathbf{r} = \mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$ and $\mathbf{r} = -\mathbf{i} + 4\mathbf{j} + 11\mathbf{k}$ were assumed to be in the plane containing point P and the line l, when in fact it was necessary to establish another vector in this plane by subtracting $\mathbf{r} = -\mathbf{i} + 4\mathbf{j} + 11\mathbf{k}$ from the position vector of some point on the line. Once this is undertaken the scalar product in **(i)** and the vector product in **(ii)** will rapidly produce the required answers. However, there are numerous suitable alternative approaches.

Answers: **(i)** $\sqrt{104}$ **(ii)** $3x - 9y + z = -28$

Question 9

Many candidates failed to state a correct general form such as $A + \dfrac{B}{2x+1} + \dfrac{C}{x+2}$. Instead they used $\dfrac{Ax+B}{2x+1} + \dfrac{C}{x+2}$ or $\dfrac{A}{2x+1} + \dfrac{Bx+C}{x+2}$ which failed to help them much towards being able to undertake the evaluation of the integral. Another common source of error was to assume that A in the correct form should be unity. While some candidates omitted A altogether. The integration of the $\dfrac{C}{x+2}$ term was usually correct, but the $\dfrac{B}{2x+1}$ term often had an incorrect coefficient. Often the detailed work to show the answer given was omitted.

Answer: $2 + \dfrac{1}{2x+1} - \dfrac{3}{x+2}$

Question 10

(i) The expression for tan $2x$ in terms of tan x was usually correct. However, many candidates then decided that when multiplying by 2 it was necessary to multiply both the numerator and the denominator. Others who clearly knew mathematically what was required, decided to ignore the requirement to move from tan x to t and hence never really started the question. Since the question gave both $t = 0$, as well as an expression for t, it was necessary to establish these results by removing a factor of t from their initial expression as opposed to just quoting the given answer $t = 0$.

(ii) Here it was necessary to look at the signs of $t - \sqrt[3]{(t+0.8)}$, as opposed to just considering values of t and of $\sqrt[3]{(t+0.8)}$.

(iii) Some good work, although several candidates still had their calculator in degree mode. Others candidates either did not follow the instructions and work to 5 decimal places or did not continue their iterations until they had shown convergence to 3 decimal places.

(iv) Most candidates knew how to find the solution of $\tan^{-1} t$ from their solution to part **(iii)**, but only found that in the first quadrant. It was rare to see the values obtained from $t = 0$.

Answers: **(iii)** 1.276 **(iv)** $-\pi$, -2.24, 0, 0.906, π

MATHEMATICS

> **Paper 9709/32**
> **Paper 32**

Key messages

- Candidates need to take care to avoid arithmetic and algebraic errors.
- A considerable proportion of the total marks depend on giving answers which are correct to three significant figures. In order to gain as many marks as possible all working should be either exact or correct to four significant figures.
- Candidates need to take care to avoid sign errors in their answers.

General comments

The standard of work on this paper varied considerably and resulted in a wide spread of marks. No question or part of a question seemed to be of undue difficulty and most questions discriminated well. However there were some parts of questions which even the strongest candidates found challenging. The questions or parts of questions that were generally done well were **Question 2** (iteration), **Questions 7 (i)** and **(ii)** (complex numbers), and **Question 9 (i)** (differentiation). Those that caused the most difficulty were **Question 5** (differential equation), **Question 7 (iii)** (complex numbers), **Question 8 (i)** (integration by substitution). **Question 9 (ii)** (volume of revolution), and **Question 10 (iii)** (vector geometry).

In general the presentation of work was good. In some questions marks were lost because candidates made basic errors with logarithms or exponentials, e.g. taking $\ln(a + b)$ to be $\ln a + \ln b$, or $\exp(c + d)$ to be $\exp c + \exp d$. In **Question 9 (ii)** the error of taking $(\ln x)^2$ to be $2\ln x$ was particularly costly. Another source of error and loss of marks was lack of control and vigilance when removing brackets, e.g. $a(b + c)$ becoming $ab + c$ and $-(b - c)$ becoming $-b - c$. These are two areas where the technique of candidates could be improved.

Where numerical and other answers are given after the comments on specific questions it should be understood that alternative forms are often acceptable and that the form given is not necessarily the only 'correct answer'.

Comments on specific questions

Question 1

This was fairly well answered. Those with a sound command of logarithms quickly reached $x^2 - x - 3 = 0$ and their only errors tended to be the failure to reject the negative root (since it was less than or equal to -1) or not giving the positive root to 3 significant figures. However there were a substantial number of candidates who made errors of principle when removing logarithms.

Answer: 2.30

Question 2

This was generally well answered. In part **(ii)** some candidates who were generating a decreasing sequence of iterates reached a point where their last two iterates rounded to 1.33 to two decimal places and presented 1.33 as the answer. Had they continued the iterative process a little further, or looked for a sign change in $\tan \theta - 3\theta$ in the interval (1.325, 1.335), they would have realised the danger of basing conclusions solely on the rounded values of two successive iterates. Before starting to use an iterative formula involving a trig function candidates need to check that their calculator is in the appropriate mode for angle measure. In this case radian mode was needed, but not all candidates realised it.

Answer: **(ii)** 1.32

Question 3

This question was quite well done and was attempted by a variety of methods. The most popular approach was to expand $(1-x)^{\frac{1}{2}}$ and $(1+x)^{-\frac{1}{2}}$, and then find their product (though some added the expansions). When forming the product most candidates took care to include all the relevant terms before combining like terms.

Some worked with $(1-x)(1-x^2)^{-\frac{1}{2}}$. Successful attempts using $(1+x)^{-1}(1-x^2)^{\frac{1}{2}}$ and $((1-x)(1+x)^{-1})^{\frac{1}{2}}$ were also seen.

Some candidates appeared to misinterpret the question and only found the term in x^2 or its coefficient. Some others stopped after presenting a fraction whose numerator and denominator were the expansions of $(1+x)^{\frac{1}{2}}$ and $(1-x)^{\frac{1}{2}}$. Attempts such as these suggest that some candidates need to be fully aware of what is meant by expanding a function as a series as far as a certain term.

Answer: $1-x+\dfrac{1}{2}x^2$

Question 4

Many with a correct understanding of the definitions of cosecant, secant and cotangent, applied the double angle formula and Pythagoras to reach the quadratic $2\sin^2\theta - 2\sin\theta - 1 = 0$, or an equivalent. Candidates who made premature approximations when obtaining θ from the negative root of this equation lost the final two accuracy marks, and sometimes a value in the wrong quadrant, for example 158.5°, was presented. Some candidates did not have correct knowledge of all the functions in the equation. Others who had correct knowledge still made the fundamental error of inverting each term of the equation so that it became $\sin 2\theta = \cos \theta + \tan \theta$. Those with an initially sound start were sometimes slow to remove common factors such as $\sin \theta \cos \theta$ and, working with more complicated expressions than necessary, were more prone to make errors in manipulation.

Answers: 201.5°, 338.5°

Question 5

Many candidates did not recognise that to separate variables they must first replace e^{2x+y} by $e^{2x}.e^y$. Those who did achieve a correct separation often made errors in the subsequent integration or in the evaluation of the constant of integration. Weakness in technique with logarithms and exponentials also prevented those who had obtained a correct solution such as $-e^{-1} = \frac{1}{2}e^{2x} - \frac{3}{2}$ from deriving a correct expression for y in terms of x.

Answer: $y = \ln\left(\dfrac{2}{3-e^{2x}}\right)$

Question 6

(i) Most candidates obtained a correct first derivative. Those who differentiated the given pair of terms correctly reached $3\cos x(1 - 4\sin x \cos x)$ quite easily. But candidates who began by using trig formulae to transform $4\cos^3 x$ into an expression such as $4(\cos x - \cos x \sin^2 x)$ or $2\cos x(\cos 2x + 1)$ made differentiating more complicated than necessary. After equating the derivative to zero the factor of $\cos x$ tended to be cancelled and the solution from $\cos x = 0$ overlooked. Those who recognised that $\sin x \cos x$ could be expressed in terms of $\sin 2x$ usually found at least $x = \frac{1}{12}\pi$. Some gave $\frac{11}{12}\pi$ as the second solution rather than $\frac{5}{12}\pi$. Candidates who squared $\sin x \cos x$ and derived quadratics in $\sin^2 x$ or $\cos^2 x$ usually failed to discard the additional spurious roots introduced by squaring.

(ii) The factorised expression 3 cos x(1 − 2 sin $2x$) for the first derivative provided an easy way for candidates to identify the nature of the stationary point at $\frac{1}{12}\pi$, either by calculating gradients either side, or by simply observing that in the neighbourhood of the point, cos x is positive and sin $2x$ is increasing, so the derivative changes from positive to negative as x increases. Hardly any candidates used this approach. Instead nearly all based their attempt on the second derivative. This involved a further round of differentiation involving the product and/or chain rule and errors, e.g. of sign, were frequent. To complete the method candidates had to substitute their least value in the second derivative and present a value on which to base a decision. All too often it was unclear what was being substituted or what value had been obtained. Many candidates simply wrote > 0 or < 0 with no supporting evidence.

Answer: **(i)** $\frac{1}{12}\pi, \frac{5}{12}\pi, \frac{1}{2}\pi$; **(ii)** Maximum

Question 7

Parts **(i)** and **(ii)** were generally well answered. In part **(iii)** only a few candidates realised that the given answer could be derived from arg(1 + 2i) − arg(1 + 3i) = arg u. The instruction forbidding the use of calculators was obeyed by all in part **(i)** but ignored by most in part **(iii)**.

Answer: **(i)** $u = -\frac{1}{2} + \frac{1}{2}i$

Question 8

(i) Candidates had considerable difficulty with this part. It is clear that more experience of substitutions that involve a reversal of limits could be beneficial for candidates. The key issue is that when the lower and upper limits for u change from 2 and 1 respectively to 1 and 2 the sign of the integrand changes. Very few solutions were consistent with this, i.e. the key step $\int_{2}^{1} f(u)du = -\int_{1}^{2} f(u)du$, or its equivalent, was rarely seen or implied. Instead candidates went to great lengths (often making several attempts) to introduce, or remove, a minus sign in some way, by for example, altering the sign of their relation between dx and du, or changing the factor (3 − u) to (u − 3) in their factorisation of the denominator of the integrand.

(ii) There were many fully correct solutions to this part, but there were also a significant number of candidates who did not realise that they needed to be using partial fractions. This question has a given answer. Most candidates who were working correctly showed sufficient manipulation of logarithms to justify it, but some did not.

Question 9

Part **(i)** was well answered on the whole. A few found the equation of the normal rather than the tangent. In part **(ii)** many started with a correct expression such as $\pi\int x(\ln x)^2 \, dx$, but then mistakenly replaced $(\ln x)^2$ by 2 ln x. Some candidates devoted considerable time and space to the evaluation of an incorrect integral.

