MARK SCHEME for the October/November 2010 question paper

for the guidance of teachers

9709 MATHEMATICS

9709/42   Paper 4, maximum raw mark 50

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

Mark Scheme Notes

Marks are of the following three types:

M   Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A   Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B   Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
              B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF   Any Equivalent Form (of answer is equally acceptable)

AG   Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD   Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO   Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO   Correct Working Only – often written by a 'fortuitous' answer

ISW   Ignore Subsequent Working

MR   Misread

PA   Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS   See Other Solution (the candidate makes a better attempt at the same question)

SR   Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1   A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through $\sqrt{}$" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1   This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1		M1		For resolving forces vertically (3 terms required)
	$R + 2000\cos15^o = 400\,g$	A1		
	$F = 2000\sin15^o$	B1		
	$[2000\sin15^o = \mu(400\,g - 2000\cos15^o)]$	M1		For using $F = \mu R$
	Coefficient is 0.25	A1	**[5]**	

	SR(max. 4/5) for candidates who either: have sin and cos interchanged or have angle 15° above the horizontal			
		M1		For resolving forces vertically
	$R + 2000\sin15^o = 400\,g$ <u>and</u> $F = 2000\cos15^o$	A1		
	$[2000\cos15^o = \mu(400\,g - 2000\sin15^o)]$	M1		For using $F = \mu R$
	Coefficient is 0.55	A1		

2	Driving force = 400/4	B1		
		M1		For using Newton's second law (either case) – 3 terms needed
	$DF - 80\,g\sin2° = 80a$ **(i)** or $DF + 80\,g\sin2° = 80a$ **(ii)**	A1		
	Acceleration is 0.9 ms^{-2} **(i)** or Acceleration is 1.6 ms^{-2} **(ii)**	A1		Accept 0.90 or 0.901 and 1.60
	Acceleration is 1.6 ms^{-2} **(ii)** and Acceleration is 0.9 ms^{-2} **(i)**	B1ft	**[5]**	ft Ans **(i)** + **(ii)** = 2.5

	SR(max. 3/5) for candidates who have sin and cos interchanged			
	Driving force = 400/4	B1		
		M1		For using Newton's second law (either case) – 3 terms needed
	$a = -8.74$ **(i)** <u>and</u> $a = 11.2$ **(ii)**	A1		

3		M1		For resolving forces in **i** and **j** directions (3 terms in at least one of the equations)
	$6\cos\alpha^o + 5\cos(90^o - \alpha^o) = F$ and $6\sin\alpha^o - 5\sin(90^o - \alpha^o) = F$	A1		
	$[6\cos\alpha^o + 5\sin\alpha^o = 6\sin\alpha^o - 5\cos\alpha^o$ $\rightarrow 11\cos\alpha^o = \sin\alpha^o]$	DM1		For attempting to solve for α^o. Dependent on 1st M1
	$\alpha = 84.8$	A1		
	$[F = 6\cos84.8^o + 5\sin84.8^o$; $F = 6\sin84.8^o - 5\cos84.8^o]$	DM1		For substituting to find F; dependent on the 1st M1
	$F = 5.52$	A1	**[6]**	

First alternative scheme

$[2F^2 = 25 + 36]$	M1		For using '(resultant of forces of magnitude F)^2 = (resultant of forces of magnitudes 5 and 6)^2'
$F = 5.52$	A1		
	M1		For using 'resultant of forces of magnitudes 5 and 6 makes angle 45° with x-axis'
	M1		For using relevant trigonometry
$\tan(\alpha^o - 45^o) = 5/6$ or $\tan(135^o - \alpha^o) = 6/5$ or $\cos(\alpha^o - 45^o)$ or $\sin(135^o - \alpha^o) = 6/\sqrt{61}$ or $\sin(\alpha^o - 45^o)$ or $\cos(135^o - \alpha^o) = 5/\sqrt{61}$	A1		
$\alpha = 84.8$	A1		

Second alternative scheme

$[6\cos\alpha^o + 5\cos(90^o - \alpha^o)$ $= 6\sin\alpha^o - 5\sin(90^o - \alpha^o)]$	M1		For using Rx = Ry
$[11\cos\alpha^o - \sin\alpha^o = 0]$	M1		For attempting to solve for α^o
$\alpha = 84.8$	A1		
For $F = 6\cos\alpha^o + 5\cos(90^o - \alpha^o)$ or $F = 6\sin\alpha^o - 5\sin(90^o - \alpha^o)$	B1		
	M1		For substituting for α
$F = 5.52$	A1		

