

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

## MATHEMATICS

9709/72
Paper 7 Probability \& Statistics 2 (S2)
May/June 2011
1 hour 15 minutes

Additional Materials: | Answer Booklet/Paper |
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| Graph Paper |
| List of Formulae (MF9) |

## READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.
Answer all the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.
At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50 .
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

1 The weights of bags of fuel have mean 3.2 kg and standard deviation 0.04 kg . The total weight of a random sample of three bags is denoted by $T \mathrm{~kg}$. Find the mean and standard deviation of $T$.
$2 X$ is a random variable having the distribution $\mathrm{B}\left(12, \frac{1}{4}\right)$. A random sample of 60 values of $X$ is taken. Find the probability that the sample mean is less than 2.8.

3 The number of goals scored per match by Everly Rovers is represented by the random variable $X$ which has mean 1.8.
(i) State two conditions for $X$ to be modelled by a Poisson distribution.

Assume now that $X \sim \operatorname{Po}(1.8)$.
(ii) Find $\mathrm{P}(2<X<6)$.
(iii) The manager promises the team a bonus if they score at least 1 goal in each of the next 10 matches. Find the probability that they win the bonus.

4 A doctor wishes to investigate the mean fat content in low-fat burgers. He takes a random sample of 15 burgers and sends them to a laboratory where the mass, in grams, of fat in each burger is determined. The results are as follows.

$$
\begin{array}{lllllllllllllll}
9 & 7 & 8 & 9 & 6 & 11 & 7 & 9 & 8 & 9 & 8 & 10 & 7 & 9 & 9
\end{array}
$$

Assume that the mass, in grams, of fat in low-fat burgers is normally distributed with mean $\mu$ and that the population standard deviation is 1.3.
(i) Calculate a $99 \%$ confidence interval for $\mu$.
(ii) Explain whether it was necessary to use the Central Limit theorem in the calculation in part (i).
(iii) The manufacturer claims that the mean mass of fat in burgers of this type is 8 g . Use your answer to part (i) to comment on this claim.

5 The number of adult customers arriving in a shop during a 5-minute period is modelled by a random variable with distribution $\operatorname{Po}(6)$. The number of child customers arriving in the same shop during a 10 -minute period is modelled by an independent random variable with distribution $\mathrm{Po}(4.5)$.
(i) Find the probability that during a randomly chosen 2 -minute period, the total number of adult and child customers who arrive in the shop is less than 3.
(ii) During a sale, the manager claims that more adult customers are arriving than usual. In a randomly selected 30 -minute period during the sale, 49 adult customers arrive. Test the manager's claim at the $2.5 \%$ significance level.

6 Jeevan thinks that a six-sided die is biased in favour of six. In order to test this, Jeevan throws the die 10 times. If the die shows a six on at least 4 throws out of 10 , she will conclude that she is correct.
(i) State appropriate null and alternative hypotheses.
(ii) Calculate the probability of a Type I error.
(iii) Explain what is meant by a Type II error in this situation.
(iv) If the die is actually biased so that the probability of throwing a six is $\frac{1}{2}$, calculate the probability of a Type II error.

7 A random variable $X$ has probability density function given by

$$
\mathrm{f}(x)= \begin{cases}k(1-x) & -1 \leqslant x \leqslant 1 \\ 0 & \text { otherwise }\end{cases}
$$

where $k$ is a constant.
(i) Show that $k=\frac{1}{2}$.
(ii) Find $\mathrm{P}\left(X>\frac{1}{2}\right)$.
(iii) Find the mean of $X$.
(iv) Find $a$ such that $\mathrm{P}(X<a)=\frac{1}{4}$.

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