Answer: **(i)** $y = x - 1$ **(ii)** $\frac{1}{4}\pi(e^2 - 1)$

Question 10

(i)   The most popular approach began by using a scalar product to show the line was perpendicular to the normal to the plane. Nearly all attempts omitted the essential second step of verifying that the line did not lie in the plane and so had to be parallel to it. The lesser used but more successful approach was to substitute components of a general point of the line in the equation of the plane and following a correct simplification of the equation in λ, reach a contradiction such as 5 = 1 and conclude that the line and plane had no points in common and were parallel.

(ii)   This was very well answered.

(iii)   Few candidates completed this correctly. To make a worthwhile start it was essential to form a modular or non-modular equation in λ by equating the perpendiculars from a general point of the line to the two planes. Some candidates reached valid equations for a point (x, y, z) such as $x - 4$ $y + 3z = 6$ and $3x - z = 8$, but did not always tie them to a general point on the line and thus convert them into equations in λ.

Answer: **(ii)** $5\mathbf{i} + 3\mathbf{j} + 3\mathbf{k}$;   **(iii)**   6

MATHEMATICS

Paper 9709/33
Paper 33

Key messages

- Candidates need to show working to obtain the full marks available for each question.
- A considerable proportion of the total marks depend on giving answers which are correct to three significant figures. In order to gain as many marks as possible all working should be either exact or correct to four significant figures.
- Candidates need to take care to avoid sign errors in their answers.

General comments

The standard of work on this paper varied considerably and resulted in a spread of marks from zero to full marks. No question or part of a question seemed to be of undue difficulty, and all questions discriminated well. The questions that candidates found relatively easy were **Question 2** (logarithms), **Question 3** (parameterisation), **Question 8** (partial fractions and integration), **Question 9 (i)**, **(ii)**, **(iii)** (vectors). Those that they found difficult were **Question 1** (binomial expansion), **Question 4** (stationary point), **Question 5** (differential equation), **Question 6** (trigonometry) and **Question 10** (complex numbers).

In general the presentation of the work was good. Some candidates appeared to waste a considerable amount of time by their approach to **Question 4 (ii)** and to **Question 9 (ii)** and **(iii)**. When attempting a question, candidates need to be aware that it is essential that sufficient working is shown to indicate how they arrive at their answers, whether they are working towards a given answer, for example as in **Question 3** and **Question 8 (ii)**, or an answer that is not given, as in **Question 5 (i)** and **Question 10 (a)**. Where numerical and other answers are given after the comments on individual questions it should be understood that alternative forms are often acceptable and that the form given is not necessarily the only 'correct answer'.

Comments on specific questions

Question 1

This question found many candidates failing to establish the starting point of $\frac{1}{2}(1+\frac{3}{4}x)^{-\frac{1}{2}}$ for the binomial expansion. Common errors were raising the term in the bracket to the power $+\frac{1}{2}$, $1+3x$ inside the bracket, and a coefficient of 2 or $\sqrt{4}$ instead of $\frac{1}{2}$. It should be stressed that at this level $\sqrt{4}$ is not an acceptable simplified form.

Answer: $\frac{1}{2} - \frac{3}{16}x + \frac{27}{256}x^2$

Question 2

(i) Some candidates dealt easily with this question. However, there were many others that finished with $2x + 3$ or $x^2 + 3$ for the RHS of their equation following the removal of the logarithms. Unfortunately many candidates who produced a correct solution then omitted to give the answer to 3 significant figures as requested.

Answer: 1.39

Question 3

Most candidates knew what to do with this question, however many were unable to differentiate successfully both x and y w.r.t. θ. The double angle formulae were usually applied correctly, although then often $\dfrac{dx}{d\theta}$ was expressed in terms of $\cos^2\theta$ as opposed to $\sin^2\theta$ The latter form was required for the factorisation of the denominator to be clearly shown, together with the cancellation of the factor $(1-2\sin\theta)$ from the numerator and the denominator.

Question 4

(i)   This question proved difficult for candidates. Some candidates had problems from the start as they applied the quotient rule to u/v with $u= e^{2x}$ and $v= x^{-3}$ instead of $v= x^3$. Even those that did use the correct quotient rule made errors such that the resulting equation could not be solved. Many others who went for the product rule failed to differentiate both e^{2x} and x^{-3} successfully. Using the quotient rule with cancellation should have provided candidates with $\dfrac{e^{2x}}{x^4}(2x-3)$, from which the stationary point is immediately available, together with a sensible starting position for **(ii)**.

(ii)   Here some candidates wasted time, for instead of saying that $\dfrac{e^{2x}}{x^4}$ was positive and then looking at the sign of $(2x{-}3)$, at say $x{=}1$ and $x{=}2$, they opted to differentiate their unsimplified expression from **(i)**. This usually took up nearly a full page of working and in most cases contained numerous errors. Alternatively, they could have numerically evaluated either the value of y at $x{=}$ 1, $\dfrac{3}{2}$ and 2 or the value of $\dfrac{dy}{dx}$ at $x{=}1$ and $x = 2$. Certainly the evaluation of $\dfrac{d^2y}{dx^2}$ at $x=\dfrac{3}{2}$ is an approach that should have been avoided.

Answers: **(i)** $x=\dfrac{3}{2}$ **(ii)** Minimum point

Question 5

(i)   Despite a number of candidates believing that it was necessary to substitute more than just for x on the RHS of the equation, that is they tried to introduce a $\dfrac{dx}{dt}$ term and quickly became confused, most were able to establish the correct differential equation and to integrate w.r.t. y and acquire ln$y$. Unfortunately they were not as successful when integrating $-3e^{-3t}$, with $9e^{-3t}$, $-e^{-3t}$ all common answers. Determining their constant of integration also proved difficult since many took e^0 to be zero and then followed this with an attempt to find an expression for y based on the exponentiation of each individual term.

(ii)   Only the occasional candidate showed any evidence of the number 100 in this solution. Even those who did usually forgot to divide their limiting value by the initial value of y.

Answers: **(i)** $y = 70\exp(e^{-3t}-1)$ **(ii)** $\dfrac{100}{e}$

Question 6

(i) Many candidates appeared to believe that it was necessary only to expand tan3*x* in terms of tan2*x* and tan*x* and then tan2*x* in terms of tan*x*, without ever establishing an equation of any form. Unless candidates found an equation of some form then no marks were available.

(ii) Usually this part was well done, although a few candidates only gave one solution, forgetting *x* = 163.2°, and others having correctly established $\tan^2 x = \dfrac{1}{11}$ failed to take the square root before evaluating their value of *x*.

(iii) The result $\tan^2 x = -\dfrac{1}{5}$ was usually seen, but then little else. It was expected that candidates would say that $\tan^2 x \geq 0$, so there were no roots.

Answers: **(ii)** 16.8°, 163.2°

Question 7

(i) This question appeared straight forward, but candidates often experienced problems early on since they failed to establish $\dfrac{\mathrm{d}x}{\mathrm{d}u}$ correctly, and reached an expression that they were unable to integrate.
Those that did obtain the correct integral following the substitution, then often made sign errors in their integration by parts. Some believed that integration by parts was not required and answers such as just sin *u* were regularly seen. The final part of this question proved challenging since candidates still had their *x* limits as the *u* limits and forgot completely that they had been told that the value of this integral was 1.

(ii) Some good numerical work was seen, although some candidates did not undertake enough iterations, others finished with the final answer of 1.246, whilst others did not work to 4 decimal places.

Answer: **(ii)** 1.25

Question 8

(i) Many candidates did not state a correct general form such as $A + \dfrac{B}{x+1} + \dfrac{C}{2x-3}$, instead used $\dfrac{Ax+B}{x+1} + \dfrac{C}{2x-3}$ or $\dfrac{A}{x+1} + \dfrac{Bx+C}{2x-3}$ which failed to help them much towards being able to undertake the evaluation of the integral. Another common source of error was to assume that *A* in the correct form should be unity, while some candidates omitted *A* altogether.

(ii) The integration of the $\dfrac{B}{x+1}$ term was usually correct, but the $\dfrac{C}{2x-3}$ term often had an incorrect coefficient. Often the detailed work to show the answer given was omitted.

Answer: **(ii)** $2 - \dfrac{2}{x+1} - \dfrac{1}{2x-3}$

Question 9

(i) Most candidates started with the correct 3 equations, but in their working to acquire the given answer numerous errors followed. Some candidates did not appear to have any strategy, such as finding λ in terms of μ from their z coordinate equation, substituting for this λ in the other two equations to acquire μ in terms of a and in terms of b respectively, and then equating their two values of μ.

(ii) Nearly all candidates obtained the correct equation for l and m to be perpendicular, with most continuing on to obtain the correct values for a and b.

(iii) Having just a single expression for λ or μ in terms of a or b meant that substituting back for either a or b should have given candidates a numerical value of λ or μ with which to acquire the point of intersection. Unfortunately, candidates chose to ignore all their previous work and substituted for a and b in their original equations and then resolved these equations. This usually resulted in a full page of working instead of a one line answer.

Answers: **(ii)** $a = 3$, $b = 2$ **(iii)** $\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$

Question 10

(a) Most candidates obtained a correct equation for u or for w but then experienced trouble solving their quadratic equation. There was no single error that caused virtually everyone to obtain an incorrect solution. The errors ranged from omitting the i from their substitution, sign errors, squaring 2i and obtaining -2, together with many other arithmetic errors. Even candidates who looked like succeeding often decided that the + and − in their u or w solution should accompany the imaginary part as opposed to the real part.

(b)(i) Some sound work was presented here, with candidates often scoring at least 4 marks. Some omitted the line Re $z \geq 1$, others had the line representing arg $z \leq -\dfrac{\pi}{4}$ starting from the centre of their circle as opposed to from the origin. However, some of this work was undone by the shading of the region inside the circle being for arg $z \geq -\dfrac{\pi}{4}$ from most of the candidates.

(ii) The consequence of the incorrect shading resulted in the greatest possible value of Re z nearly always being $x = 4$, instead of that corresponding to the largest of the two x coordinates where the line arg $z = -\dfrac{\pi}{4}$ meets the circle.

Answers: **(a)** $u = 1 +2i$ and $w = 1 −2i$; $u = −1 +2i$ and $w = −1 −2i$ **(b) (ii)** $2 + \sqrt{2}$

MATHEMATICS

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Paper 9709/41

Paper 41

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Key messages

- It is important for candidates to read the questions carefully and answer as required.
- Working needs to be presented in a clear and readable form.
- Candidates need to recognise the direction of the frictional forces and normal reaction in a given situation. Accurate force diagrams could have helped some candidates in answering questions.