4	**(i)** $[\frac{1}{2}\ 20(2.5^2 - 1.5^2),\ 20\times10\times10\sin 4.5^o]$	M1		For using KE loss = $\frac{1}{2}\ m(u^2 - v^2)$ or PE gain = mg($L\sin\alpha$)
	KE loss = 40 J or PE gain = 157 J	A1		
	PE gain = 157 J or KE loss = 40 J	B1	**[3]**	

(ii)	[WD = 157 – 40 + 50]	M1		For using WD by pulling force = PE gain – KE loss + WD against resistance
	Work done is 167 J	A1ft	**[2]**	ft incorrect PE gain + 10, even if –ve

(iii)	$[167 = F\times10\cos15^o]$	M1		For using WD = $FL\cos 15^o$
	Magnitude is 17.3 N	A1ft	**[2]**	

SR (max. 1/2) for candidates who (implicitly) make the unjustifiable assumption that acceleration is constant and apply Newton's second law

For magnitude is 17.3 N from $F\cos 15^o - 20g\sin4.5^o - 50/10 = 20 \times (-0.2)$	B1	

5	(i)	$[15 = 20t - 5t^2 \rightarrow 5(t^2 - 4t + 3) = 0]$	M1		For use of $h = ut - \frac{1}{2}gt^2$
		$t = 1, 3$	A1		
		Duration is 2 s (accept $1 < t < 3$)	B1ft	[3]	ft $t_2 - t_1$
	(ii)		M1		For using $h_P = h_Q$ at time t after P's (or Q's) projection
		$20t - 5t^2 = 25(t - 0.4) - 5(t - 0.4)^2$ (or $20(t + 0.4) - 5(t + 4)^2 = 25t - 5t^2$ or $(20 \times 0.4 - 5 \times 0.4^2) + 16t - 5t^2 = 25t - 5t^2$)	A1		
		$t = 1.2$ (or $t = 0.8$)	A1		
		$[v_P = 20 - 10 \times 1.2; v_Q = 25 - 10 \times (1.2 - 0.4)$ (or $v_P = 20 - 10 \times (0.8 + 0.4); v_Q = 25 - 10 \times 0.8]$	M1		For using $v = u - gt$ for both v_P and v_Q
		Velocities are $8\,\text{ms}^{-1}$ and $17\,\text{ms}^{-1}$	A1	[5]	
6	(i)	$[\frac{1}{2} 2.5(\text{speed}_{max}) = 4]$	M1		For using area property for distance
		Greatest speed is $3.2\,\text{ms}^{-1}$	A1	[2]	
		SR (max. 1/2) for candidates who (implicitly) make the unjustifiable assumption that speed_{max} occurs when $t = 1.25$			
		Greatest speed is $3.2\,\text{ms}^{-1}$ from $2 \times \frac{1}{2} 1.25(\text{speed}_{max})v = 4$	B1		
	(ii)	$[V = 3 \times 2]$	M1		For using $a = (V - 0)/(4.5 - 2.5)$ or $V = 0 + at$
		$V = 6$	A1	[2]	
	(iii)		M1		For using area property for distance from $t = 2.5$ to $t = 14.5$
		$\frac{1}{2} 6(12 + T) = 48$ or $\frac{1}{2} 6 \times 2 + 6T + \frac{1}{2} 6(10 - T) = 48$ or $\frac{1}{2} 6 \times 2 + 6(10 - \tau) + \frac{1}{2} 6\tau = 48$	A1ft		
		$t = 8.5$	A1	[3]	from $4.5 + T$ or $14.5 - \tau$
	(iv)		M1		For using $a = (0 - V)/(14.5 - 8.5)$ or $0 = V + a(14.5 - 8.5)$
		Deceleration is $1\,\text{ms}^{-2}$	A1ft	[2]	

7	(i)	$a(t) = 0.006t^2 – 0.24t + 1.8$	B1	
		$[0.006(t^2 – 40t + 300) = 0]$	M1	For solving $a(t) = 0$
		$T_1 = 10$, $T_2 = 30$	A1	
			M1	For integrating $v(t)$
		$s(t) = 0.0005t^4 – 0.04t^3 + 0.9t^2 + 5t + (C)$	A1	
		$[405 – 1080 + 810 + 150]$	M1	For using limits 0 to T_2 or equivalent
		Distance is 285 m	A1 [7]	
	(ii)	Velocity is 5 ms^{-1}	B1	
		For curve with v increasing from a +ve value at $t = 0$ to a maximum	B1	
		Then decreases to a +ve minimum and thereafter increases	B1 [3]	