General comments

The paper allowed candidates of all abilities to gain marks where they were able. There were few candidates who coped successfully with all, or nearly all, of the demands of the paper. The majority found some questions, particularly **Questions 2**, **5** and **7** and parts of **Question 3**, **4** and **6**, very demanding. A significant minority of candidates had difficulty with basic facts and techniques required by the syllabus.

Standards of presentation varied between scripts. There were some pleasing scripts that by their presentation made it easy for the Examiner to assess the worth of the answers. Many Centres and candidates do need to heed the instructions on the front of the question paper and remember "the need for clear presentation".

Comments on specific questions

Question 1

Most candidates scored well. They identified the need to first use Newton's Second Law and then knew the relationship between power, force and speed required to reach the answer. The most common error was to omit the resistance force from the application of Newton's Second Law.

Answer: 20 000 W

Question 2

This question was a challenge for many candidates. Few candidates were able to demonstrate a correct relationship between the two given forces and the forces resultant, either in a diagram or written working.

(i) Correct solutions were divided equally between those obtained by resolving vertically and horizontally and by those obtained by employing a triangle of forces and use of the cosine formula.

(ii) Success in part **(i)** was almost invariably followed by success in part **(ii)**.

Answers: **(i)** 67.4° **(ii)** 9 N

Question 3

(i) and **(ii)** Most candidates found little difficulty in the application of the routine standard formulae for potential and kinetic energies.

(iii) Unfortunately most candidates then assumed that the given "work done against the resistance to motion" was the only work involved, completely ignoring their successful answers to the preceding parts. Candidates must read questions carefully and treat each part on its merits.

Answers: **(i)** 32 000 J **(ii)** 125 J **(iii)** 1250 W

Question 4

The majority of candidates realised that this question required the use of the calculus. Few attempted to make use of the equations of motion with constant acceleration.

(i)   The initial differentiation was almost always accurate. The resulting expression was always equated to zero and commonly correctly solved. Those who obtained two solutions, 0 and 8, did not always specify which value was to be their answer.

(ii)   The initial integration was almost always accurate. Those using indefinite integration usually just ignored the resulting constant. Whether indefinite or definite integration was used the result was almost always the same – as in the previous question a majority of candidates took the first available figure and used it without thought. Few realised that the time of 8 seconds obtained in the first part of the question had nothing to do with the time when the direction of motion changed. The value 8 was substituted in the distance formula and no attempt made to find the actual value when the change took place. Those who realised that the velocity formula must be equated to zero successfully found and then used the required time of 12 seconds.

Answers: **(i)** $t = 8$ s   **(ii)** 108 m

Question 5

Only the last part of this question proved to be a source of marks for most candidates.

(i)   Most candidates simply assumed, erroneously, that the particle fell vertically under gravity, despite the text and diagram. Many of those who correctly realised this question was about did not correctly compute the potential energy lost. The vertical displacement from O to A was the difficulty, either because of the trigonometry involved or confusion between the displacements from O to A and from A to B.

(ii)   Newton's Second Law was correctly identified as the approach to use. This was not often successfully done. Either the force of friction or the weight was omitted. The normal reaction was often incorrect due to faulty trigonometry as was the required component of the weight.

(iii)   Few candidates had any difficulty in substituting their answers to the previous parts into the required equation of motion.

Answers: **(i)** 12 ms^{-1} **(ii)** 2 ms^{-2} **(iii)** 13.6 ms^{-1}

Question 6

The switch from the usual vertical strings to non-vertical strings in a pulley situation proved very difficult for some candidates.

(i)   Most candidates who offered a solution attempted initially to apply Newton's Second Law to each particle. Most did not realise that a component of the particle weights rather than the whole weight would be involved. The subsequent solution of the resultant simultaneous equations made demands for algebraic accuracy that many could not meet. Those who opted to apply Newton's Second Law to the whole system had the same problem with components of the weights. These candidates usually proved subsequently to be unable to apply the law successfully to a single particle.

(ii)   If an acceleration had been found in part **(i)** an answer to this part was produced. This was usually just the time taken for P to reach the ground. Less frequently it was just the time from then until Q reached its highest point – and often assumed that the acceleration involved would simply, and wrongly, be that due to gravity. Rarely both times were found and added together. Candidates should be encouraged to think carefully about what will actually happen in the more involved contexts of later questions in the paper.

Answers: **(i)** Tension = 3.84 N, acceleration = 1.6 ms^{-2} **(ii)** 1.5 s

Question 7

Only the very best candidates earned marks on this question. Few candidates offered anything substantial, normally resulting from a difficulty in applying Newton's Second Law to the situation. This commonly resulted from a lack of a structured approach. Diagrams were usually non-existent. Diagrams with forces and their points of application clearly indicated even rarer.

(i) Most candidates offering a solution produced one equation, often correct, for the forces acting at point *B* – usually a horizontal resolution. The other equation mixed forces acting at point *B* with some of those acting at point *A*. The majority of equations had the same value for tension in each part of the string. Many additionally found it difficult to handle the required trigonometry correctly. Similarly, those with two equations in two unknowns found it difficult to solve them correctly.

  The most efficient and successful solutions were those offered by a small number of able candidates who simply resolved twice at *B* in the directions of *BC* and of *BA*.

(ii) Those who offered a solution understood the relationship between force of limiting friction, coefficient of friction and normal reaction. The difficulty lay in fully identifying the forces acting on the ring and their directions. Again this appeared to be related to the lack, in most cases, of a clear diagram with forces clearly indicated.

Answers: **(i)** Tension in *AB* = 6.4 N, tension in *BC* = 4.8 N   **(ii)** 0.359

MATHEMATICS

Key messages

- Candidates need to be careful to read the question in detail and answer as required.
- Candidates need to take care to avoid arithmetic and sign errors.
- Candidates need to show working to obtain the full marks available for each question.
- Candidates should check working ensure high accuracy.

General comments

As usual with the Mechanics papers the early questions were better answered than the later questions; this was anticipated as the later questions are designed to be more demanding than the earlier ones.

Some candidates had misinterpreted of the requirements within some questions, most notably in **Question 2**, **Question 5**, **Question 6(ii)** and **Question 7(i)**.

Another unfortunate feature of some candidates' work arises in writing down an equation such as one deriving from Newton's Second Law or an equation representing a balance of work and energy. The feature is that a linear combination of terms is dimensionally unbalanced. Common examples of the feature in Newton's Second Law is to have 'weight x acceleration' instead of 'mass x acceleration' or 'mass + tension + air resistance' instead of 'weight + tension + air resistance'. Such errors prevented the candidate from gaining relevant method marks.

Another factor is errors of signs in linear equations. Generally the method mark is not withheld for errors in signs but the associated accuracy mark is not scored. An exception to this is when a quantity A needs to be found, say, just by the addition of B and C. If the candidate uses $A = B - C$ the method mark will be withheld. Also for a method mark for a linear equation, it is required that each of the terms that should be present are represented in such a way that they indicate the correct intent. All such terms should be represented and there should be no extraneous terms.

Comments on specific questions

Question 1

This question was very well attempted and almost all candidates scored all three of the available marks.

Answer: $F = 30$ N

Question 2

A significant minority of candidates gave the answer for α as 45°, without any working. This may be due to candidates deciding this just by looking at the diagram in the question paper.

Candidates should realise that if $\alpha = 45°$ the directions of the forces of magnitudes 12 N and F N should be seen to be symmetric about the y-axis. There are indications in the diagram that suggest otherwise. The angle between the force and the x-axis should be the same in the two cases, although it may be necessary to put a ruler edge along the x-axis to be able to see that is not the case. More clearly the magnitudes of the two forces must be the same if $\alpha = 45°$.

The number of candidates who gave the answer $\alpha = 45°$ was small. Very many candidates answered correctly and completely to score all four marks.

Answers: $\alpha = 53.1°$, $F = 9$ N

Question 3

(i)   Almost all candidates recognised the need to integrate $a(t)$ and most remembered to introduce a constant of integration into the expression for $v(t)$. This is a very important factor in both parts of this question. Very many candidates scored all four marks in this part.

(ii)   Almost all candidates scored the method mark for attempting to integrate again to obtain $s(t)$. For candidates without a constant of integration in part **(i)**, the marks for accuracy were unavailable. Candidates should be aware that in Mechanics questions for which integration is involved, it is almost certain that the constant of integration is an essential ingredient.

Answers: **(i)** $t^{5/3}$ = 5/6   **(ii)** OP = 2.13 m

Question 4

(i)   Many candidates, who basically knew the Mechanics required, were challenged by this question, and were unable to find the components of the force exerted on the ring.

(ii)   This part of the question was very well attempted. However, a minority of candidates who answered part **(i)** correctly and completely, proceeded to the erroneous $T\cos25^{\circ}$ = 0.4 x 40.

Answers: **(i)** Horizontal 0.906T, Vertical 40 + 0.423T **(ii)** T = 21.7 N

Question 5

A significant minority of candidates gave answers as required in part **(i)** and then carried forward these answers into part **(ii)** as values of tensions acting on particles A and B respectively. From the wording of the question in part **(ii)** the tension in S_2 is zero, and that the magnitude of the tension acting on B is the same as that acting on A.

(i)   Many candidates wrote down incorrect answers in this part.

(ii)   Most candidates applied Newton's Second Law to both particles, or to only one particle and applied the formula $(m_A + m_B)g - (R_A + R_B) = (m_A + m_B)a$. However, many candidates made mistakes in application, the most common being incorrect signs, the absence of the tension and the absence of the air resistance.

Answers: **(i)** S_1 = 30 N, S_2 = 50 N   **(ii)** a = 8.88 ms^{-2}, T = 1.76 N

Question 6

A significant number of candidates did not obtain the required degree of accuracy in both parts of the question. Many such candidates used sin 7.2° for sin α instead of the given value 0.125. A smaller number of candidates used sin 0.125 for sin α.

(i)   This part of the question was very well attempted and many candidates scored all four marks.

(ii)   Candidates found this part of the question more challenging than part **(i)**. Many candidates who realised the need to find the speed of the car at the top of the hill did not know how to find this speed.

Many candidates who found a value, correct or otherwise, used it with the constant acceleration formula $v^2 = u^2 + 2as$ to find a value of a. However there is no indication in the question paper that the acceleration of the car is constant, as it would need to be to justify the use of the formula quoted. Such candidates then applied Newton's Second Law to find a value for the driving force, which is three times greater at the top of the hill than it is at the bottom. Thus, it cannot possibly be constant throughout the uphill motion and the value of the driving force found is invalid. It follows therefore that the value of the acceleration cannot be constant.

This part of the question requires the use of a work/energy equation to find the work done by the car's engine.

Answers: **(i)** Work done = 945 000 J   **(ii)** Work done = 985 000 J

Question 7

(i) From the wording in the question the kinetic energy loss happens instantaneously when the block hits the wall and reverses its direction of motion. Despite this the majority of candidates interpreted the given loss of 0.072 J as the block's loss of kinetic energy as it moves for two seconds from X to Y and found an incorrect value of v at Y.

There are unfortunate consequences arising from this error. Firstly $v = u + at$ is applied to the motion from X to Y to find an incorrect value for a by using the incorrect value of v. This makes redundant the frictional force, and there becomes no reason to apply Newton's Second Law. The first seven marks are unavailable to candidates who have misinterpreted in this way. Each of the final two marks are available.

Among the candidates who did use Newton's Second Law to find the acceleration a large proportion obtained $a = 0.8$ instead of -0.8 (or instead of writing 'deceleration = 0.8'). The resulting increase in the speed of the block as it reached Y, did not lead to candidates questioning their previous calculations.

Among the candidates who reached ½ $0.15(1.4^2 – v^2) = 0.072$ ➔ $v^2 = 1$ correctly, the majority gave the required answer as $v = 1$ instead of $v = -1$. The symbol v is precisely defined in the question.

Very few candidates sketched a velocity-time graph that showed v to be negative for $v > 2$.

(ii) Only a few candidates were able to sketch a valid graph in this part.

Answers: **(i)** $v_{Approach} = 1.4$ ms^{-1}, $v_{Return} = -1.0$ ms^{-1}, $t = 3.25$ s

MATHEMATICS

> **Paper 9709/43**
> **Paper 43**

Key messages

- Candidates need to ensure that answers are given to the required accuracy of 3 significant figures and they should be aware that early approximation can affect the accuracy of final answers (e.g. **Question 4** and **Question 5(ii)**).

- When answering a 'show that ...' question such as **Question 6** parts **(i)** and **(ii)** candidates should check that their solution leads accurately to the answer given.

- Candidates who omit to use one or more relevant forces in their solution (e.g. in **Question 2** or **Question 5**) may benefit from the use of a complete force diagram before attempting equations.

General comments

Many of the scripts showed a high standard of work both in content and in presentation. Most candidates were able to make some progress with all questions whilst some parts of the later questions challenged even the stronger candidates. **Questions 1** and **3** were the most well answered questions. **Question 5** part **(ii)**, **Question 6** parts **(i)** and **(ii)** and **Question 7** parts **(iii)** and **(iv)** were found to be more difficult.

Comments on specific questions

Question 1

The majority of candidates calculated the work done by the tension accurately. A few treated the acceleration rather than the speed as constant and calculated a distance of e.g. ½ (0+0.5)x8 = 2 m. Other errors included the occasional use of sine instead of cosine. An answer of 22 J either from early approximation or from rounding correct to 2 significant figures was sometimes seen.

Answer: 21.9 J

Question 2

Most candidates understood in part **(i)** the need to resolve forces in two perpendicular directions. Those who formed two equations in $T\sin\theta$ and $T\cos\theta$ usually continued in part **(ii)** to attempt the solution using a suitable method. The most common method was to find numerical values for $T\sin\theta$ and $T\cos\theta$ and then to eliminate T. Surprisingly often the situation was oversimplified at the start with the omission of either the weight leading to $T\sin\theta = T\cos\theta$ or the omission of the tension in the string between B and R leading to $T\sin\theta = 1.6$. Equations such as $T\cos\theta + T\cos(90-\theta) = 11.2$ or $T_1\cos\theta + T_2\sin\theta = 11.2$ sometimes left candidates unsure how to progress further.

Answers: T=8 θ=53.1

Question 3

This question was generally answered well with complete and accurate solutions seen frequently.

(i)   Candidates knew that they needed to integrate to find the distance travelled by P and the integration was usually completed correctly. A few candidates left their answer as a function of t or used an incorrect value of t such as $t=20/3$, but the majority solved $v=0$ and substituted $t=10$ to obtain the distance OA successfully. Weaker candidates occasionally attempted erroneously to use a constant acceleration formula in their solution instead of using integration.

(ii)   Candidates also knew that they needed to use $a = dv/dt$ and any errors in this part of the question were usually 'numerical' from obtaining an incorrect value of t from 'their' solution of $dv/dt=0$.

Answers: **(i)** 22.5 m   **(ii)** 4 ms^{-1}

Question 4

(i)   Whilst many candidates calculated the two accelerations correctly, some were confused by the 'average' acceleration and tried, for example, to do an additional calculation using their two answers. The answer of 0.244 ms^{-2} was quite often given as 0.24 ms^{-2} which then affected accuracy in part **(ii)**. Some candidates calculated $\Delta t/\Delta v$ rather than $\Delta v/\Delta t$ for the second time interval.

(ii)   Newton's Second Law was often used effectively to obtain the values of P and R, showing understanding that the driving force could be represented by P/v. Erroneous solutions included the use of '$P/v = ma - R$' instead of '$P/v = ma + R$' and the use of $P=mav$ instead of $P=$driving force x v.

Answers: $P=30800$ W $R=1240$ N

Question 5

Candidates found this question more challenging particularly in part **(ii)**.

(i)   A correct solution involved either a work / energy equation or an equation of motion and both methods were used successfully. Some equations formed had the wrong number of terms with, for example, the omission of the resisting force or with the introduction of an extra term by treating potential energy and work done due to weight separately. Some equations were also seen with a mixture of work and force terms. Occasionally candidates misinterpreted the distance 500 m showing it as a horizontal or vertical distance.

(ii)   Although a work / energy equation was expected for this solution some candidates attempted to use Newton's Second Law making an assumption that acceleration was constant and also sometimes omitting to consider the component of weight. Many of the work / energy equations formed had one or more missing terms (often the gain in potential energy). Some equations contained sign errors and others used an approximation of 1.7^{o} for the angle, thus losing accuracy in their final answer.

Answers: **(i)** $\alpha = 1.7$ (ii) $v = 30$ ms^{-1}

Question 6

In this question the answers for parts **(i)** and **(ii)** were provided which did not always benefit the candidate. Most were able to gain some marks, but full marks required a clear understanding of frictional force in a variety of situations.

(i)   Candidates needed to recognise that at rest, for a frictional force (F) and normal reaction (R), $F \leq \mu R$. Candidates often assumed $F = \mu R$ and then stated the answer given $\mu \geq 4/5$ rather than forming and solving a suitable inequality.

(ii)   Part **(ii)** required the use of $F = \mu R$ and 'net downward force'>0. A common solution seen included $7 = 6\mu$ treating the net downward force and the frictional force as equal despite the motion of the particle.

(iii) Most candidates applied Newton's Second Law to form an equation which was then solved with greater success than the other parts of this question. Errors included missing a term from the equation and the occasional use of mga rather than ma. 0.993 was sometimes seen rather than 0.994.

Answer: $\mu = 0.994$

Question 7

The first two parts of this question were found to be accessible to most candidates whilst the rest of the question was more problematic.

(i)   The acceleration was found either by forming an equation of motion for each particle or by forming a single equation for the acceleration of the system. In both cases sign errors led to incorrect values of a. For those forming a single equation, $a = (3.8 - 1.2)/3.8 = 6.84$ was sometimes seen.

(ii)   Candidates were usually able to use appropriate constant acceleration formulae with 'their' value of a to obtain the speed of B and also the time taken to reach the ground although some candidates did not find the time.

(iii) Many candidates misinterpreted the meaning of T often calculating the time taken for two rather than three stages of motion. Sketches were thus often incomplete illustrating either the motion from release until the highest point was reached by particle A or motion from the moment at which the string became slack until the same position was reached for a second time. Correct sketches were not often seen as even the best candidates sometimes sketched a speed-time graph instead of a velocity-time graph.

(iv)   Although candidates knew the methods for finding distance by using either area or formulae, the calculations were often incomplete due to the misinterpretation of T. Some candidates also believed that the initial distance travelled by A was 1.3 m rather than 0.65 m.

Answers: **(i)** 5.2 ms^{-2} **(ii)** 2.6 ms^{-1} 0.5 s   **(iii)** 1.02   **(iv)** 1.326 m (1.33 m)

MATHEMATICS

Key messages

- It is important for candidates to read the questions carefully, and answer as required.
- Candidates should check working to ensure high accuracy and to avoid arithmetic errors.
- Candidates need to make full use of the supplied formula booklet.

General comments

Many of the candidates were challenged by this paper. On occasions candidates did not understand the question set.

Questions 1, 3(ii), 6(i) and **7(i)** were well answered. Other questions were more challenging.

Comments on specific questions

Question 1

Most candidates scored both available marks. A few candidates used the incorrect formula $\omega = rv$, instead of the correct formula $v = r\omega$.

Answer: $\omega = 0.5$ rads^{-1}

Question 2

(i) The centre of mass formulae are quoted in the formula booklet. Some candidates used incorrect formulae. The idea of taking moments about O was often used. This resulted in errors for many candidates.

(ii) The required angle $\tan^{-1}(0.0371/0.7) = 3.0°$ was rarely seen. Most candidates stated that the lowest point was in the lamina.

Answers: **(i)** $OG = 0.0371$ m   **(ii)** Required angle is 3.0, Lowest point is on the lamina

Question 3

(i) Candidates knew that they had to resolve horizontally and vertically. Errors were made when trying to eliminate the force. Only a few candidates scored all 4 marks.

(ii) This part of the question was well answered and the final answer $\omega = 5.37$ rads^{-1} was often seen.

Answers: **(i)** $v = 1.86$ ms^{-1} **(ii)** $\omega = 5.37$ rads^{-1}

Question 4

(i)   Candidates recognised that an energy equation was needed. The correct equation was rarely seen because the wrong extension was calculated. The extension should be $(1.2^2 + 1.6^2)^{\frac{1}{2}} - 1.2$. There should be 3 terms in the equation and these should be initial KE, final PE and final EE.

(ii)   In this part of the question Newton's Second Law is needed with the components of T and the weight being used. The extension required here is $(1.2^2 + 0.5^2)^{\frac{1}{2}} - 1.2$.

Answers: **(i)** $m = 0.2$ **(ii)** $a = 3.27$

Question 5

This question proved a particular challenge for candidates.

(i)   Newton's Second Law down the slope results in the equation $0.4v dv/dx = 0.4g\sin30 - 0.6x$. When integrated this gives $0.2v^2 = (0.4g\sin30)x - 0.3x^2$ and leads to $v^2 = 10x - (3x^2/2)$. The maximum speed occurs when $a = 0$ so $0.4g\sin30 - 0.6x = 0$, $x = 3\frac{1}{3}$ and this gives $v = 4.08\,\text{ms}^{-1}$.

(ii) $v = 0$ is now used leading to $10x - (3x^2/2) = 0$ and hence $x = 6\frac{2}{3}$ or 6.67.

Answers: **(i)** Maximum speed is $4.08\,\text{ms}^{-1}$ **(ii)** Distance travelled by P is 6.67 m

Question 6

(i)   Most candidates found the position of the centre of mass without difficulty.

(ii)   This part of the question proved to be more difficult. To establish the set of values of F it was necessary to take moments about O and A. Moments about A: Fx1.5 = 120x0.6375, F = 51. Moments about O: Fx1.5 = 120(0.6375 − 0.4), F = 19. Hence $19 \leq F \leq 51$

Answers: **(i)** Distance of centre of mass from OE is 0.6375 m   **(ii)** $19 \leq F \leq 51$

Question 7

(i)   This part of the question was well done by most candidates. A few candidates did not express x and y in terms of t and simply used the trajectory equation to give the required equation.

(ii)   By using $y = 0.869x - 0.0390x^2$ with $x = 1.5$ to find y and then by adding 1.6 this resulted in the height of the fence being 2.82 m. The distance from the fence to A can be found by substituting $y = -1.6$ into $y = 0.869x - 0.0390x^2$ leading to a quadratic equation which when solved gives $x = 23.99$. When 1.5 is subtracted this gives 22.5 m the required distance of the fence from A.

Answers: **(i)** $y = 0.869x - 0.0390x^2$ **(ii)** Height of fence is 2.82 m and distance of fence from A is 22.5 m

MATHEMATICS

Key messages

- Care needs to be taken to avoid arithmetic and sign errors.
- Candidates need to make full use of the supplied list of formulae.

General comments

Candidates scored a range of marks on this paper. The easier questions were **1**, **3**, **4**, **5(i)**, **6(iii)** and **7(i).** The harder questions were **2(ii)**, **6(i)** and **7(ii)**.

Comments on specific questions

Question 1

This question was generally well done with many candidates scoring all 4 marks. Occasionally a negative sign was seen attached to the $0.3x$ term.

Answer: $v = 6$ ms^{-1}

Question 2

(i) The centres of mass of both the solid hemi-sphere and the hemi-spherical shell have to be found. Sometimes the incorrect formulae were used. These formulae are given in the formula booklet. To find the centre of mass of the sphere, moments had to be taken about the centre of the sphere. The common mistake was not to realise that one of the moments should be negative.

(ii) To find the force, F, moments had to be taken about the point of contact. A common error was to use Fx0.2 instead of Fx0.4 for the moment of F about the point of contact.

Answers: **(i)** Distance is 0.005 m   **(ii)** $F = 0.25$

Question 3

(i) This part of the question was quite well done. The most common error was not to use the components of the tension. Sometimes the wrong extension was used.

(ii) To solve this part of the question a four term energy equation was required. Too often only three terms appeared.

Answers: **(i)** $m = 0.2$ kg   **(ii)** $v = 2.47$ ms^{-1}

Question 4

This question proved to be a very popular one with many candidates scoring all 7 marks.

(i) A number of candidates used $+3t$ instead of $-3t$. Also some candidates did not introduce a constant of integration.

(ii) If the constant of integration was omitted then only method marks could be scored. No accuracy marks were available.

Answers: **(i)** Initial speed is 54 ms^{-1} **(ii)** Distance travelled is 108 m

Question 5

(i) This part of the question was generally well done and many candidates scored all 3 marks. Some candidates only found half the time and half the distance.

(ii) Many candidates scored all 4 marks. Those candidates failing to score full marks usually used $t = 4.7$, the time for travelling from O to P and not the time for the first bounce.

Answers: **(i)** Time to travel from O to P is 4.7 s and the distance OP is 40.2 m
(ii) The speed when leaving P is 13.2 ms^{-1}

Question 6

(i) Having found the required centres of mass the candidates then attempted to take moments about O. Sometimes the incorrect formula was used for the centre of mass of a semi-circular lamina and too often a sign error occurred in the moment equation.

(ii) This part of the question was very well done. A few candidates used $OG = 1 - r$ instead of $OG = r$.

(iii) This part of the question was very well done.

Answers: **(i)** Distance of centre of mass from O is $4(1 + r + r^2)/3\pi(1 + r)$

(ii) r = 0.494   **(iii)** Angle is 26.3°

Question 7

(i) This part of the question was generally well done.

(ii) Many candidates scored well on this part of the question. $T = 10$ was found and $T\sin 60 = 0.5Rx6.25^2$ leading to $R = 0.4434\ldots$ The last 3 marks were lost by quite a number of candidates.

Answers: **(i)** Radius of path of P is 0.16 m and speed of P is 1 ms^{-1}
(ii) Total length of string is 0.832 m

MATHEMATICS

Paper 9709/53
Paper 53

Key messages

- Candidates need to be careful to read the question in detail and answer as indicated.
- Candidates need to take care to avoid arithmetic and sign errors.
- Candidates need to take care to avoid algebraic errors.

General comments

This paper was well answered by most candidates.

Questions **1**, **2**, **3(ii)**, **4(i)** and **7(i)** were accessible to most candidates. Questions **5**, **6(iii)** and **7(ii)** were more challenging.

Comments on specific questions

Question 1

This question was generally well answered, with candidates calculating horizontally and vertically. A few candidates only found *OX* and went no further. Some candidates when finding *OY* had a positive *g* term.

Answer: *OP* = 43.6 m

Question 2

The centre of mass of the arc was usually correctly calculated. Some errors occurred when taking moments about *A*.

Answer: *F* = 0.537 N

Question 3

(i) Some candidates omitted to use the negative sign when setting up the differential equation and others did not introduce a constant of integration.

(ii) This part proved to be accessible provided that the candidate had the correct integration in part **(i)**.

Answers: **(i)** KE = 1.25 J **(ii)** *x* = 2.06

Question 4

(i) This part was well answered, with many candidates scoring all 3 available marks.

(ii) The tension was often found correctly. Newton's Second Law was then used. If a mistake occurred it was usually a sign error.

(iii) Many candidates did not realise that the horizontal force was zero.

Answers: **(i)** $v = 2 \text{ ms}^{-1}$ **(ii)** $m = 0.5$ **(iii)** Least value of $\omega = 6.24$

Question 5

(i) By resolving parallel to the slope it is possible to say that $T_A = T_B + 6\cos60$ i.e. $T_A = T_B + 3$. It is then necessary to choose some fixed point and also use $T = \lambda x / L$. The extension for each string can be found and also the tension. These values are then substituted into the earlier equation. This allows the determination of AP.

(ii) This part of the question requires a correct energy equation to be set up with 5 different terms.

(iii) Here the candidate needs to set up another energy equation and show that the velocity is zero when $AP = 1.6$.

Answers: **(i)** $AP = 2.05$ m   **(ii)** $v = 4.5\,\text{ms}^{-1}$ **(iii)** $v = 0$ when $AP = 1.6$ m

Question 6

(i) Many candidates tried to take moments about BE. Errors were made due to sign errors or faulty algebraic manipulation.

(ii) It is necessary here to recognise that the centre of mass lies on BE and so $x = (3 - 4h^2)/(12 + 12h) = 0$, leading to $h = 0.866$

(iii) If θ is the angle between EG and EB where G is the centre of mass then $\tan\theta = x/(a/2) = (a/2)/h$. When $h = 0.5$ $x = 2/18$ and so $(2/18)/(a/2) = (a/2)/0.5$, $a = 0.471$

Answers: **(i)** Distance of centre of mass from BE is $(3 - 4h^2)/(12 + 12h)$ **(ii)** $h = 0.866$ **(iii)** $a = 0.471$

Question 7

(i) This part of the question was well answered. By comparing $y = 0.6x - 0.017x^2$ with the trajectory equation it is possible to say that $\tan\theta = 0.6$ and $10/[2(v\cos31)^2] = 0.017$ leading to $\theta = 31.(0)$ and $v = 20$

(ii) Vertical motion gives $5.2 = 20t\sin31 - 10t^2/2$ leading to $t = 1.17$ $v_{vertical} = 20\sin31 - 10 \times 1.17 = -1.372$. Resultant speed $= = [(20\cos31)^2 + 1.372^2]^{\frac{1}{2}} = 17.2$ ms^{-1}. If α is the direction with the horizontal then $\tan\alpha = 1.372/17.15$, $\alpha = 4.6$

Answers: **(i)** $v = 20$ ms^{-1} **(ii)** Speed is 17.2 ms^{-1} and the direction is 4.6° to the horizontal

MATHEMATICS

Paper 9709/61

Paper 61

Key messages

- Candidate need to ensure they do not make approximations during their working, as this can impact on the accuracy of the final result.
- A considerable proportion of the total marks depend on giving answers which are correct to 3 significant figures. In order to gain as many marks as possible all working should be either exact or correct to at least 4 significant figures.
- Candidates need to make full use of the supplied list of formulae.

General comments

The overall proportion of scripts of a satisfactory standard has increased, with fewer candidates being unprepared. However, there were many candidates poorly prepared for either the Binomial or the Normal Distributions. Questions involving probabilities caused particular problems to candidates in differentiating between Distributions and whether to add or multiply probabilities.

The general presentation was satisfactory. Marks were lost by approximating prematurely, truncating values and not giving answers to the required accuracy as detailed in the comments below.

Comments on specific questions

Question 1

There is a lot of confusion between variance and standard deviation, with common use of 4.5 instead of $\sqrt{4.5}$. A Normal Distribution is continuous and the use of a Continuity Correction is wrong. Candidates who used a diagram were more successful in determining the correct probability. The accuracy mark was lost by some candidates in their use of tables.

Answer: 0.729

Question 2

(i) This part was usually successfully attempted. Candidates who were not successful invariably insisted on using their value in **(ii)** rather than the given value.

(ii) A large minority of candidates did not recognise the need for Conditional Probability and a quotient. A simple tree-diagram would assist candidates in determining the probabilities.

Answers: **(i)** p=0.3 **(ii)** 0.785

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Question 3

(i) Most candidates obtained the correct value of *p*. In **(ii)**, some candidates identified the possible outcomes and then erroneously gave the required probabilities as 3/15 and 10/15 respectively, ignoring the different probabilities associated with the values of *X* and *Y*.

(ii) (a) This part was generally well done. The main sources of error were in assuming P(*X*=1, *Y*=3) = P(*X*=3, *Y*=1) and to include P(*X*=2, *Y*=2) twice in the calculation.

(b) Most attempts were performed by evaluating the 10 different outcomes, with the anticipated errors in multiplying 2 decimal probabilities often occurring. The outcomes (4,1) and (5,1) were often omitted. Few candidates attempted the method of using [1 – P(product ≥8)]

Answers: **(i)** p=0.1   **(ii)(a)** 0.225   **(ii)(b)** 0.765

Question 4

Many candidates were challenged by both parts of this question.

(i) Marks were lost through omitting or inserting P(*X*=5) through not understanding P(*X*<5), and in premature approximating. Probabilities used in a compilation require to be at least to 4 significant figures if the answer is expected to be to 3.

(ii) Few correct responses were seen, with P(at least 1) not being recognised or evaluated. Those successful responses then correctly used P(*X*=3) from the distribution Bin(4, 0.808).

Answers: **(i)** 0.994   **(ii)** 0.405

Question 5

There were very few high marks for this question with each part either presenting specific difficulties or candidates not working carefully.

(i) 'Back-to-back' was ignored by some candidates and marks were lost by not having integers in the stem, leaves in columns, leaves in ascending order from the stem and a key containing all the relevant information.

(ii) The median was usually found successfully, but many candidates were either unable to find the Quartiles (i.e. the values, in this example, of the 2.5th and 7.5th measurements) or to attempt to evaluate [Q_3 - Q_1].

(iii) The mark for the mean was lost if the answer was presented as 0.93 without any evidence of an uncorrected value. Few candidates obtained the mark for the standard deviation because of the incorrect use of the formula [e.g. $\sum(x^2) \neq \{\sum(x)\}^2$], by prematurely approximating the mean to 0.927 or 0.93, or by not square rooting.

Answers: **(i)**

Flat screen		Conventional
	6	5 7 9
6	7	1 4 5 7
9 5	8	5 6
6 4 2 1	9	
7 4	10	

Key 5│8│4 means 0.85 m for flat screen and 0.84 m for conventional screen.

(ii) Median = 0.74, IQR = 0.13   **(iii)** Mean = 0.927, SD = 0.0882

Question 6

Many candidates were unable to progress with either part of this question.

(i) A diagram with the relevant information would assist in diagnosing what is required. There was confusion between probabilities and 'z'-values. Other sources of error were in poor reading off from tables, the signs of the 'z' values and premature approximations.

(ii) $z = \pm 1$ was rarely used and the wrong area evaluated. Again, a simple diagram could be useful. Many candidates quoted '68% of the area is within 1 sd from the mean', but this is an approximation and resulted in the loss of an accuracy mark.

Answers: **(i)** $\mu = 9.95$, $\sigma = 3.15$ or 3.16 **(ii)** 317

Question 7

Responses were varied, with candidates often being successful in **(a)** or **(b)**, but not in both.

(a) (i) Very few candidates realised that each couple could be in 2 ways and consequently 2^7 was rarely seen.

  (ii) Likewise, the doubling of (7! x 7!) was overlooked by most candidates.

(b) (i) Many candidates preferred to consider separate outcomes instead of using $_7C_2$.

  (ii) Most candidates realised the two cases, but many forgot to add 1 to the 5.

  (iii) Consideration of the different outcomes of 2 and 3 girls was recognised but most candidates again opted to consider the compilation of all the separate combinations between boys and adults rather than $_6C_1$ and $_6C_1 \times _3C_2$ and in consequence often omitted a possible outcome.

Answers: **(a) (i)** 645120   **(ii)** 50 803 200 (50 800 000)   **(b)(i)** 21   **(ii)** 6   **(iii)** 51

MATHEMATICS

Paper 9709/62

Paper 62

Key messages

- To do well in this paper candidates must work with 4 significant figures or more in order to achieve the accuracy required.
- Candidates should show all working, so that in the event of a mistake being made, credit can be given for method.
- Candidates should label graphs and axes, show dotted lines for finding the median or quartiles from a graph, and choose sensible scales.

General comments

This paper was well attempted by the majority of candidates who had worked and prepared for it. Most candidates appeared to have covered all the topics and were able to make a start on all of the questions. A few candidates used long alternative solutions, especially in **Questions 1**, **3**, **6** and **7**, instead of shorter more standard approaches.

Comments on specific questions

Question 1

This question was well attempted. Almost all candidates obtained the mean and variance. Many candidates stopped there and were unable to recognise that $\dfrac{\Sigma(x-\bar{x})^2}{n}$ is also the variance despite it being in the formula sheet, and thus an easy matter to obtain $\Sigma(x-\bar{x})^2$. Accuracy was lost by some candidates who used the standard deviation to be 6.06 instead of 6.063. Some candidates chose to expand the brackets. This method is perfectly acceptable although longer and more difficult, with many candidates not including the sigma notation in all three terms.

Answer: 5514

Question 2

Many candidates did not read this question carefully and hence found this question more challenging than necessary. The question was a good discriminator as most candidates who did answer this question tended to do well on the paper as a whole. Some candidates did not use the given probabilities, instead using 1/9ths for each result in a 3 x 3 table. Another common mistake was not to check that their probability sum in the table was 1.

Answers: **(i)** P(Y = 0) = 0.42, P(Y = 2) = 0.48, P(Y = 4) = 0.1   **(ii)** E(Y) = 1.36.

Question 3

(i) The majority of candidates recognised the binomial situation and used appropriate binomial probabilities. Some candidates used the sum of P(3, 4, 5, 6, 7, 8, 9, 10, 11) whilst others used the quicker way of 1 – P(0, 1, 2, 12). Both methods were given full credit. Some candidates used the normal approximation on the grounds that np = 7.8 was > 5. They did not check that nq = 4.2 which is < 5 and so the approximation was not valid.

(ii) The second part of this question appeared familiar to approximately one third of the candidates. The rest either attempted to use the normal approximation or left this part out.

Answers: **(i)** 0.993   **(ii)** 22

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Question 4

Almost all candidates were successful in finding the correct median, and knew how to find the interquartile range. Most appreciated that with 33 data values the median would be the 17th value, the lower quartile the 8.5th value and the upper quartile the 25.5th value when put in order, but found difficulty in finding the 25.5th value, which was midway between 178 and 180. When drawing the box-and-whisker plot it is sensible to choose a scale which makes it possible to plot the quartiles accurately. Many candidates chose a scale of 7, 8, 9, 12 units for 2 cm on the graph paper, which means they lost accuracy marks. Candidates also are expected to use a pencil and a ruler.

Answers: **(i)** Median = 0.186, IQ range = 0.019

Question 5

This question was well answered by the majority of candidates, especially parts **(i)** and **(ii)**. Candidates are encouraged to explain (in one or two words only) what they are attempting to do, for example 'qn 1 and 2 in' and 'qn 1 and 2 out' in part **(iii)**.

Answers: **(i)** 462 **(ii)** 406 **(iii)** 210

Question 6

In spite of the instruction to "copy and complete" the table, some candidates did not reproduce the table in their answer books. However, the large majority of candidates did this question well, understood what was happening and gained 4 or 5 marks. Part **(v)** clearly said that 4 biscuits were taken without replacement from the box. This should have alerted candidates to the fact that probabilities were changing, and it was not a binomial situation. Credit was given for appreciating that there were 6 options ($_4C_2$) and for multiplying by 4 factors with denominators 30, 29, 28, 27. Most candidates who attempted this managed to get either 1, 2 or 3 method marks followed by an accuracy mark if they found the correct answer. Some candidates used the permutations and combinations method, and this was a very neat way of doing this type of problem.

Answers: **(ii)** 2/5 **(iii)** 5/9 **(iv)** 10/17 **(v)** 0.368

Question 7

The normal distribution was well covered by almost all the candidates, most of whom were able to use normal tables properly. The binomial situation in part **(i)** arising out of the normal distribution was also recognised by some candidates. Some candidates still, wrongly, used a continuity correction and others did not work to 4 significant figures thus losing an accuracy mark at the end. Part **(ii)** involved finding two z-values backwards from the tables and solving two simultaneous equations to give values of μ and σ. A good proportion of candidates managed this successfully, and had clearly practised with this situation before. In part **(iii)** some candidates lost valuable time in recalculating the given probability of 95%. Those who did evaluate the probabilities were, for the most part successful in multiplying them to give the required answer.

Answers: **(i)** 0.433 **(ii)** μ = 30.0, σ = 2.77 **(iii)** 0.0266

MATHEMATICS

Key messages

- Candidates should be encouraged to show all necessary working. Too many candidates do not show sufficient working to make their approach clear.
- A large proportion of the 50 marks available for this examination paper depended on the use of appropriate methods.
- Whilst full marks are usually awarded when the answer is correct, an incorrect answer with no indication of method cannot be given any credit.

General comments

Answers to **Questions 4**, **5** and **6** were usually stronger than answers to the other three questions.

Whilst some candidates adopt a strategy of attempting the questions in decreasing order of the marks allocated, the order in which questions were answered varied considerably more than usual. Several candidates delayed attempting **Questions 1** and **2**, or both, until after they had attempted the other questions. Nearly every candidate did answer all six questions.

Some scripts contained quite a lot of deleted work. Questions and their parts should be clearly numbered. This was particularly true for **Question 5**.

Comments on specific questions

Question 1

Many candidates gained full marks on this question. Others appeared unfamiliar with this topic. Most candidates recognised the significance of the number of prices, with only a few giving the same answer to both parts. There was some confusion between 'diagrams' and 'distributions'. Candidates found it easier to name a type of diagram than to give a reason for their choice. Using a fact given in the question is not sufficient.

(i) The appropriate diagram for the 21 headphone prices is a stem-and-leaf diagram. This allows all of the original prices to be seen. It would also allow the way in which the prices are spread to be seen, as well as facilitating the calculation of several statistics (e.g. median, range). A box-and-whisker plot would also be appropriate. A histogram could be used, although this is more appropriate for a larger amount of data.

(ii) With 163 headphone prices a histogram would be appropriate. This diagram would allow the modal class to be seen. Alternatively, a cumulative frequency graph or box-and-whisker plot could be drawn. All three types of diagram would show how the prices are spread. The larger amount of data is an appropriate reason for using a histogram or cumulative frequency graph.

Answers: **(i)** Stem-and-leaf diagram **(ii)** Histogram

Question 2

Strong candidates had little difficulty with this question. Weaker ones appeared unfamiliar with the ideas of a working mean and were uncertain which, if any, of the given formulae were appropriate.

(i) The majority of answers gained both marks for this part of the question. Weaker candidates worked with 72 and 104.8 only.

(ii) The most popular method was to calculate the standard deviation/variance of y, where $y = x - 100$, and equate this to the standard deviation/variance of z, where $z = x - 104.8$. Most errors arose through not realising that $\Sigma(x - 104.8) = 0$ since the mean of x is 104.8. A few candidates either expanded as $\Sigma((x - 100) - 4.8)^2$ or used $\Sigma(x - 100)^2$ to calculate the value of Σx^2 and used this in the expansion of $\Sigma(x - 104.8)^2$.

Answers: **(i)** 15 **(ii)** 154

Question 3

There were many fully correct answers to this question. Incorrect answers often consisted of figures with little or no explanation. Several candidates used permutations instead of combinations in parts **(ii)**, **(iii)** and **(iv)**.

(i) The most frequent error was to omit the factor of 2.

(ii) This question was usually answered correctly.

(iii) A frequent error was to multiply the required answer of $_6C_3$ (= 20) by $_3C_1$.

(iv) The majority of candidates realised that having found the number of different selections with no Es in part **(ii)** and the number of different selections with one E in part **(iii)**, the number of different selections when there are no restrictions should be obtained by summing these two answers with those for two Es and three Es. A common error was to give the answer $_9C_4$, which would apply if there were nine different letters.

Answers: **(i)** 1680 **(ii)** 15 **(iii)** 20 **(iv)** 56

Question 4

(i) This part was usually answered well. Quite often the final answer was given as 0.904, following the use of 1.83 for the mean. Other frequent errors were to give the variance rather than the standard deviation or to omit the subtraction of $\left(\frac{11}{6}\right)^2$. Those candidates who produced a probability distribution table nearly always gained full marks.

(ii) This part was usually answered well. Many candidates would have gained more marks had they shown their working. Those using a possibility space were generally more successful than those using a tree diagram or probabilities without any diagram. Some candidates gave probabilities which did not sum to 1, whilst others did not give their probabilities in a table. A few candidates assumed that the die for this part of the question had faces numbered 1, 2, 3, 4, 5 and 6.

(iii) Several candidates used $P(Y = 3) = \frac{1}{6}$ instead of $\frac{1}{3}$. Quite often 'np' was used for the expectation of a binomially distributed variable, but this was not followed by '$np(1 - p)$' for the variance.

Answers: **(i)** $\frac{\sqrt{29}}{6}$ (0.898) **(iii)** $5\frac{1}{3}$ (5.33)

Question 5

(i) This question was usually answered well. The most frequent error was to assume that Suzanne has 8 pairs of sports shoes with designer labels. Sometimes the row of totals was omitted.

(ii) This question was usually answered correctly. A few candidates used the row total of 4 in their denominator instead of the overall total of 20.

(iii) This question was nearly always correct.

(iv) This question was usually answered correctly. Some candidates attempted to use Bayes' theorem. This required more work than usual for 1 mark and quite frequently an arithmetic error resulted in an incorrect answer.

(v) Some candidates omitted this part of the question, but generally the condition for two events to be independent was understood. A few candidates used the sum rather than the product whilst others compared their sum or product with 0 or 1. Methods involving conditional probability were much less popular and often included wrong probabilities

(vi) Nearly all candidates recognised that the binomial distribution should be used. The majority of answers involved the correct binomial terms and there were few arithmetic errors. The most frequent mistake was to calculate the probability of Suzanne wearing a pair of shoes with designer labels on 4 rather than "at most 4" days.

Answers: **(ii)** $\frac{1}{20}$ **(iii)** $\frac{1}{2}$ **(iv)** $\frac{1}{4}$ **(v)** Not independent   **(vi)** $\frac{14121}{15625}$ (0.904)

Question 6

Most candidates gained the majority of the marks available for parts **(i)** and **(iii)** of this question. Several candidates used $\sqrt{\sigma}$ or, less often, σ^2 when standardising. A few candidates used their answers for the mean and standard deviation of the trout in part **(i)** for the mean and standard deviation of the salmon in parts **(ii)** and **(iii)**. This resulted in part **(iii)** being impossible to answer. Only rarely did a candidate realise that they had made an error and needed to read the question again. Greater use of small normal distribution curve diagrams, with parts shaded to illustrate the probabilities under consideration, might have helped more candidates decide on the correct arithmetic to be used.

(i) There were many correct solutions. A common error was to use 0.524 instead of −0.524 in the second simultaneous equation. Some candidates used probabilities instead of z-values.

(ii) A large number of candidates calculated either the probability that a salmon was less than 34 cm long or the probability that a salmon was more than 34 cm long. Several candidates used an upper boundary of 34.4 instead of 34.5. Some attempts involved the binomial distribution, which is inappropriate in this context.

(iii) Some candidates produced very good solutions with the correct notation throughout. Weaker candidates appeared confused between z-values and probabilities, with both often appearing in the same equation. Several candidates assumed that 0.5 was the probability in the centre of the normal distribution, hence 0.25 was the probability in the upper tail and so used $z = 0.674$.

Answers: **(i)** Mean 28.4 cm, Standard Deviation 3.25 cm   **(ii)** 0.149   **(iii)** 35.1

MATHEMATICS

Key messages

- A considerable proportion of the total marks depend on giving answers which are correct to 3 significant figures. In order to gain as many marks as possible all working should be either exact or correct to at least 4 significant figures.
- Candidates need to be careful to read the questions in detail and answer as indicated.
- Candidates need to show working to obtain the full marks available for each question.

General comments

On this paper, candidates were able to demonstrate and apply their knowledge in the situations presented. Whilst there were some good scripts, there were also candidates who appeared unprepared for the paper. In general, candidates understood the method required on **Questions 1** and **7**, whilst **Questions 4(iii)** and **5(b)** proved particularly demanding.

Accuracy continues to incur loss of marks. It is important for candidates to realise that for a final answer to be accurate to 3 significant figures, then all working out up to the final answer needs to be accurate to at least 4 significant figures.

Detailed comments on specific questions follow. Whilst the comments indicate particular errors and misconceptions, it should be noted that there were also some good and complete answers.

Comments on specific questions

Question 1

Candidates, in general, used the correct formula for the confidence interval and the correct value of z was usually found. It is important to note that the final answer must be written as an interval rather than two separate values.
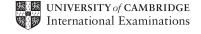
Answer: 490 to 498

Question 2

The candidates who attempted this question were generally successful in finding the mean of the final marks, but the correct method for finding the variance was not always used. Confusion between standard deviation and variance was evident. It is important for candidates to read the question carefully; the question required the standard deviation to be given and some candidates gave the variance. The question gave the standard deviation of M and P; some candidates used these values rather than calculating the variance of M and P.

Answers: Mean = 61.7, Standard Deviation = 12.1

Question 3

When performing a hypothesis test it is important that all steps are clearly followed. The null and alternative hypotheses must be clearly stated; a one-tail test should have been used here. A clear comparison between the z calculated (2.345) and 1.96 should have been shown either as an inequality statement, or clearly shown on a diagram. Alternative valid comparisons (of area) could have been made. If a valid comparison is not shown, the conclusion is not justified, and cannot score marks even if correct. Better candidates gave their final conclusion as a statement related to the question rather than just stating 'Reject H_0'.

Answer: Evidence that the proportion is higher for the new plan.

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Question 4

Probability density function questions are usually well answered by candidates. The integration required here was not always well attempted, but the general methods required to find k and a were usually understood. It is important for candidates to note that to 'show' that the value of k was 2 in part **(i)** it is necessary to show all steps of working. Marks may not be gained if there is a lack of all necessary working. Part **(iii)** was not well attempted. Whilst many candidates realised that $m<0.5$ they were unable to successfully explain why this was the case.

Answers: **(ii)** $a = 1/9$ **(iii)** $m<0.5$

Question 5

The probability **(i)(a)** was usually successfully found by candidates, but the method for finding the required conditional probability in **(i)(b)** was not always understood. Part **(ii)** required candidates to describe fully the distribution of the sample mean (N(3.2,3.2/120)). It appeared that some candidates did not realise what was required, as they failed to give this answer, but successfully used this correct distribution in part **(b)**.

Answers: **(i)(a)** 0.62(0)   **(b)** 0.359   **(ii)(a)** (Approx) Normal with mean 3.2 and variance 3.2/120   **(b)** 0.730

Question 6

In part **(i)** of this question, many candidates successfully found the unbiased estimate of the population mean. Calculation of the unbiased estimate of the population variance, however, was not as straightforward, in particular there was much confusion shown in the calculation of $\Sigma x^2 f$. The two formulae for the unbiased estimate were often confused, and some candidates calculated the biased estimate. In part **(ii)** a significance test was required, and, as in **Question 3**, it was important that all steps were carried out. Failure to show the relevant comparison either as an inequality statement or on a diagram means that the conclusion is not fully justified. A conclusion, in context, is preferable to the statement 'Accept H_0'. Part **(iii)** needed to refer to the answer found in **(ii)**. Few candidates were able to give the set of values of the test statistic.

Answer: **(i)** Mean = 1.96   Variance = 1.26(37)   **(ii)** No evidence that the mean has changed
        (iii) No because H_0 was not rejected   **(iv)** State mean has not changed when it has
        -1.96<test statistic<1.96

Question 7

This question was well attempted by the majority of candidates. The correct value of λ (5) was usually used, and a correct Poisson expression used to find the probability of more than two emails. An occasional misinterpretation of 'more than two' caused some candidates to calculate 1-P(0,1) rather than the correct expression 1-P(0,1,2). Part **(ii)** required a Normal approximation (N(120,120)), this was often correctly used, but a continuity correction was not always applied. In part **(iii)**, P(3,4,5) with $\lambda=1.75$ was required; candidates often misinterpreted this.

Answer: **(i)** 0.875   **(ii)** 0.0285   **(iii)** 0.247

MATHEMATICS

> **Paper 9709/72**
>
> **Paper 72**

Key messages

- A considerable proportion of the total marks depend on giving answers which are correct to 3 significant figures. In order to gain as many marks as possible all working should be either exact or correct to at least 4 significant figures.
- Candidates need to be careful to read the questions in detail and answer as indicated.
- Candidates need to show working to obtain the full marks available for each question.
- Candidates need to be careful to avoid algebraic and integration errors.

General comments

On this paper, candidates were able to demonstrate and apply their knowledge in the situations presented. There was a complete range of scripts from good ones to poor ones. In general, candidates scored well on **Questions 3**, **4** and **5(i)**, whilst **Question 7** proved particularly demanding.

Accuracy was not as great an issue as has been reported in the past, though it should still be noted by candidates that for a final answer to be accurate to 3 significant figures, all working out up to the final answer needs to be accurate to at least 4 significant figures.

Timing did not appear to be a problem for candidates.

Detailed comments on specific questions follow. Whilst the comments indicate particular errors and misconceptions, it should be noted that there were also some good and complete answers.

Comments on specific questions

Question 1

The correct null and alternative hypotheses were given by many candidates. It should be noted that the hypotheses should refer specifically to the 'population mean' and not just to the 'mean'.

In part **(ii)** a clear comparison of 0.0683 and 0.05 was required (either as an inequality statement or clearly marked on a diagram) in order to justify the conclusion. Conclusions that were not clearly justified in this manner where not accepted (even if correct). Some candidates did unnecessary and incorrect calculations at this stage, not realising that it was only the comparison with 0.05 that was required to reach a conclusion. Better candidates gave conclusions using the context of the question rather than just concluding 'accept H_0'.

Answers: **(i)** H_0 : Population mean=3 H_1 : Population mean>3
        (ii) No evidence that pop. mean has increased.

Question 2

This question required an understanding of the Central Limit Theorem. Part **(i)** was to identify the mean and standard deviation of the distribution of the sample mean. Many candidates correctly stated the mean, but the standard deviation was not so successfully found. Candidates should always ensure that they read the question carefully, as some gave a correct variance for the distribution, but did not state the standard deviation, as required. In part **(ii)** it was necessary to consider the distribution of the underlying parent population. This was not recognised by many candidates.

Answers: **(i)** Mean = 7, Standard Deviation = $3/\sqrt{n}$ **(ii)(a)** Population is Normal   **(b)** Large sample

Question 3

Many candidates successfully found the correct confidence interval for the proportion of candidates, and clearly wrote their answer as an interval, as required, rather than two separate values. The correct z value was usually found, and the correct level of accuracy adhered to. However, there was confusion shown by some candidates on how to calculate the confidence interval for a population proportion.

Part **(ii)** required a comment about the sample, which was not always fully realised.

Answers: **(i)** 0.202 to 0.518 **(ii)** Sample random

Question 4

This question was well attempted by the majority of candidates. The correct value of λ (7.96) was usually found, and a correct Poisson expression used to find the probability of at least three bacteria. An occasional misinterpretation of 'at least three' caused some candidates to calculate 1-P(0,1,2,3) rather than the correct expression 1-P(0,1,2). This illustrates the need for candidates to make sure they carefully interpret the requirements of the question.

Part **(ii)** was also well attempted. Candidates realised they were required to use a Normal approximation N(49.6,49.6). Not all candidates used the correct variance, and some omitted the necessary continuity correction, but the correct general method was usually understood and followed.

Answers: **(i)** 0.986 **(ii)** 0.016(0)

Question 5

Part **(i)** of this question was well attempted, with the correct parameters usually used. In part **(ii)** this was not always the case. The question required the probability F-$2J$>0; many candidates misinterpreted the wording of the question and calculated $2F$-J>0, illustrating, as in **Question 4**, the need for careful reading and interpretation of the question. The calculation of the variance also caused problems for some candidates.

Answers: **(i)** 0.942 **(ii)** 0.0268

Question 6

In part **(i)**, candidates were required to show that k=1/6. In questions where the answer is given, it is important that candidates show all necessary steps in their working to reach the given answer. In this question many candidates successfully reached the required answer. Part **(ii)** required candidates to attempt $\int xf(x)\ dx$. Whilst this was usually understood and clearly stated by candidates (though some candidates confused mean and median), algebraic and integration errors were often made. Part **(iii)** required candidates to integrate f(x) from 20 to 25, errors made here included incorrect limits (21 to 25, or 4 to 20 without the necessary 1-$\int$f(x)dx). On the whole candidates made a good attempt at this question, though part **(iv)** caused some problems. Here, there were many suggestions as to why the model may not be realistic, and valid reasons were accepted. However, answers that were too vague were not.

Answers: **(ii)** 13 **(iii)** 0.176 **(iv)** Weekly demand may be >25 (or <4)

Question 7

This question, particularly part **(ii)**, was a challenge for candidates. Part **(i)** required candidates to carry out a significance test. The hypotheses needed to be stated, and values for the unbiased estimates of the population mean and variance calculated. This was reasonably well done, though some candidates calculated the biased variance. The calculation using standardisation of a test statistic was then required, and it was important that a clear comparison of values was then shown, either as an inequality statement or clearly shown on a diagram so that the conclusion made was fully justified. Candidates who scored well on part **(i)** followed all the necessary steps.

Part **(ii) (a)** required a statement about a Type II error. This had to be in the context of the question, which meant that text book definitions were not acceptable. The statement needed to refer to the mean weight of the bags of carrots. In part **(b)** many candidates failed to realise what was required to calculate the probability of a Type II error.

Answers: **(i)** No evidence that $\mu \neq 2.0$ **(ii)** Concluding that $\mu = 2.0$ although not true

MATHEMATICS

Key messages

- Candidates need to show working to obtain the full marks available for each question.
- A considerable proportion of the total marks depend on giving answers which are correct to 3 significant figures. In order to gain as many marks as possible all working should be either exact or correct to at least 4 significant figures.
- Candidates need to be careful to read the questions in detail and answer as indicated.

General comments

This paper was well answered by the majority of candidates, who presented all their solutions clearly and to at least the accuracy required. **Question 4** involving the application of the Poisson distribution was particularly well done as were **Questions 2** and **3** involving the use of the Normal distribution. Candidates need to ensure that when an answer is given in the question for instance in **Question 7** that all working is shown. Numerical working throughout the paper was of a high standard with nearly all candidates working with at least 4 significant figures to ensure that their answers were correct to 3 significant figures. Significance testing proved to be the most challenging question on the paper.

Comments on specific questions

Question 1

(i) The correct confidence interval was calculated by most candidates, and correctly expressed as an interval. The correct value of z was used in the majority of solutions with incorrect values such as 2.326 seen rarely. Few calculations gave only one of the two key values.

(ii) The correct probability for the whole of a 99% confidence interval lying below the mean was correctly calculated by few candidates. A significant number of candidates did not make sufficient use of the symmetry of the distribution in order to identify 0.5% as the required probability.

Answers: **(i)** 12.0, 13.0 **(ii)** 0.005 or 0.5%

Question 2

This question was answered particularly well by many candidates. They correctly calculated the mean and variance of 3X-Y, standardised and used normal tables to find the required probability. Candidates needed to calculate both the mean and variance of the new variable. To correctly calculate the new variance the variance of X needed to be multiplied by 3^2 (not 3) and added to the variance of Y. The second parameter of a Normal distribution is the Variance so should not be squared in this calculation

Answer: 0.747

Question 3

(i)   A large number of candidates obtained correct values for the unbiased estimates of the mean and variance. Candidates used the correct formula in nearly all cases showing that they understood the difference between biased and unbiased estimates. Substitution into the formula for Variance was almost always correctly done.

(ii)   Nearly all candidates then standardised using these figures in order to estimate the probability that a new sample of 80 would have a mean greater than 53 mm. The need to use $\sqrt{80}$ in the standardisation was recognised by most candidates. Some errors were made interpreting greater than 53 as requiring a continuity correction, but the majority of candidates were able to use their correct standardisation to find the required probability within the correct range of allowable answers.

Answers: **(i)** Mean 50.1, Variance 245 (both 3sf) **(ii)** An answer between 0.0488 to 0.0509

Question 4

(i)   Most candidates identified that the required probability was found by $1 - (P(0) + P(1))$, and correctly worked with the Poisson mean for a standard safari of 0.8. Nearly all candidates who used the correct method obtained the answer accurate to at least 3 significant figures.

(ii)   As in the first part of the question a large number of candidates obtained the correct answer. They recognised that the new mean was found by $3 \times 0.8 + 2 \times 2.7$, and then with this Poisson mean identified that the required probability was the sum of the probabilities from 0 to 4. A minority of candidates incorrectly included $P(5)$ and nearly all used the correct distribution, which was Poisson.

(iii)   Many correct solutions were seen to the final part of this question. Candidates usually worked with an inequality of $e^{-0.8n}$ and 0.1 and correctly solved the inequality including reversing the inequality when dividing by a negative number. Most then moved to the correct whole number value of 3. A number used a correct trial and improvement method to obtain this answer. A number of candidates were less successful, setting up an incorrect equation/inequality with either 0.9 or 0.01. There were only a few incorrect uses of methods connected to the Normal distribution.

Answers: **(i)** 0.191 **(ii)** 0.112 **(iii)** $n=3$

Question 5

(i)   Many candidates correctly identified that they need to work with the number of heads an unbiased coin would show in 12 throws and use this to calculate the probability of 10, 11 or 12 heads. Because a significance level is required they then need to convert this number to a percentage. This question as a whole proved the most challenging on the paper and a number of incorrect approaches were seen. Candidates need to use the value of p which is consistent with the coin being unbiased. The required region is for 9 or more heads so must be less than 50%. The Binomial distribution is appropriate for the values of the parameters, $n=12$ $p=0.5$

(ii)   In the second part of the question a far larger number of trials are involved. To answer this question there was a need to identify that the Normal approximation to the Binomial was essential and that standardisation with an unknown value for the region was needed with the equation of this standardisation to 1.645. Candidates needed to apply a continuity correction, and the region must start with an integer as the number of heads is a discrete variable. Many candidates identified some of these requirements. Errors were made with the variance often dividing by 100. The continuity correction was not always included, or the incorrect one used. A significant number of otherwise correct solutions indicated that the final region was greater than or equal to 58.7 rather than 59. Despite the high demand of this question a good number of fully correct solutions were seen.

Answers: **(i)** 1.93% or 1.9% (2% allowed with correct working) **(ii)** Region is ≥ 59

Question 6

(i) The fact that Samir was testing to see if his journey time had increased was identified by most candidates who then correctly concluded that the test was one-tailed. An answer that $H_{1:\ \mu} > 45.7$ was also accepted.

(ii) Candidates were asked to use the given data to show that there was no evidence of an increase in journey times. As this is a hypothesis test it is expected that a full solution is shown. This requires the statement of a hypothesis, the calculation of the sample mean; the calculation using standardisation of a test statistic, and a clear comparison of a probability with 0.05 or of the calculated z value with 1.645. Finally a clear conclusion should be drawn. The best answers showed all of these requirements clearly. It remains the case that some candidates do not perform the test in an acceptable way. The test can be performed by using the two key values and an inequality, or by showing the two values on a Normal Curve. Stating a critical value without performing the comparison is not sufficient. It is also crucial that the Hypotheses are stated.

(iii) A number of candidates correctly identified that as in part **(ii)** H_0 had been accepted then a Type II error was possible. The response that as the sample mean was more than 45.7 then the null hypothesis was false was not accepted as justification that a Type II error must have been made.

Answers: **(i)** One-tailed as testing for an increase **(ii)** Accept H_0 based on a value of z in the range 1.481 to 1.503 **(iii)** Type II as H_0 accepted

Question 7

(i) This question on Continuous Random Variables was well answered by many candidates. The need to integrate the function having equated to 1 to obtain the given value of k, was correctly performed by the majority of candidates. Because the answer was given, there was a minimum requirement that the evaluation of the integral $(-\cos x)$ was shown for upper and lower limits.

(ii) The majority of candidates then went on to correctly show that the median was 1.32 to 3 significant figures. The majority of candidates performed all the necessary stages, which are equating the integrand to 0.5, using one of the boundaries of the range with the other limit as an unknown. This leads to an equation in $\cos m$, which is usually expressed as $\cos m = 1/4$. Candidates who identified the correct method almost always completed it.

(iii) The calculation of $E(X)$ required the integration of $x \sin x$. There were some excellent solutions involving the technique of integration by parts. Those who identified the correct method usually carried it out with no errors. A significant number of candidates moved from $x \sin x$ to $\sin x^2$ and could not therefore obtain the correct value of $E(X)$

Answers: **(i)** $k = 2/3$ (AG) **(ii)** Median $= 1.32$ (3sf) (AG) **(iii)** 1.28 or equivalent surd form.